Modelling trend: Part 1

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Exponential smoothing

Exponential smoothing

- We have the data $x_1, \ldots x_n$ and we want to (1) smooth them and (2) make predictions
- Firstly we assume that there is **no trend and no seasonality**
- Model:

$$x_t = \mu_t + w_t$$

where μ is a smoothed value and w are independent variables with zero expected value

- We denote by a_t our estimate of the expected value μ_t

Exponential smoothing

- Basic idea: **weighed average of previous estimate and realized value**:

$$a_{t+1} = \alpha a_t + (1 - \alpha) x_t$$

  where α is between 0 and 1
- We can write

$$a_{t+1} = \alpha x_t + \alpha(1 - \alpha) x_{t-1} + \alpha(1 - \alpha)^2 x_{t-2} + \dots$$

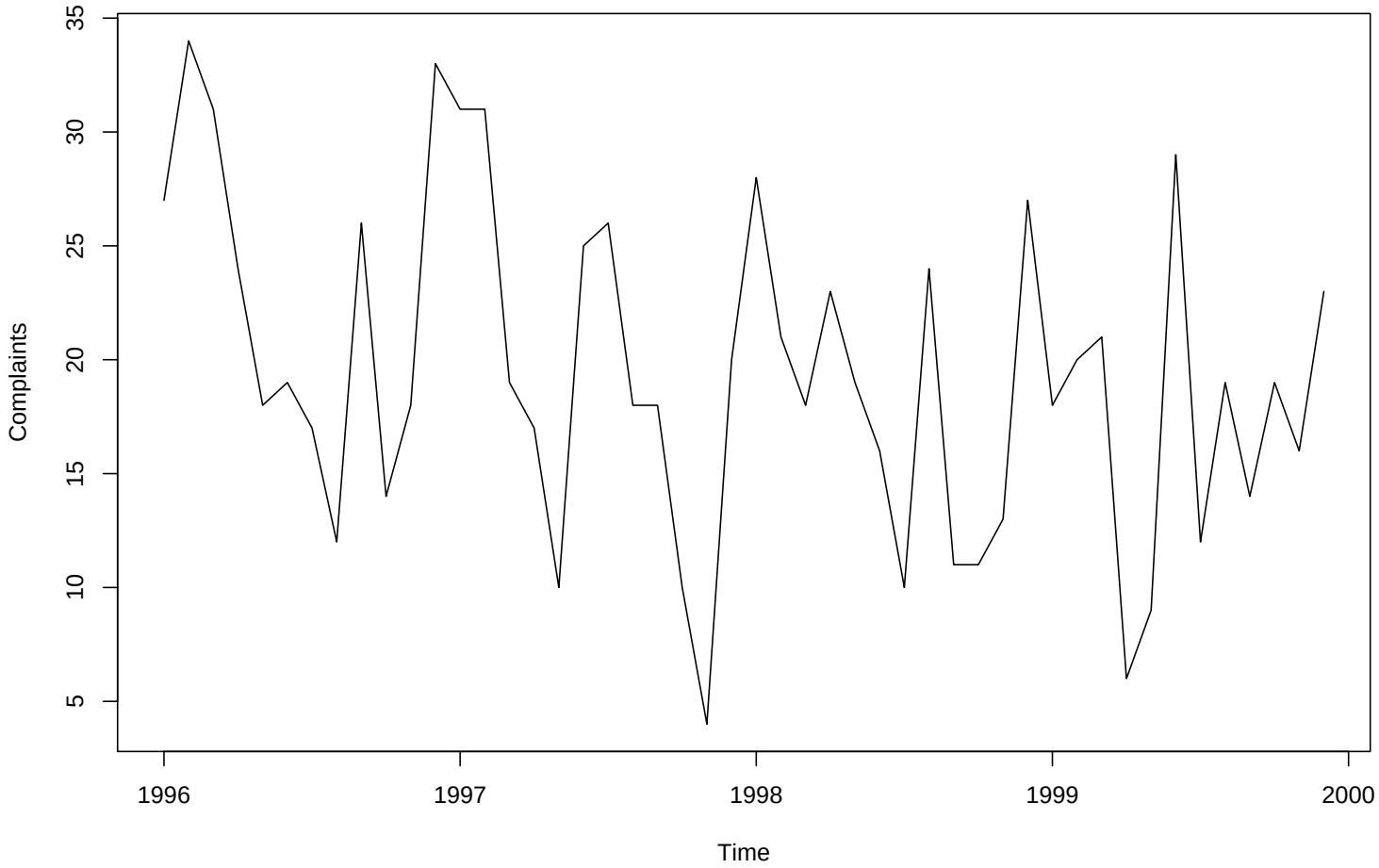
  therefore the name *exponential smoothing*
- α chosen to minimize the sum of squred errors.
- Predictions: $\hat{x}_{n+k|n} = a_n$

Example: Number of complaints

P. S. P.Cowpertwait, A. V. Metcalfe: Introductory Time Series with R. Springer, 2009. ***Complaints to a motoring organization, pp. 56-58***

```
x <- read.table("motor.txt", col.names=c("Complaints"))
x <- ts(x, frequency=12, start=c(1996,1))
plot(x)
```

Example: Number of complaints



Example: Number of complaints

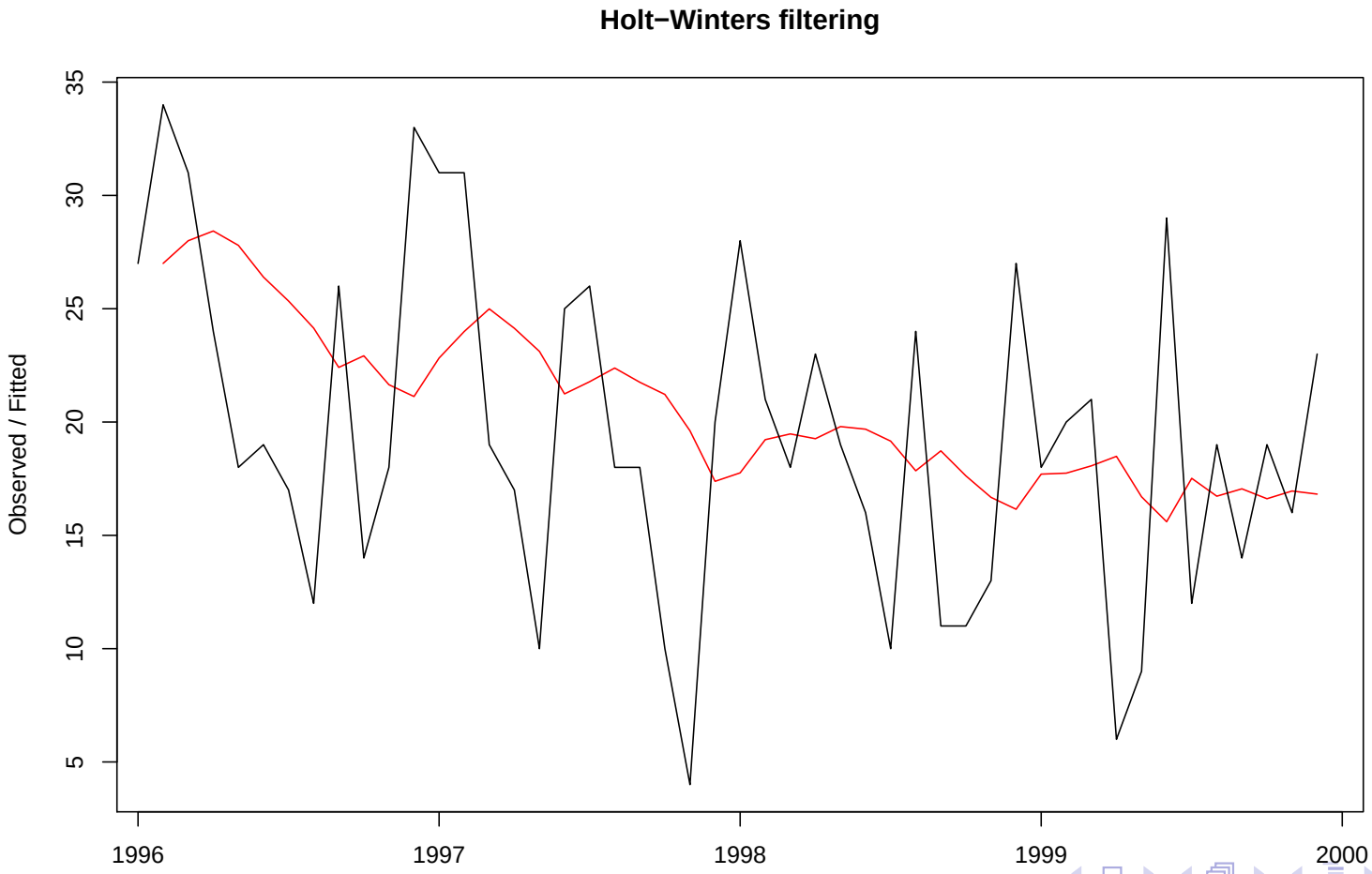
Function `HoltWinters` with `beta=FALSE, gamma=FALSE` (we estimate only `alpha`)

```
model1 <- HoltWinters(x, beta=FALSE, gamma=FALSE)
model1
```

```
## Holt-Winters exponential smoothing without trend and wit
##
## Call:
## HoltWinters(x = x, beta = FALSE, gamma = FALSE)
##
## Smoothing parameters:
##  alpha: 0.1429622
##  beta : FALSE
##  gamma: FALSE
##
```

Example: Number of complaints

```
plot(model1)
```



Example: Number of complaints

How to obtain `SSE` (i.e. *sum of squared errors*)

```
model1$SSE
```

```
## [1] 2502.028
```

Plot the dependence of `SSE` on parameter `alpha` and compare with the estimated optimal value of this parameter.

Holt-Winters filtering

Holt-Winters filtering

We characterise the time series with:

- a_t = **level** = seasonaly adjusted expected value
- b_t = **slope** = change in the level from one period to the next one, models different (also short term) trends
- c_t = **seasonal part**

Holt-Winters filtering

Predictions:

- for additive seasonality:

$$\hat{x}_{n+k|n} = a_n + kb_n + s_{n+k-p}$$

  for $k \leq p$ (for example $p = 12$ for monthly data)

- for multiplicative seasonality:

$$\hat{x}_{n+k|n} = (a_n + kb_n)(1 + s_{n+k-p})$$

Holt-Winters filtering

Model for additive seasonality:

- Level: $a_t = \alpha(x_t - s_{t-p}) + (1-\alpha)(a_{t-1} + b_{t-1})$
- Slope: $b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1}$
- Seasonality: $c_t = \gamma(x_t - a_t) + (1-\gamma)s_{t-p}$

where α, β, γ are between 0 and 1.

In R: function `HoltWinters` with estimation of all of its parameters

Example: Wine in Australia

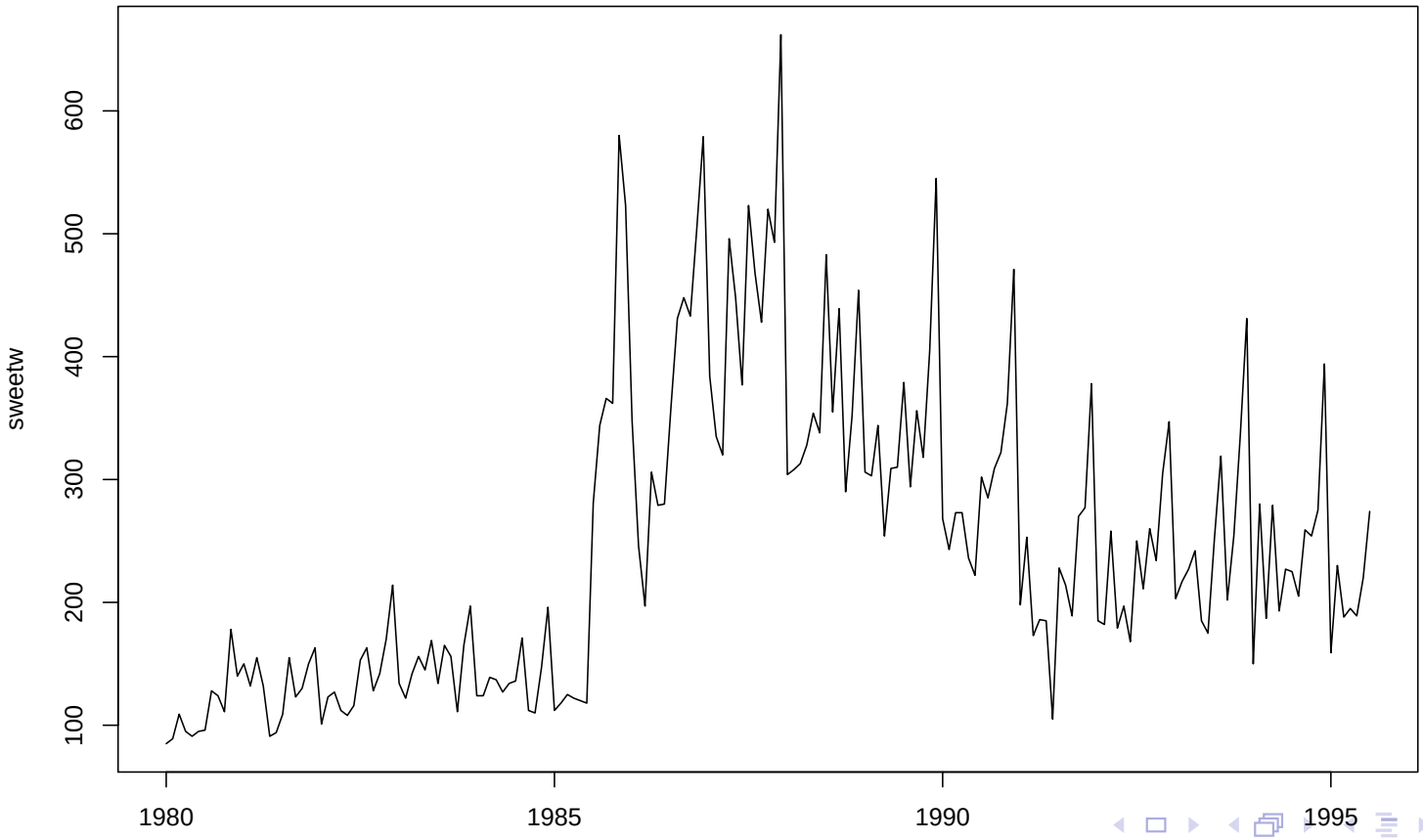
P. S. P.Cowpertwait, A. V. Metcalfe: Introductory Time Series with R. Springer, 2009. ***Sales of Australian wine, pp. 60-62***

```
data <- read.table("wine.txt", header=TRUE)
attach(data)
head(data)
```

```
##   winet fortw dryw sweetw  red rose spark
## 1     1  2585 1954     85  464  112  1686
## 2     2  3368 2302     89  675  118  1591
## 3     3  3210 3054    109  703  129  2304
## 4     4  3111 2414     95  887   99  1712
## 5     5  3756 2226     91 1139  116  1471
## 6     6  4216 2725     95 1077  168  1377
```

Example: Wine in Australia

We will model `sweetw<-ts(sweetw, frequency=12, start=c(1980,1))`



Example: Wine in Australia

Holt-Winters method:

```
HWsweet <- HoltWinters(sweetw, seasonal="mult")
plot(HWsweet)
```

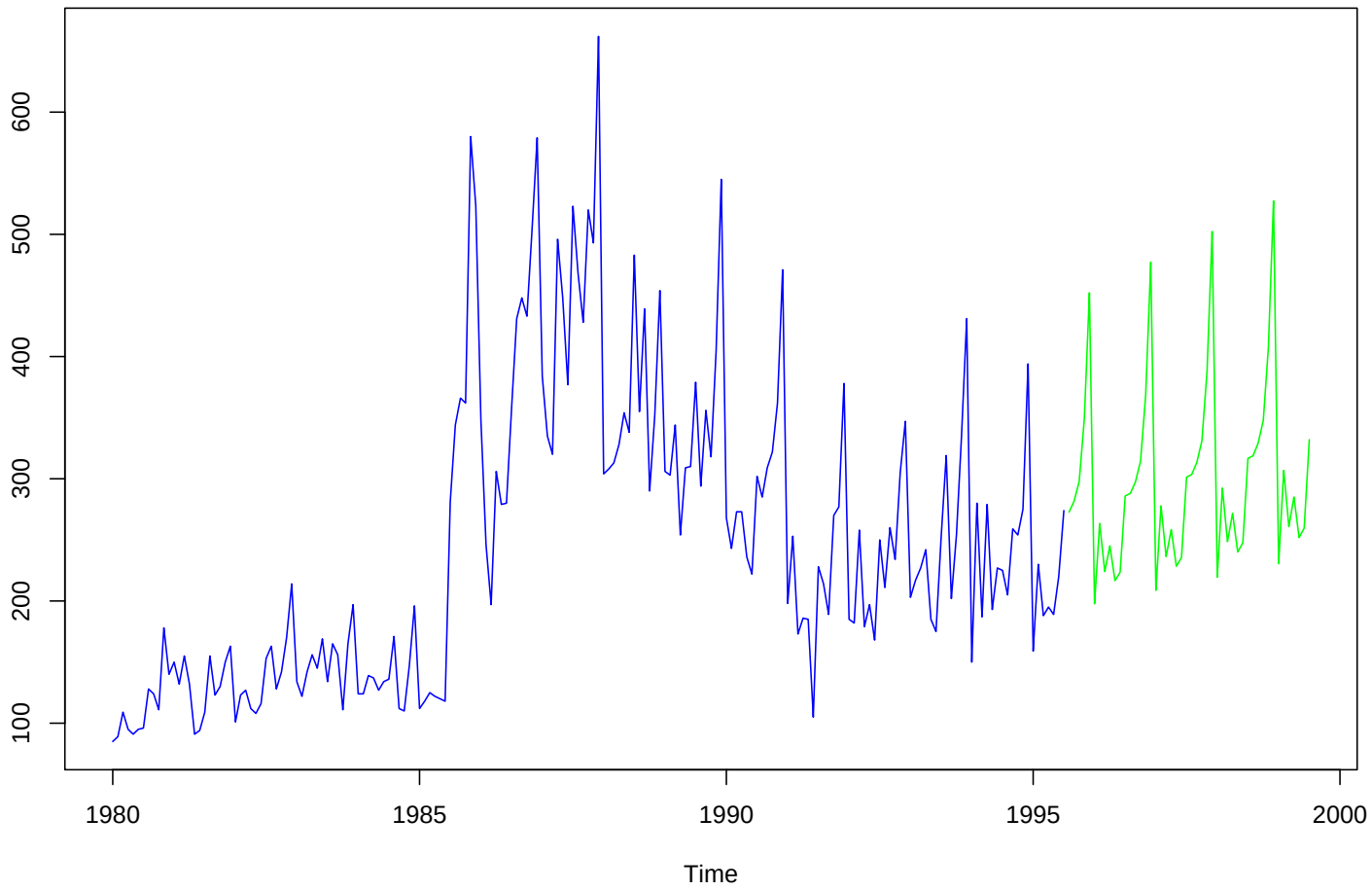
Predictions:

```
sweetP <- predict(HWsweet, n.ahead=48)
```

`ts.plot` - a useful function when making a plot of several time series

```
ts.plot(sweetw, sweetP,                          # time series
        gpars=list(col=c("blue","green")))       # parameters
```

Example: Wine in Australia



Exercise with multiplicative seasonality

Number of airline passengeres.

```
data(AirPassengers)
y <- AirPassengers
```

Modell `y` using Holt-Winters method and make predictions

Exercise with multiplicative seasonality

The data: