1. **Comparing Wold representations.** Consider the following data:

```
library(astsa); y<-gas
```

   (a) Show that both AR(4) and MA(3) are good models for the differences of our data.

   (b) How is it possible that different models can fit the same data? In fact they are not "so different". Compute the Wold representations of these two models (up to some lag) and compare them. In R, you can use `ARMAtoMA` function.

   Graphical comparison:

```
k <- as.factor(rep(1:10, 2))
psi <- NA
model <- rep(c("ar4","ma3"), each=10)
wold <- data.frame(k, psi, model)
```

   After computing the values of `psi`:

```
library(ggplot2)
ggplot(wold, aes(x=k, y=psi, fill=model)) + geom_bar(stat="identity",
position="dodge")
```

   (c) Derive the Wold representations by hand, without using `ARMAtoMA` function.

2. **Observing AR(1) process with an error.** We simulate the AR process y:

```
set.seed(12345) # for reproducibility
y <- arima.sim(n=200, list(ar = c(0.9)), sd=0.1) # theoretical
```

   which we will observe with an error:

```
eps <- rnorm(200)/15 # observed
x <- y+eps
```

   (a) Plot the theoretical and observed process into one graph.

   (b) Show that AR(1) is a good model for the data y (as it is supposed to be)

   (c) Show that, however, AR(1) is not a good model for the data x.

   (d) Show that ARMA(1,1) is a good model for the data x.

   (e) Show that the previous property holds in general (i.e., if x is an AR process plus an error of this form, then it can be written as an ARMA(1,1) process).

3. Identification of ARMA processes: Shumway & Stoffer, exercise 3.4

4. Problems 1, 2, 3, 4 from the Midterm 1 at UC Berkeley.
   `http://www.stat.berkeley.edu/ bartlett/courses/153-fall2010/`

5. *From previous term exams.* Derive (i.e., not just substitute into the formula from the lecture) the explicit form of the autocorrelation function of the ARMA process

$$x_t = \frac{1}{2}x_{t-1} + u_t - \frac{1}{3}u_{t-1}.$$

6. *From previous term exams.* Find a process which has the following property. In each case, prove that the required property is indeed satisfied:

   - AR process with expected value 20
   - random process whose Wold representation has only finitely many nonzero terms
   - AR process which has all values of the ACF greater than zero
   - ARMA(1,1) process which is stationary but not invertible
   - a process, whose differences are stationary AR(1) process
   - AR(2) process which has a negative PACF of order 1
   - a process which has the PACF of order 3 equal to zero

   At each exam, there were 5 problems of this kind, together for 10 points (the same this term).