Modelling volatility - ARCH and GARCH models

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Time series analysis

Stock prices

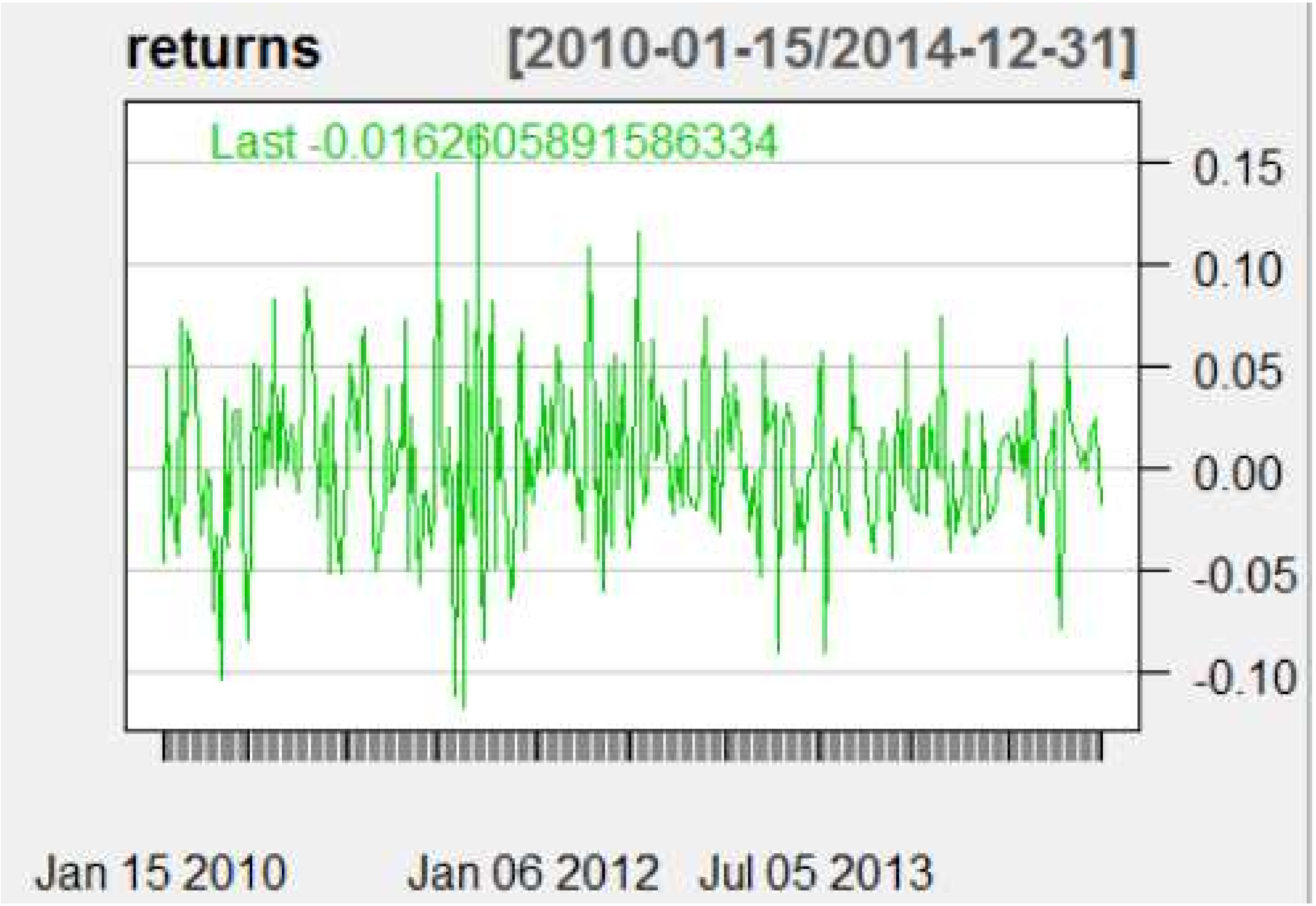
- Weekly stock prices (library quantmod)
- Continuous returns:

```
library(quantmod)
library(astsa)

getSymbols("EBAY", from="2010-01-01", to="2014-12-31", auto.assign=TRUE)
EBAY <- to.weekly(EBAY)
returns <- diff(log(EBAY$EBAY.Adjusted))[-1] # omit NA at the beginning
chartSeries(returns, theme="white")

```

- At the beginning of the term we analyzed their autocorrelations in a HW

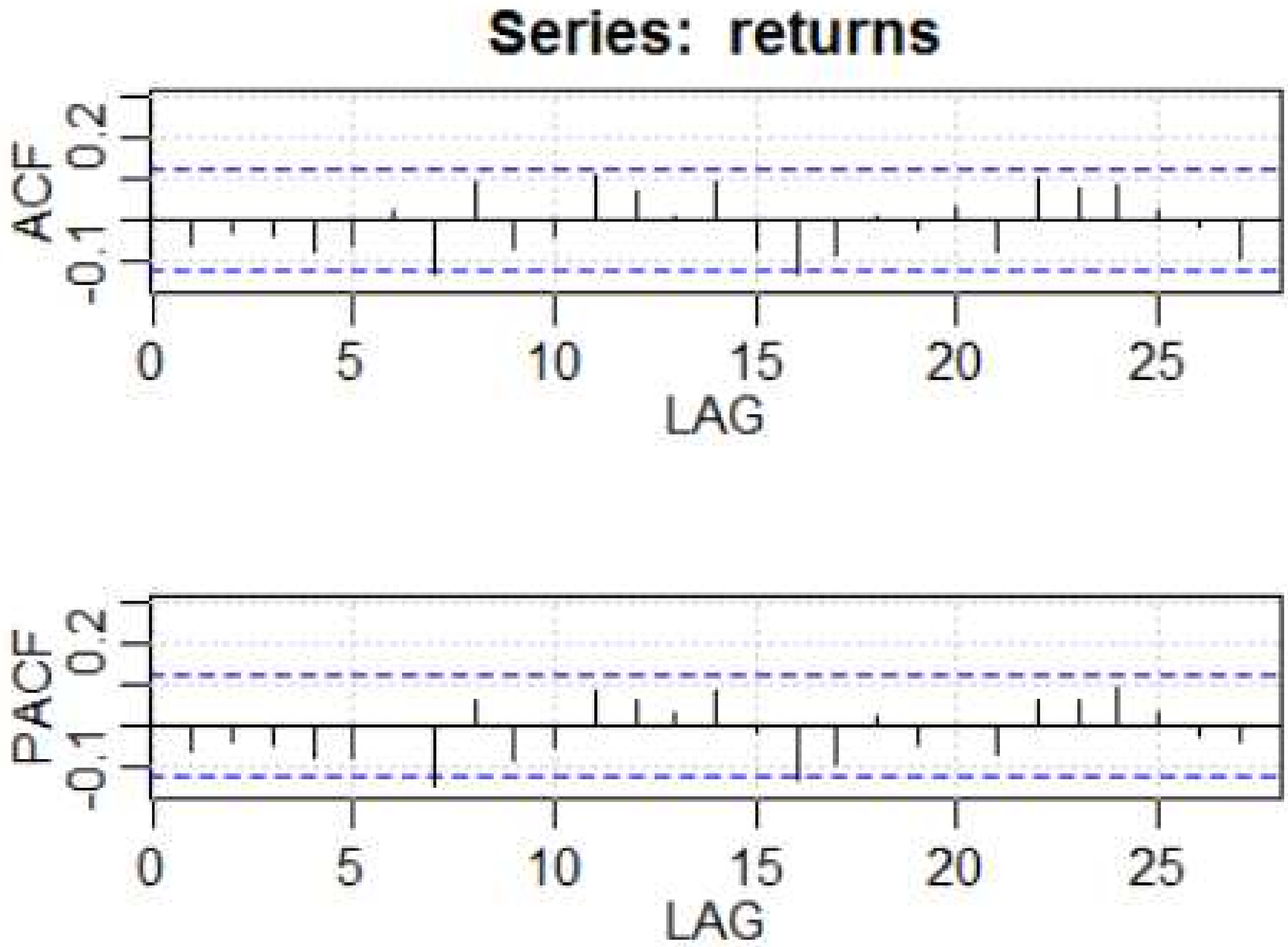
Returns

- Time evolution:



Returns

- Based on ACF, they look like a white noise:



Returns

- We model them as a white noise:

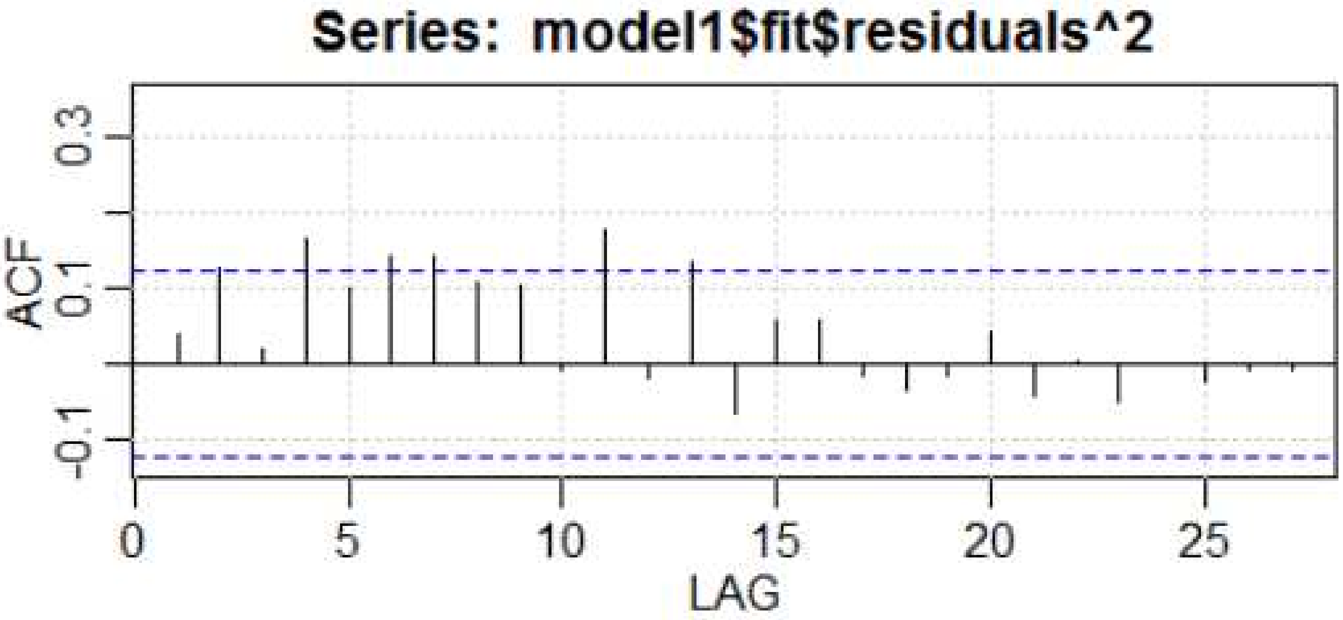
```
model1<-sarima(returns,0,0,0, details=FALSE)
acf2(model1$fit$residuals)   # the same as acf2(vynosy) -> ok
acf2(model1$fit$residuals^2) # -> PROBLEM
```

  $\rightarrow$ residuals are just - up to a contant - the returns

- If the absolute value of a residual is small, usually follows a residual with a small absolute value
- Similarly, after a residual with a large absolute value, there is often another residual with a large absolute value - it can be positive or negative, so it cannot be seen on the ACF
- Second powers will likely be correlated (but this does not hold for a white noise)

Returns

- ACF of squared residuals:

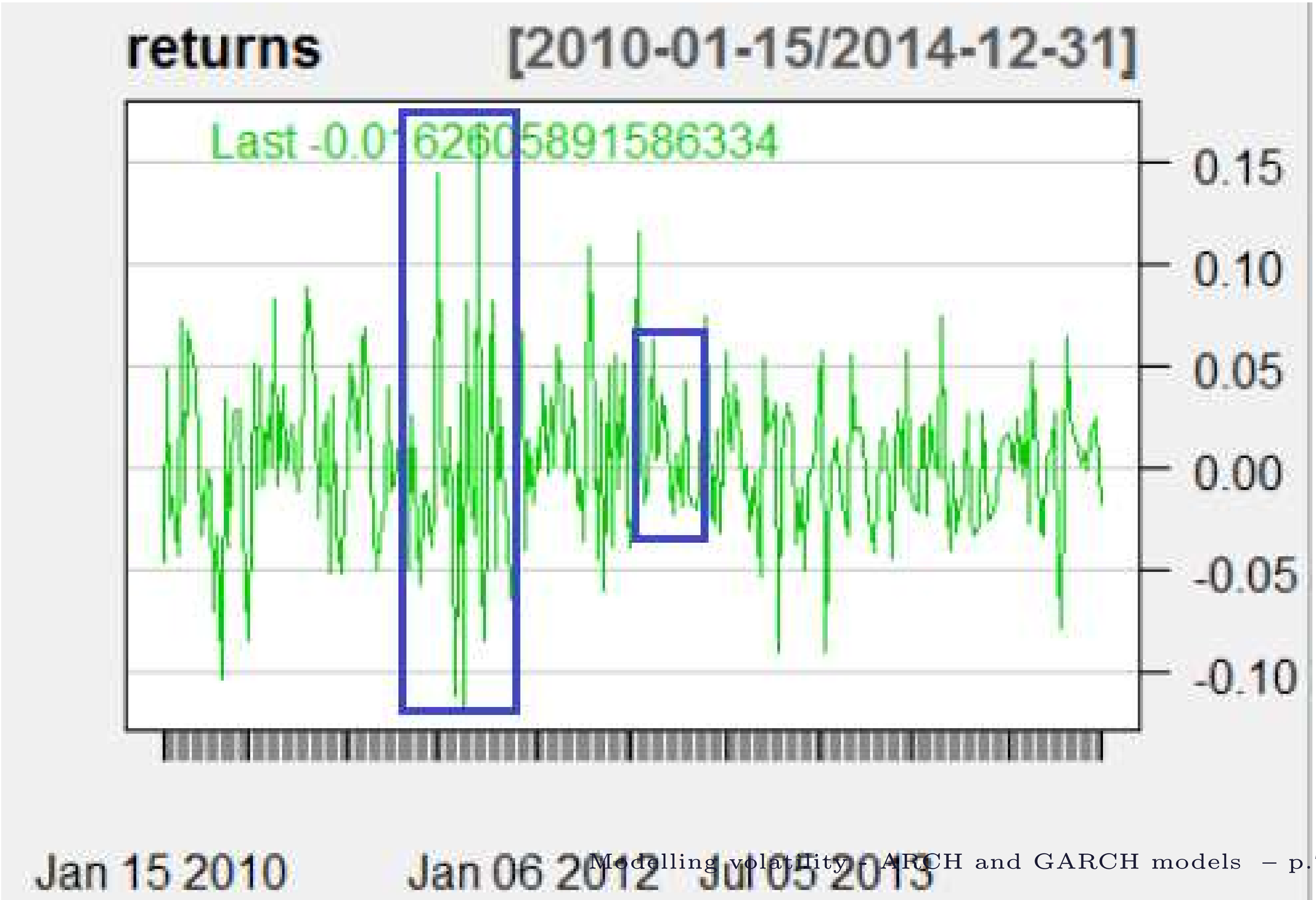


$\rightarrow$ significant autocorrelation

- QUESTION:
  Which model can capture this property?

Returns

- Possible explanation: nonconstant variance



ARCH and GARCH models

- u is not a white noise, but

$$u_t = \sqrt{\sigma_t^2}\,\eta_t,$$

  where η is a white noise with unit variance, i.e.,

$$u_t \sim N(0, \sigma_t^2)$$

- ARCH model (autoregressive conditional heteroskedasticity) - equation for variance σ_t^2:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \ldots \alpha_q u_{t-q}^2$$

- Constraints on parameters:
  - ◇ variance has to be positive:

$$\omega > 0, \alpha_1, \ldots, \alpha_{q-1} \geq 0, \alpha_q > 0$$

  - ◇ stationarity:

$$\alpha_1 + \ldots + \alpha_q < 1$$

ARCH and GARCH models

- Disadvantages of ARCH models:
  - a small number of terms u_{t-i}^2 is often not sufficient - squares of residuals are still often correlated
  - for a larger number of terms, these are often not significant or the constraints on paramters are not satisfied
- Generalization: GARCH models - solve these problems

ARCH and GARCH models

- GARCH(p,q) model (generalized autoregressive conditional heteroskedasticity) - equation for variance σ_t^2:

$$\begin{aligned}\sigma_t^2 &= \omega + \alpha_1 u_{t-1}^2 + \ldots + \alpha_q u_{t-q}^2 \\ &\quad + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2\end{aligned}$$

- Constraints on parameters:
  - ◇ variance has to be positive:

$$\omega > 0, \alpha_1, \ldots, \alpha_{q-1} \geq 0, \alpha_q > 0$$

$$\beta_1, \ldots, \beta_{p-1} \geq 0, \beta_p > 0$$

  - ◇ stationarity:

$$(\alpha_1 + \ldots + \alpha_q) + (\beta_1 + \ldots \beta_p) < 1$$

- A popular model is GARCH(1,1).

GARCH models in R

- Modelling YHOO returns - continued
- In R:
  - ◇ library fGarch
  - ◇ function garchFit, model is writen for example like

    arma(1,1)+garch(1,1)
  - ◇ parameter trace=FALSE - we do not want the details about optimization process
- We have a model constant + noise; we try to model the noise by ARCH/GARCH models

ARCH(1)

- Estimation of ARCH(1) model:

```
# arch(1) = garch(1,0)
model10 <- garchFit(~garch(1,0), data=returns, trace=FALSE)
stand.res <- model10@residuals/model10@sigma.t # standardized reziduals
acf2(stand.res)
acf2(stand.res^2)
summary(model10)
```

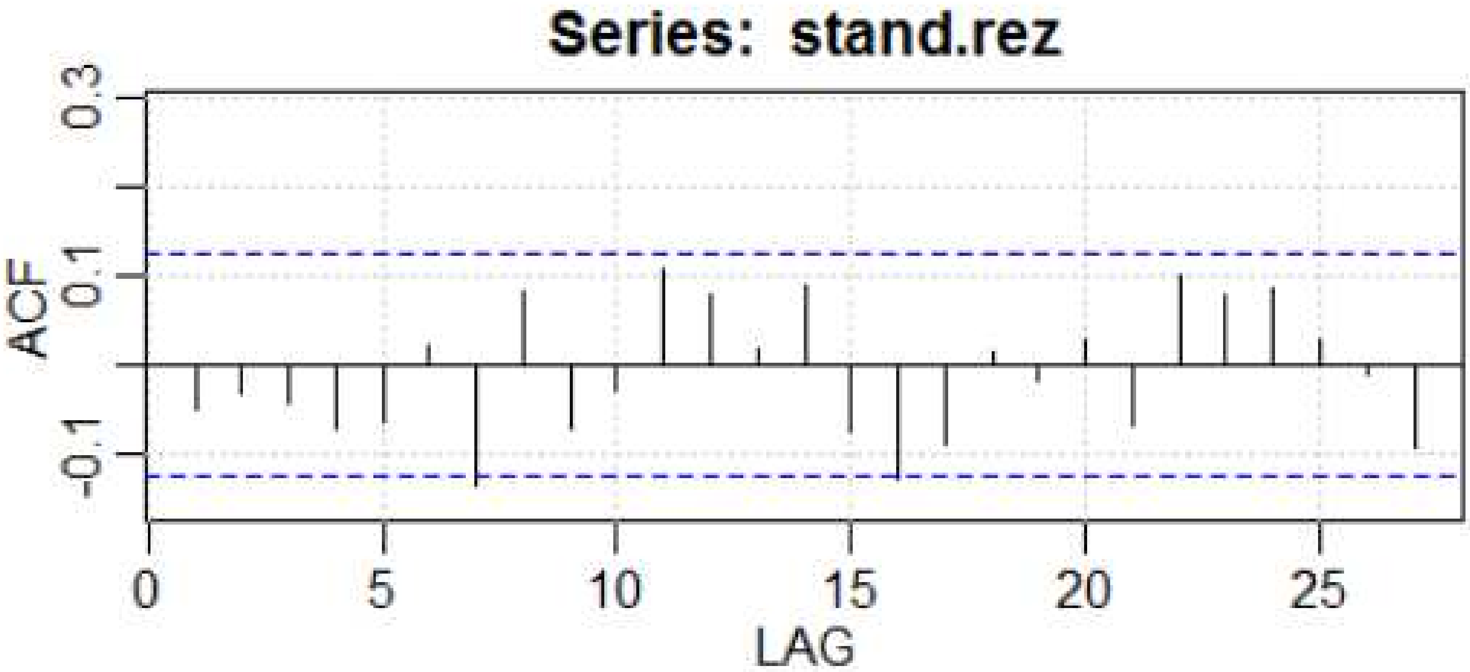
- We check
  1. ACF of standardized residuals
  2. ACF of squared standardized residuals
  3. summary with tests about standardized residuals and their squares

GARCH models in v R

- Useful values:
  - @fitted - fitted values
  - @residuals - residuals
  - @h.t - estimated variance
  - @sigma.t - estimated standard deviation
- Standardized residuals - residuals divided by their standard deviation rezíduá vydelené ich štadardnou - should be a white noise
- Also their squares should be a white noise

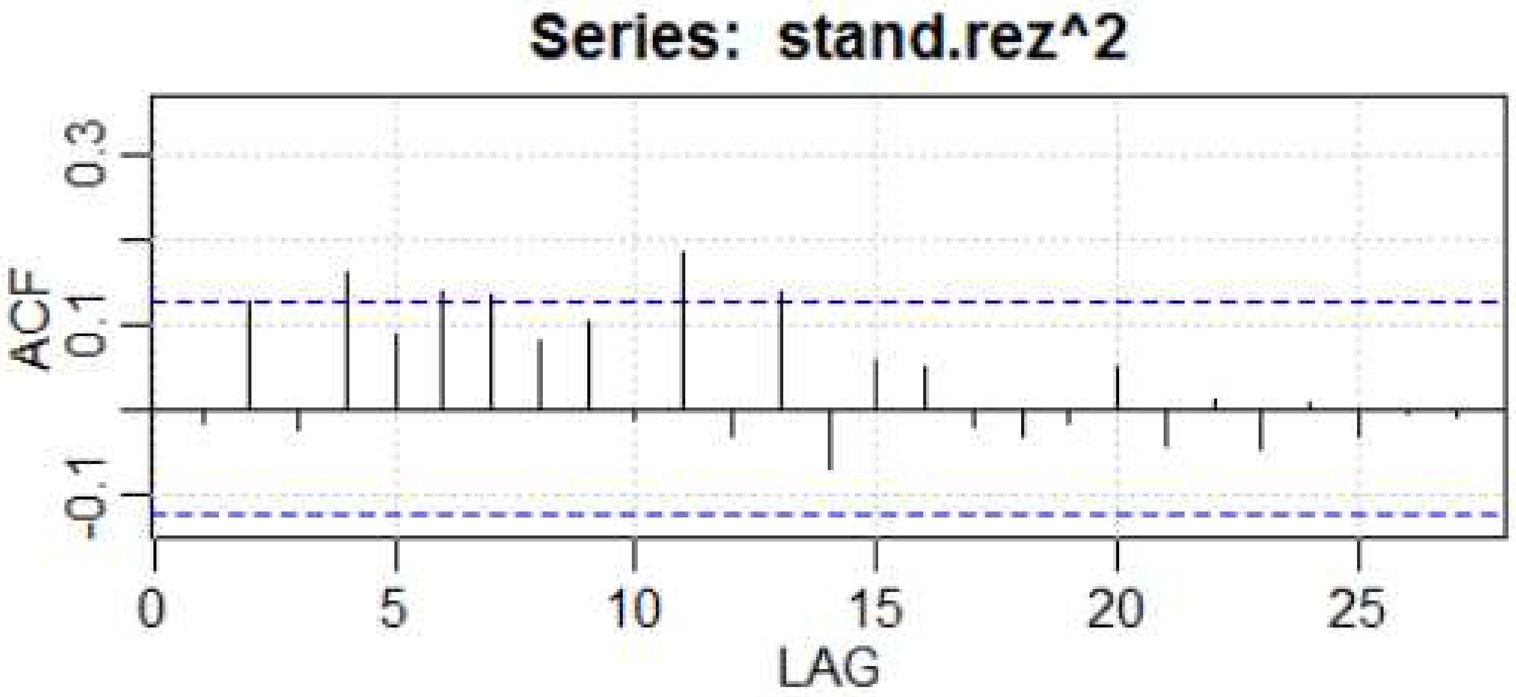
ARCH(1)

- Residuals:



ARCH(1)

- Squares:



Tests about residuals

- Tests:

```
Standardised Residuals Tests:
                                  Statistic p-Value
 Jarque-Bera Test    R    Chi^2  29.18907  4.588523e-07
 Shapiro-Wilk Test   R    W      0.9821336 0.002392983
 Ljung-Box Test      R    Q(10)  12.29485  0.2658079
 Ljung-Box Test      R    Q(15)  21.08263  0.1342099
 Ljung-Box Test      R    Q(20)  28.34245  0.1015375
 Ljung-Box Test      R^2  Q(10)  27.55866  0.002123322
 Ljung-Box Test      R^2  Q(15)  44.23218  0.0001011277
 Ljung-Box Test      R^2  Q(20)  46.02944  0.0007985573
 LM Arch Test        R    TR^2   25.98164  0.01079828
```

- We have: normality test, Ljung-Box for standardized residuals and their sqaures
- What is new: testing homoskedasticity for the residuals

ARCH(2)

- We try ARCH(2) - results of the tests:

Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	34.65834	2.978778e-08
Shapiro-Wilk Test	R	W	0.9814854	0.001841151
Ljung-Box Test	R	Q(10)	10.75724	0.3767411
Ljung-Box Test	R	Q(15)	19.54396	0.1901334
Ljung-Box Test	R	Q(20)	26.93409	0.1371285
Ljung-Box Test	R^2	Q(10)	14.11704	0.1677196
Ljung-Box Test	R^2	Q(15)	27.9337	0.02198778
Ljung-Box Test	R^2	Q(20)	30.13048	0.06776678
LM Arch Test	R	TR^2	20.09535	0.06530357

ARCH(3)

- ARCH(3) - results of the tests:

Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	33.88186	4.391851e-08
Shapiro-Wilk Test	R	W	0.9816512	0.001968254
Ljung-Box Test	R	Q(10)	10.70552	0.3809165
Ljung-Box Test	R	Q(15)	19.44674	0.1941989
Ljung-Box Test	R	Q(20)	26.77638	0.1416733
Ljung-Box Test	R^2	Q(10)	14.01256	0.1724194
Ljung-Box Test	R^2	Q(15)	27.74511	0.02322011
Ljung-Box Test	R^2	Q(20)	29.97015	0.07033882
LM Arch Test	R	TR^2	20.00903	0.06691536

ARCH(4)

- ARCH(4) - results of the tests:

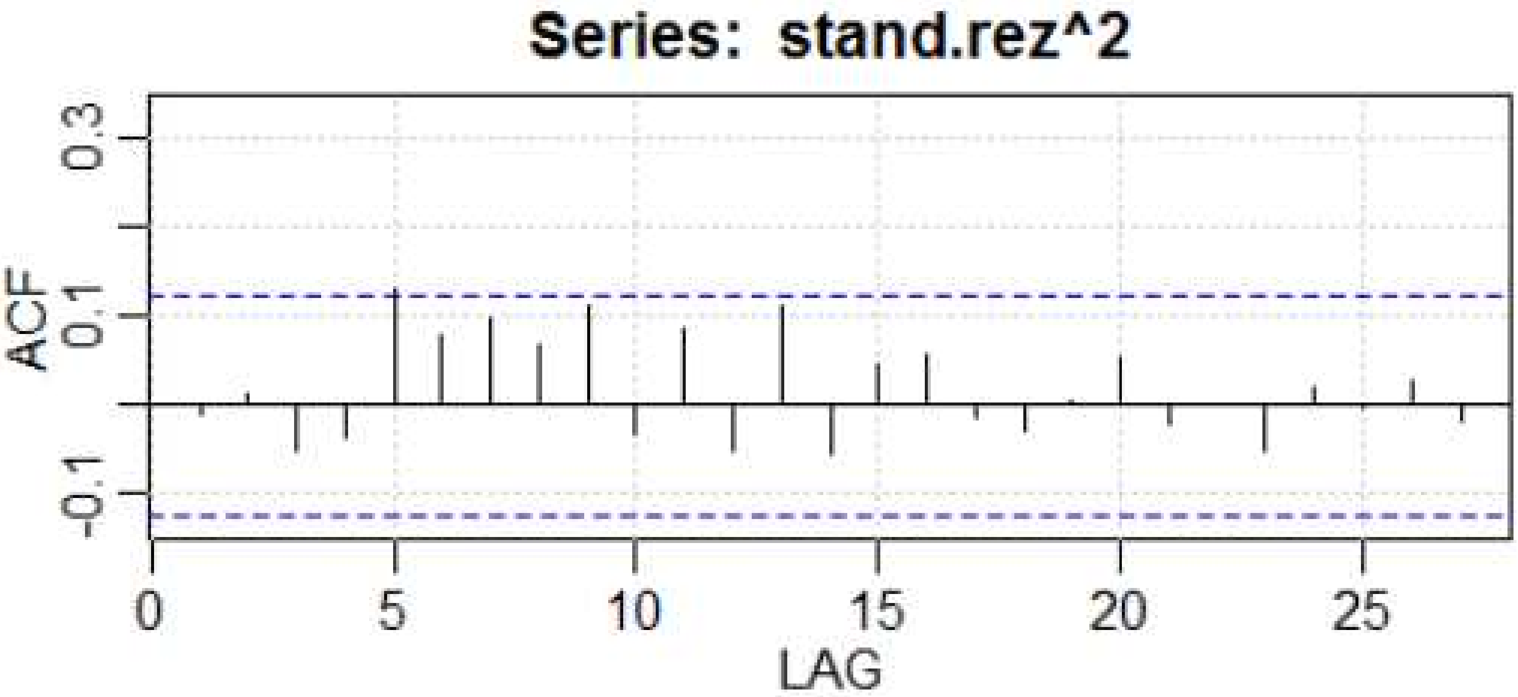
Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	12.79082	0.001669201
Shapiro-Wilk Test	R	W	0.9880808	0.0302226
Ljung-Box Test	R	Q(10)	10.54746	0.3938435
Ljung-Box Test	R	Q(15)	18.98255	0.2145258
Ljung-Box Test	R	Q(20)	26.02379	0.1650277
Ljung-Box Test	R^2	Q(10)	14.41393	0.1549343
Ljung-Box Test	R^2	Q(15)	21.97894	0.1083574
Ljung-Box Test	R^2	Q(20)	23.88737	0.2473466
LM Arch Test	R	TR^2	18.14053	0.1114896

- No autocorrelation in residuals and their squares.

ARCH(4)

- ACF of squared residuals:



$\rightarrow$ without significant correlation

ARCH(4)

- But ARCH coefficients α_i are not significant:

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.022e-03   2.433e-03    1.653   0.0983 .
omega  1.170e-03   2.003e-04    5.842 5.16e-09 ***
alpha1 3.839e-02   1.250e-01    0.307   0.7587
alpha2 1.259e-01   8.058e-02    1.563   0.1181
alpha3 1.000e-08   1.475e-01    0.000   1.0000
alpha4 1.098e-01   8.052e-02    1.364   0.1726
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

GARCH(1,1)

- We try GARCH(1,1)
- Tests:

Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	11.47005	0.003230806
Shapiro-Wilk Test	R	W	0.987312	0.02152423
Ljung-Box Test	R	Q(10)	10.26988	0.4171434
Ljung-Box Test	R	Q(15)	17.86043	0.2700744
Ljung-Box Test	R	Q(20)	23.83093	0.2498554
Ljung-Box Test	R^2	Q(10)	9.549341	0.4808792
Ljung-Box Test	R^2	Q(15)	17.0167	0.3178669
Ljung-Box Test	R^2	Q(20)	19.01346	0.5209512
LM Arch Test	R	TR^2	12.16882	0.4322184

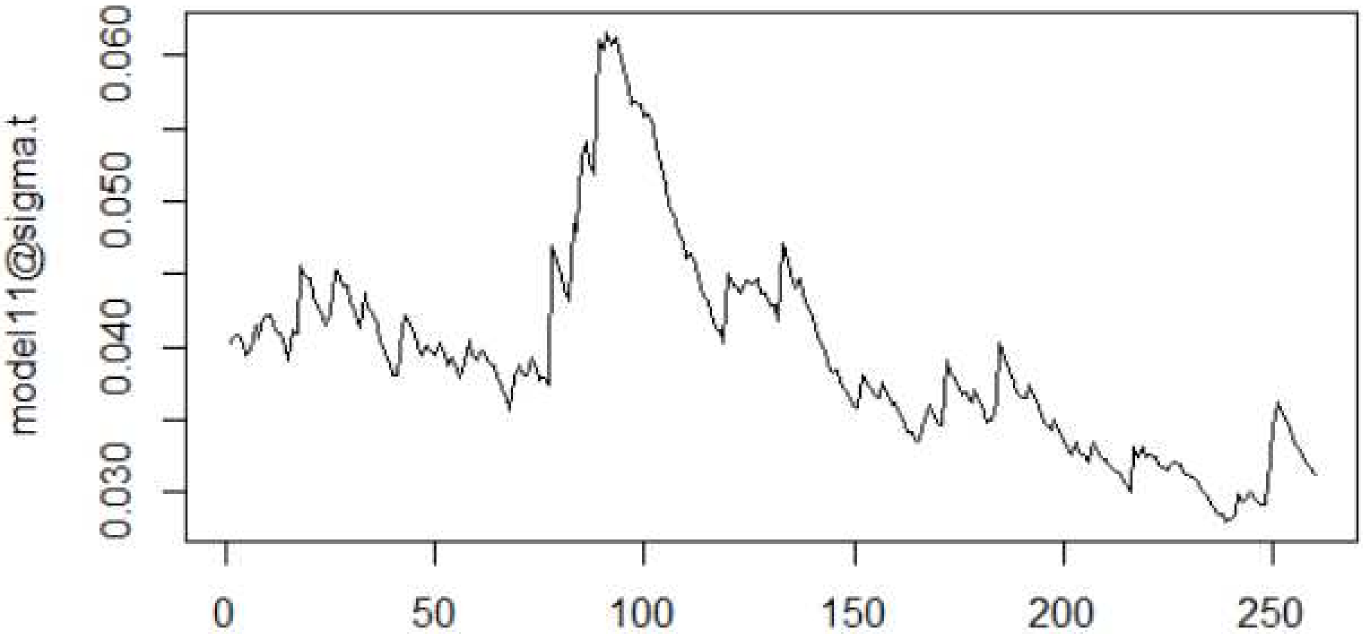
GARCH(1,1)

- Estimates:

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     3.262e-03   2.351e-03    1.388   0.1652
omega  2.428e-05   3.596e-05    0.675   0.4994
alpha1 4.334e-02   2.401e-02    1.805   0.0711 .
beta1  9.396e-01   3.923e-02   23.954   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Estimated standard deviation

- We obtain it using @sigma.t :



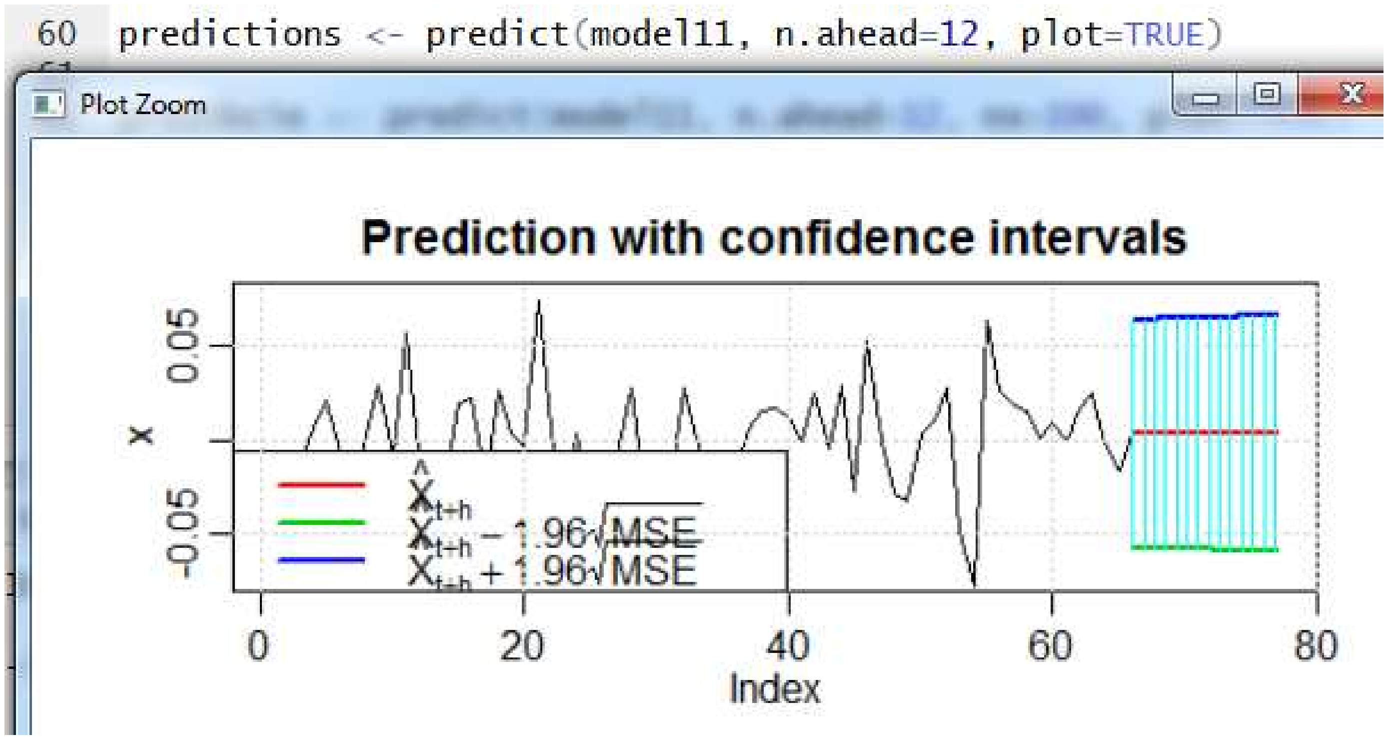
Estimated standard deviation

- Another access to the graphs - plot(model11):

```
> plot(model11)

Make a plot selection (or 0 to exit):

 1:   Time Series
 2:   Conditional SD
 3:   Series with 2 Conditional SD Superimposed
 4:   ACF of Observations
 5:   ACF of Squared Observations
 6:   Cross Correlation
 7:   Residuals
 8:   Conditional SDs
 9:   Standardized Residuals
10:   ACF of Standardized Residuals
11:   ACF of Squared Standardized Residuals
12:   Cross Correlation between r^2 and r
13:   QQ-Plot of Standardized Residuals

Selection:
```

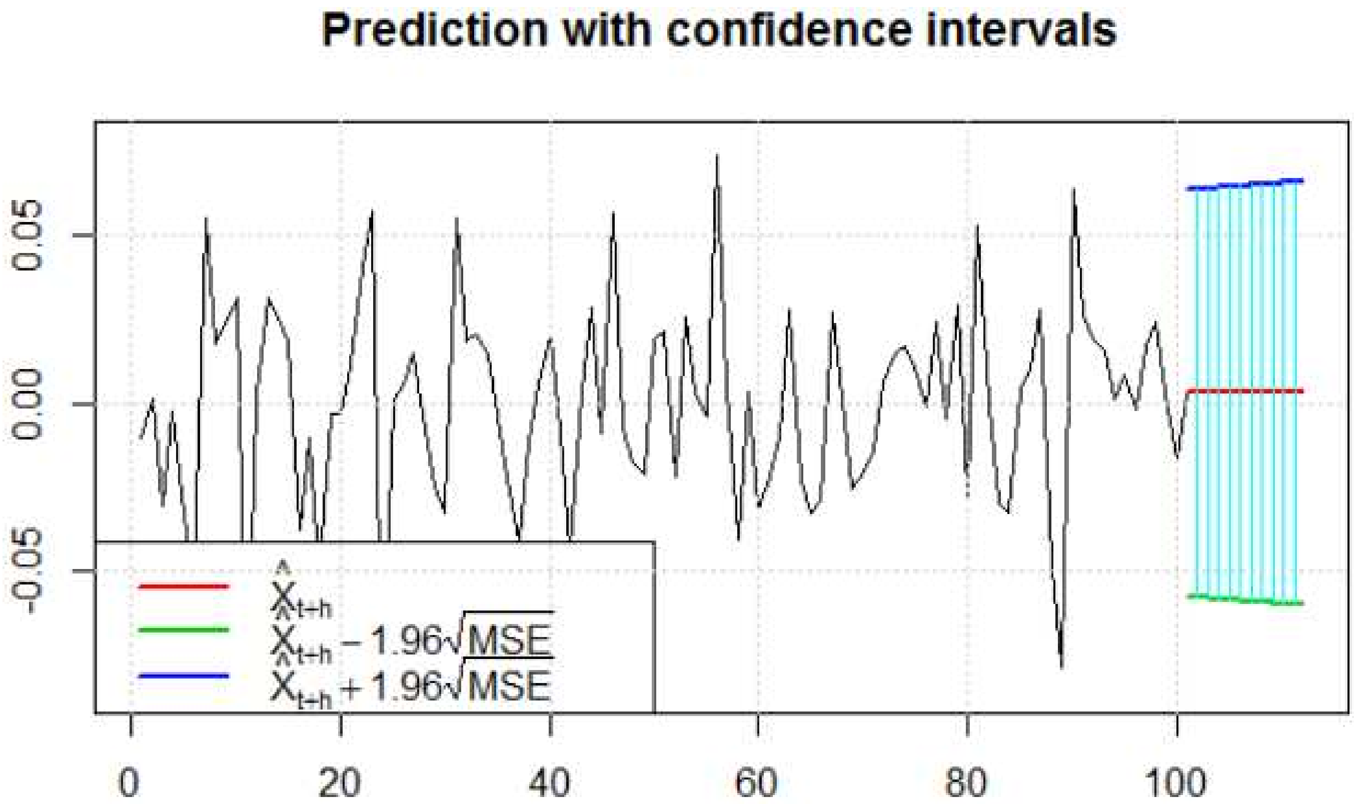
Predictions

- We use the function predict with parameter n.ahead (number of observations)



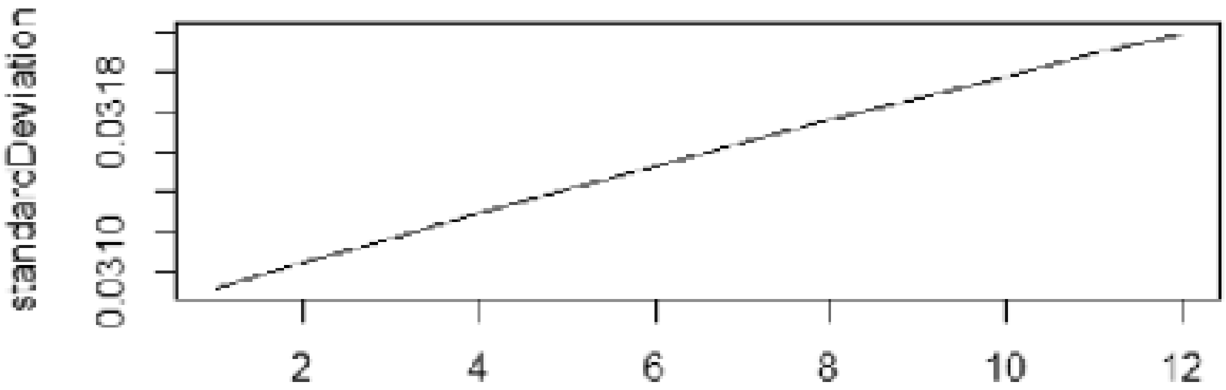
Predictions

- Parameter nx - we can change the number of observations from the data which are shown in the plot (here nx=100:



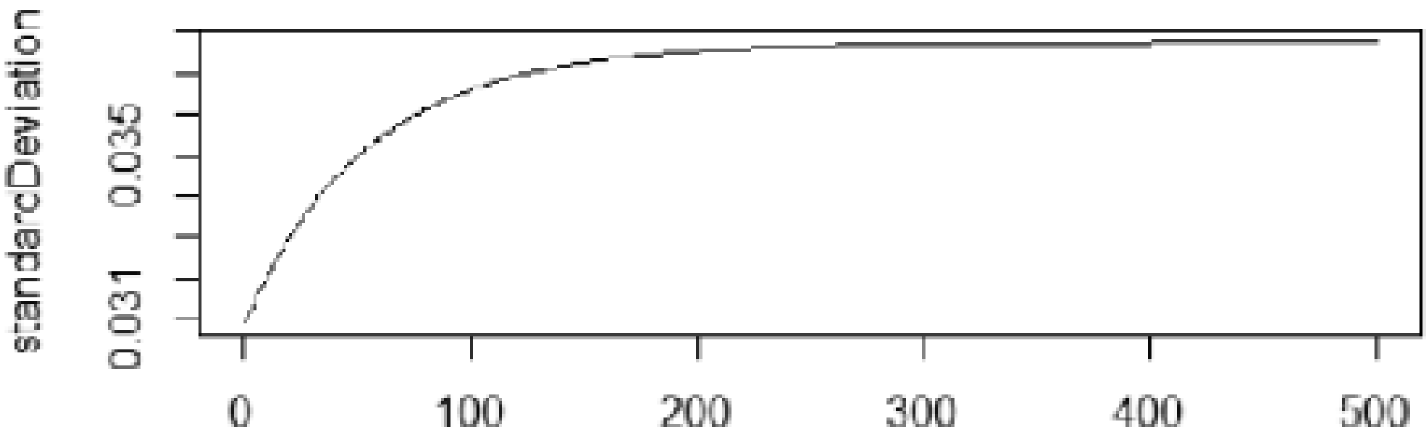
Predictions

- Predicted standard deviation: plot(ts(predictions[3]))



Predictions

- For a longer time (exercise: compute its limit):



Application: Value at risk (VaR)

- Value at risk (VaR) is basicly a quantile
- Let X be a portfolio value, then

$$\mathcal{P}(X \leq VaR) = \alpha,$$

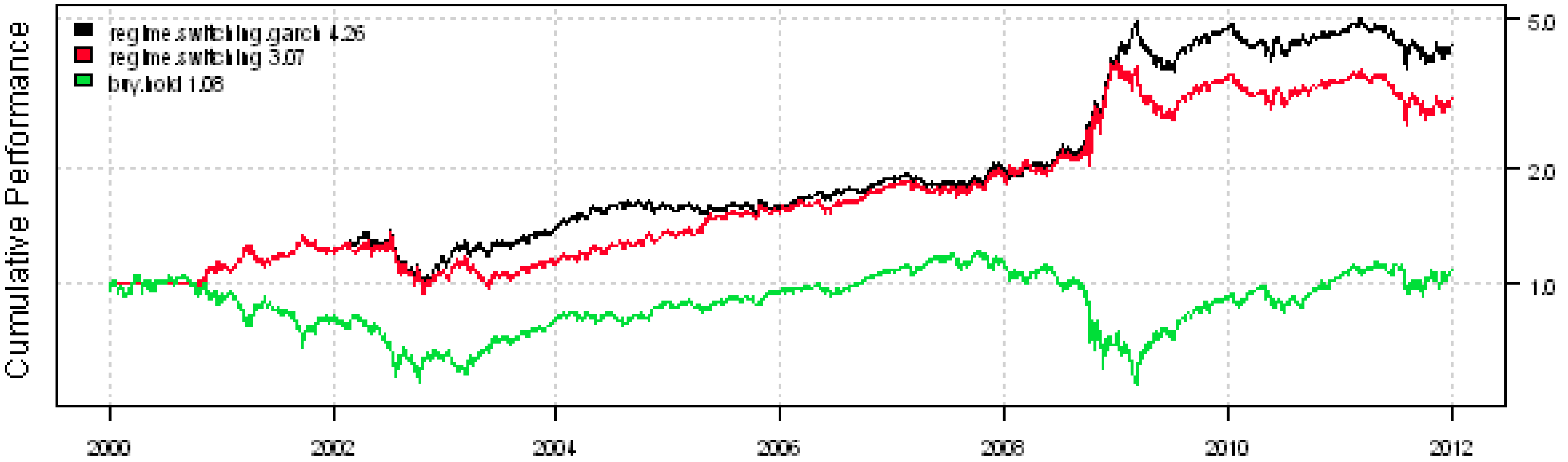
  for example for $\alpha = 0.05$
- A standard GARCH assumes normal distribution - we can compute quantiles
- Shortcomings:
  - ◇ normality assumptions
  - ◇ there are also better risk measures than VaR

Apication: Value at risk (VaR)

- WHAT WE WILL DO:
  - Start with N observations of returns
  - Estimate the GARCH model.
  - Make a prediction for standard deviation and using the prediction we construct *VaR* for returns for the following day
  - Every day move the window with data (we have a new observation), estimate GARCH again and compute thes new *VaR*

Not required - for those interested

- https://systematicinvestor.wordpress.com/2012/01/06/trading-using-garch-volatility-forecast/
- *"... Now, let's create a strategy that switches between mean-reversion and trend-following strategies based on GARCH(1,1) volatility forecast."* + R code
- From the website:



Other models for volatility

- Threshold GARCH:
  - $u_t > 0$ - *"good news"*, $u_t < 0$ - *"bad news"*
  - TARCH can model their different effect on volatility
  - *leverage effect*: *bad news* have a higher impact
- We do not model variance (as in ARCH/GARCH models), but
  - its logarithm $\rightarrow$ exponential GARCH
  - any power of standard deviation $\rightarrow$ power GARCH
- and others...