Autoregressive processes II.

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Stationarity of a given AR(p) process

Lecture: AR(1) and AR(2) processes

For the process

$$x_t = \delta + \alpha x_{t-1} + u_t$$

we have shown that stationarity is equivalent to $|\alpha| < 1$.

Writing the process in terms of the lag operator L:

$$(1 - \alpha L)x_t = \delta + u_t$$

Stationarity condition: the root of

$$1 - \alpha L = 0$$

are in absolute value greater than 1.

Lecture: AR(1) and AR(2) processes

The process

$$x_t = \delta + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + u_t$$

can be written as

$$(1 - \alpha_1 L - \alpha_2 L^2) x_t = \delta + u_t$$

Stationarity condition: the roots of

$$1 - \alpha_1 L - \alpha_2 L^2 = 0$$

are in absolute value greater than 1, i.e., they are **outside of the unit circle**.

General AR(p) process

The process

$$x_t = \delta + \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + u_t$$

can be written as

$$(1 - \alpha_1 L - \cdots - \alpha_p L^p) x_t = \delta + u_t$$

Stationarity condition: the roots of

$$1 - \alpha_1 L - \ldots \alpha_p L^p = 0$$

are in absolute value greater than 1, i.e., they are **outside of the unit circle**.

Stationarity in R

We will use the libary `fArma` and the function `armaRoots`

```
library(fArma)
```

Roots of

$$1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p = 0$$

can be found by:

```
armaRoots(c(alfa_1,alfa_2, ..., alfa_p))
```

Stationarity in R

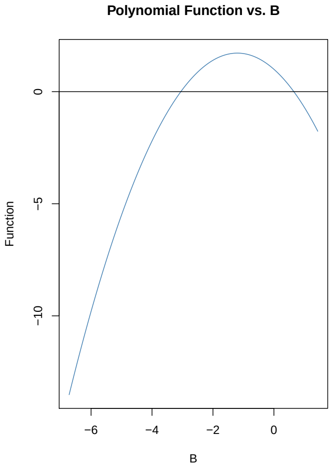
For example:

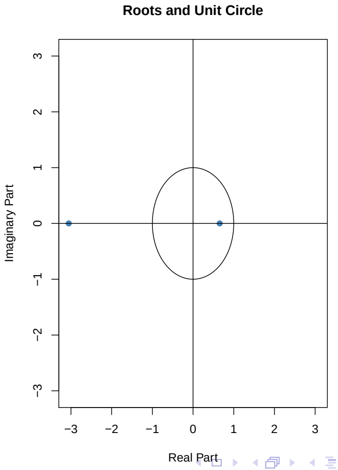
```
# in we want both graphs in the same figure
# (otherwise they will be in separate figures):
par(mfrow = c(1, 2))

# roots of 1 - 1.2 L - 0.5 L^2 =0
armaRoots(c(1.2,0.5))

# restore original settings
par(mfrow = c(1, 1))
```

The output is graphical as well as given in a table (real part, imaginary part, absolute value)

Stationarity in R

Graphically:





Stationarity in R

One of the roots is not outside of the unit circle, therefore the process

$$(1 - 1.2L - 0.5L^2)x_t = \delta + u_t$$

is not stationary.

Stationarity in R

Exercises:

- Check the stationarity of the AR(2) model for the `spread` from last week lesson
- Find another example of a stationry AR(2) process
- Find an example of a nonstationary AR(2) process.
- Part of *fundamentals* on exam: find out whether the given process is stationary

Autocorrelation function of an AR process

Solving the difference equation

We will solve the difference equations for ACF of the two AR(2) processes from the lecture.

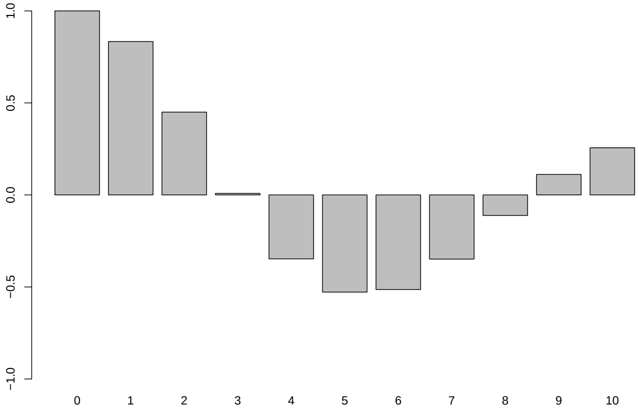
Function ARMAacf

For the process $x_t = 1.5x_{t-1} - 0.8x_{t-2} + u_2$:

```
r <- ARMAacf(ar=c(1.5, -0.8), lag.max=10)
```

Plot the ACF (for example using `barplot`)

Result



Exercise: Modelling long-term unemployment

Data

```
library(WDI)
WDIsearch('long.*term.*unemployment')
```

```
##       indicator
## [1,] "SL.UEM.LTRM.FE.ZS"
## [2,] "SL.UEM.LTRM.MA.ZS"
## [3,] "SL.UEM.LTRM.ZS"
##       name
## [1,] "Long-term unemployment, female (% of female unempl
## [2,] "Long-term unemployment, male (% of male unemployme
## [3,] "Long-term unemployment (% of total unemployment)"
```

Data

```
data <- WDI(indicator='SL.UEM.LTRM.ZS',
          country=c('CA'),
          start=1982)

data <- data[order(data$year),] # 20 years
y <- data$SL.UEM.LTRM.ZS
y <- ts(y, start=1982, frequency=1)
```

Exericise

Find a suitable AR model for y. Check its stationary and residuals. Does it have a periodical character (as suggested by the plot)?