Modelling seasonality

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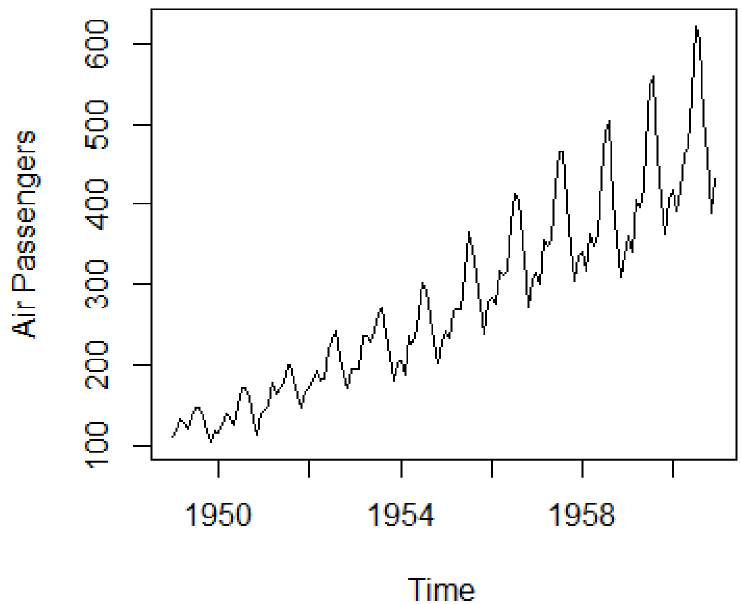
Time series analysis

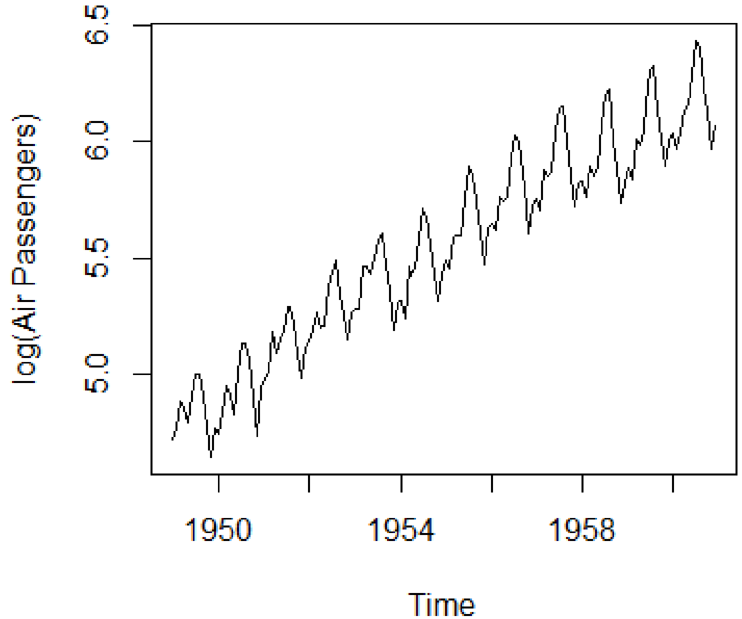
Modelling seasonality

- We have seen models with seasonal character - for example AR(2) model with complex roots
- These models are, however, not sufficient to model all seasonal data
- Also, we might want to include seasonality (quarterly data, monthly data) into specification
- There are models which are specifically for modelling seasonal data - SARIMA models (seasonal ARIMA models)

Example - data

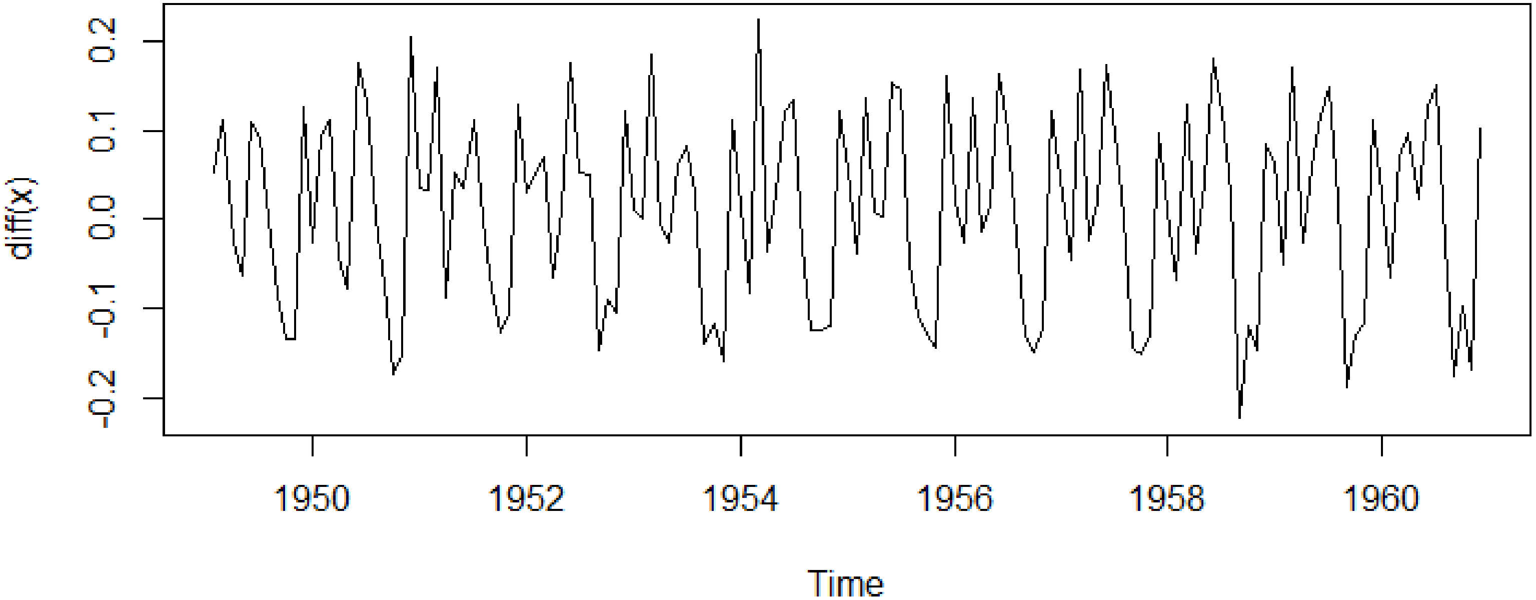
- Number of airline passengers by Box and Jenkins - founders of ARIMA modelling
- Monthly data, January 1949 - December 1960
- We work with logarithms, they stabilize variance
- In R: data(AirPassengeres); x <- log(AirPassengeres)





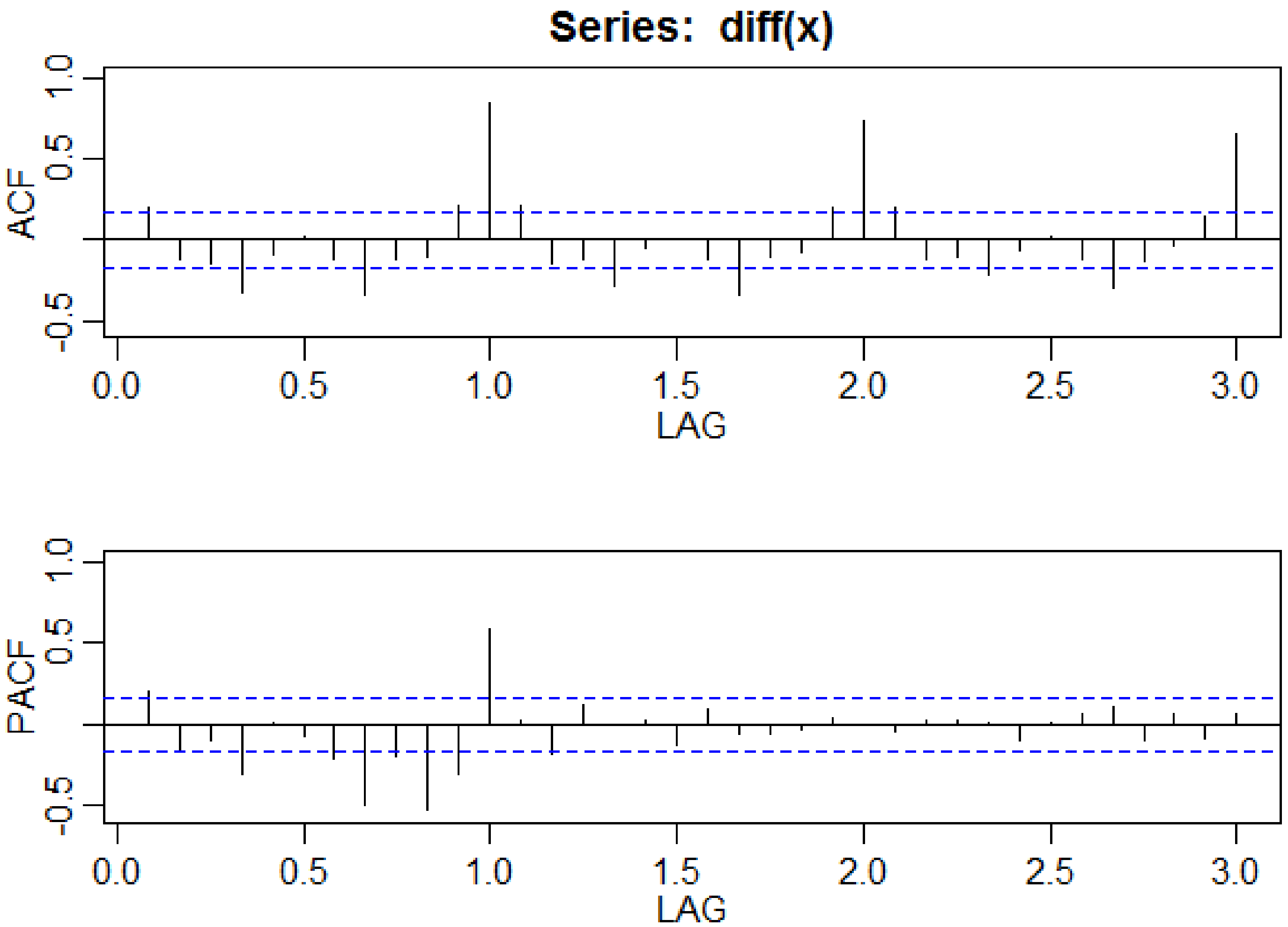
Example - differences

- We take differences - in R: diff(x) - they also have seasonality:



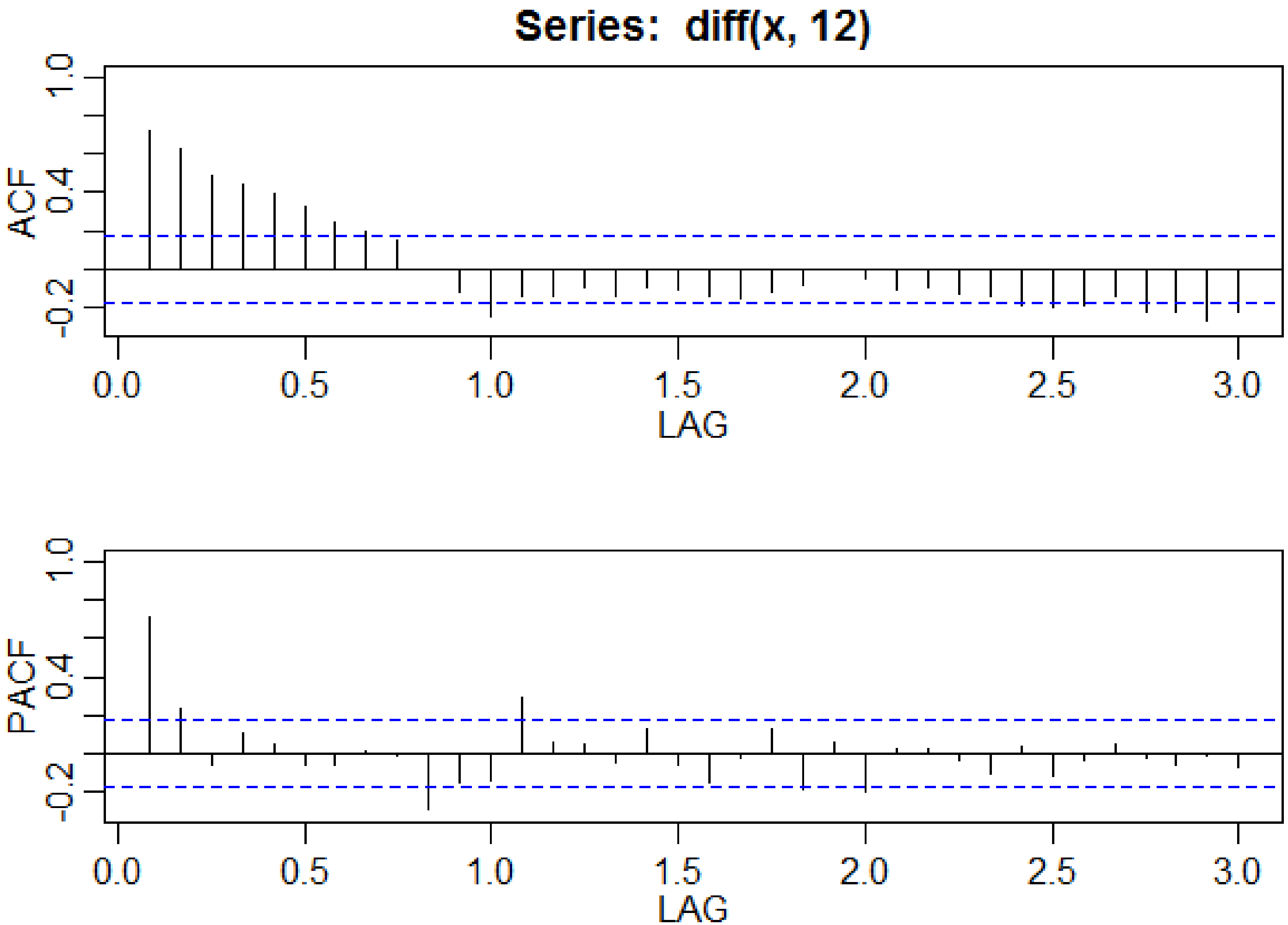
Example - differences

- ACF and PACF for these differences - in R
  acf2(diff(x)):



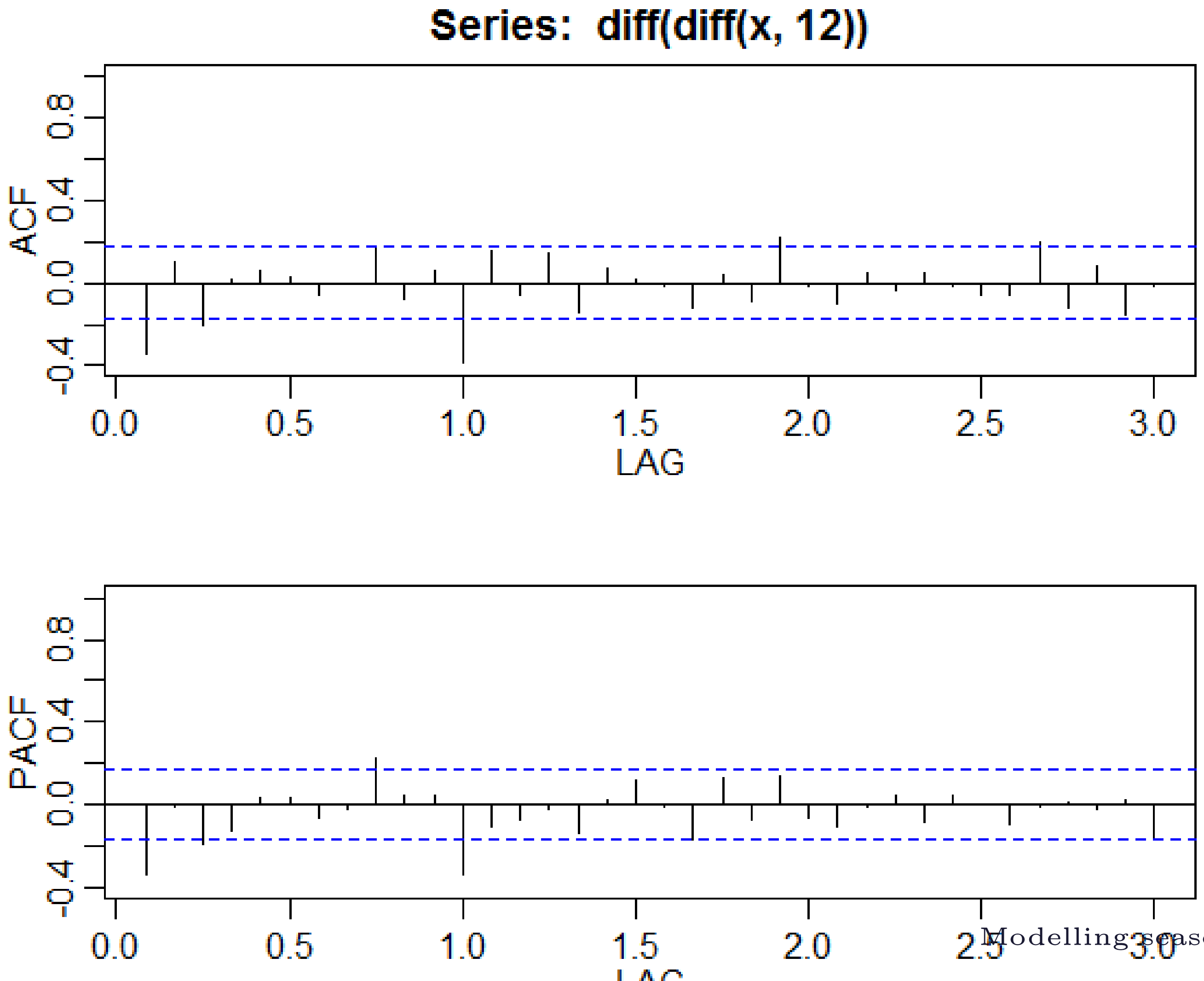
Example - differences

- We can tke seasonal differences $x_t - x_{t-12}$ - in R diff(x,12):



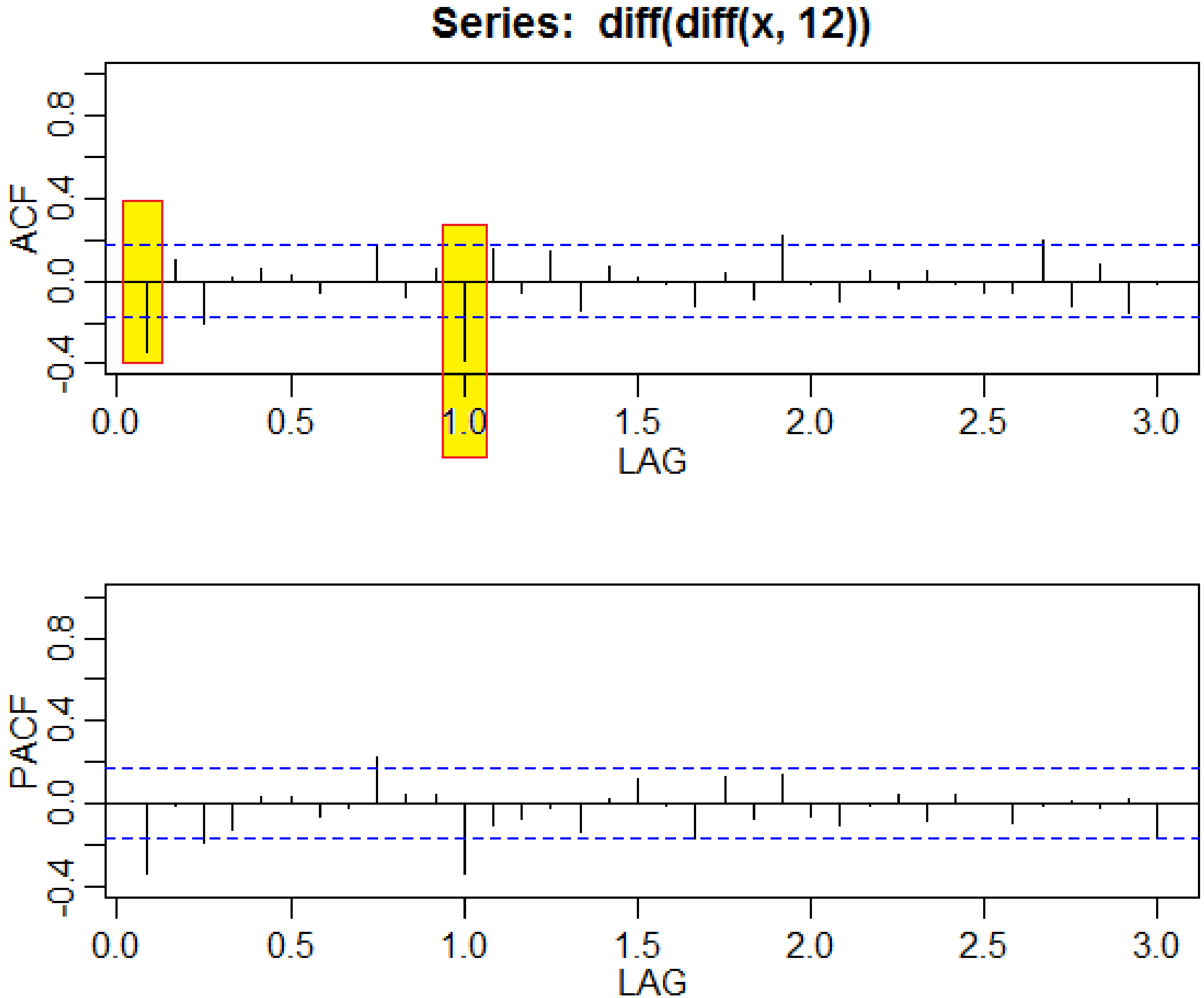
Example - differences

- Both classical and seasonal differences - classical because of the trend, seasonal because of the differences - in R diff(diff(x,12)):



Example - differences

- What to do with that:



Example - seasonal AR and MA terms

- Based on ACF we may trey using terms up to **ma(12)**
- Box and Jenkins:
  - not all **ma(1), ma(2), ..., ma(12)**
  - neither only **ma(1)** a **ma(12)**
  - but multiply polynomials of order 1 and 12:

$$(1-\beta L)(1-\theta L^{12})u_t$$

  - we get 13 **ma** terms but we need only 2 coefficients
- In the same way + we can combine them:
  - seasonal **ma** terms of higher order: $1-\theta_1 L^{12}-\theta_2 L^{24}$
  - seasonal **ar** term with an ordinary one: $(1-\alpha L)(1-\theta_1 L^{12})x_t$

SARIMA models - terminology

- Recall ARIMA (p, d, q) models:
  - ◇ p - number of AR terms
  - ◇ d - how many times we take a difference
  - ◇ q - number of MA terms
- SARIMA $(p, d, q) \times (P, D, Q)_s$ has also:
  - ◇ P - number of seasonal AR terms
  - ◇ D - how many seasonal differences
  - ◇ Q - number of seasonal AR terms
  - ◇ s - period of the data
- We need to check that the data we use - after differencing - does not have a unit root

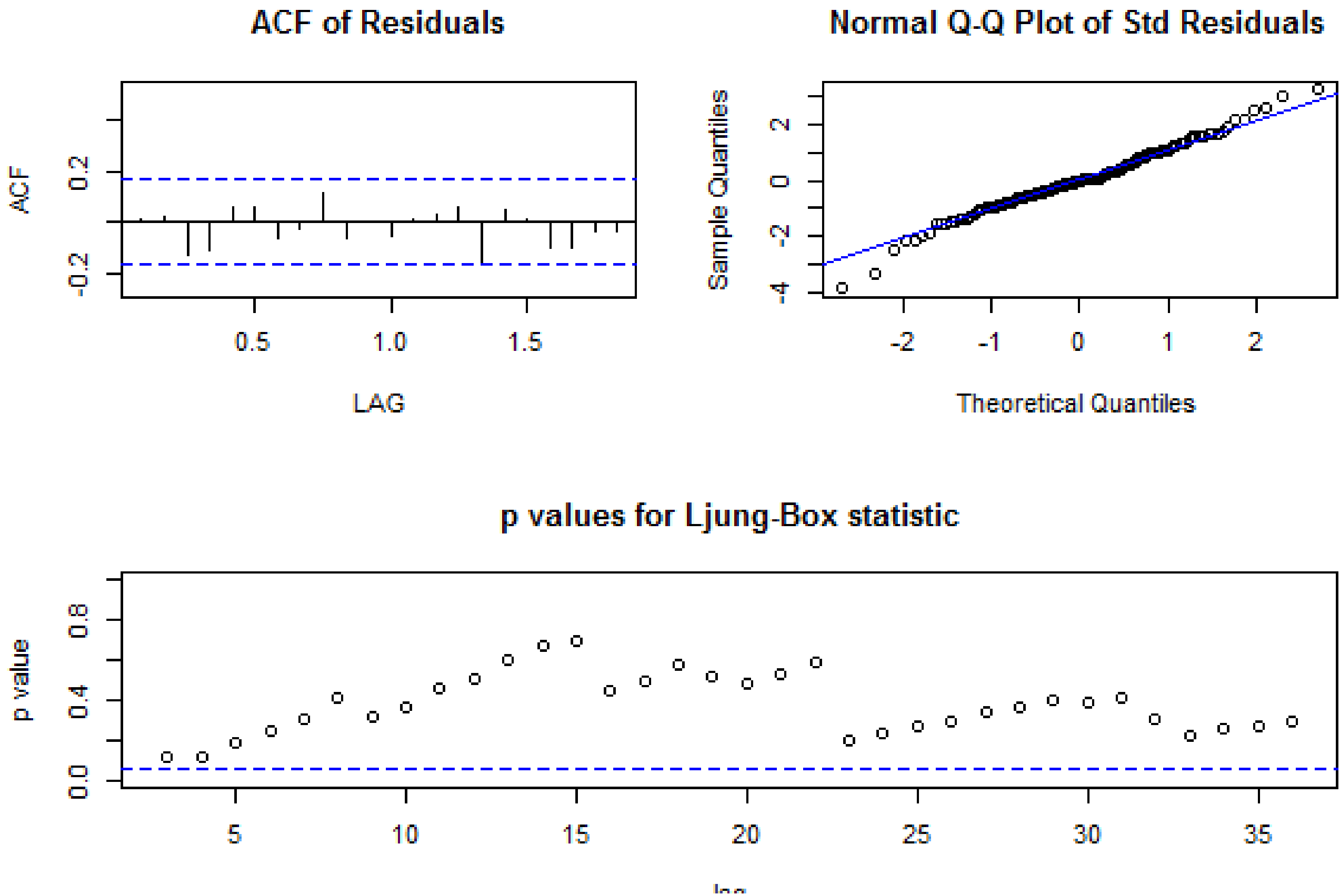
Example - model in R

- For our data: SARIMA $(0,1,1) \times (0,1,1)_s$, where $s = 12$
- Time series diff(diff(x,12)) does not have neither trend nor a unit root
- In R: sarima(x,0,1,1,0,1,1,12)
- We get:

```
> sarima(x,0,1,1,0,1,1,12,details="FALSE")
$fit
Series: xdata
ARIMA(0,1,1)(0,1,1)[12]

Coefficients:
          ma1     sma1
      -0.4018  -0.5569
s.e.   0.0896   0.0731

sigma^2 estimated as 0.001348:  log likelihood=244.7
AIC=-483.4   AICc=-483.21   BIC=-474.77

$AIC
[1] -5.58133
```

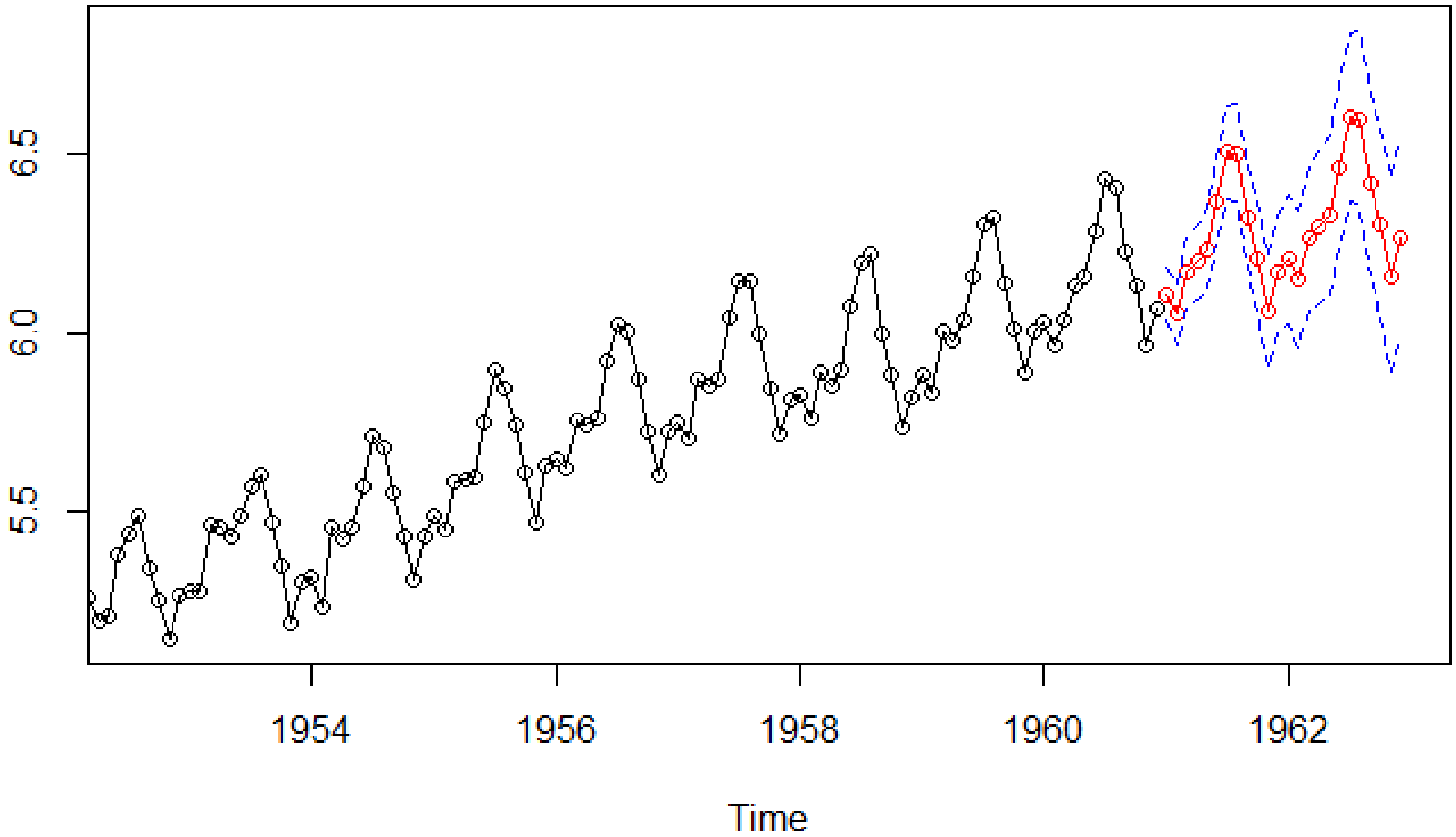
Example - model in R

- Residuals are ok:



Example - predictions in R

- We have a model SARIMA $(0,1,1)\times(0,1,1)_s$
- Prediction for the following 2 years:
  sarima.for(x,24,0,1,1,0,1,1,12)



Exercises

Find a suitable SARIMA model (data on course webpage):

- `spain.txt` - number of tourists in Spain, mothly dat from January 1970 to March 1989
- `souvenirs.txt` - sales in a souvenir shop on a beach in Australia, monthly data from January 1987 to December 1993