I. POUŽITÁ LITERATÚRA

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II. PRÍLOHA A

Definícia: (Markovovský proces)

Proces $\{X_t\}_{t\in T}$ sa nazýva Markovovský, ak pre

$\forall n \in N, n > 2, \forall t_0 < t_1 < \cdots < t_n \in T, \forall a_0, \ldots, a_{n-2}, i, j \in R$ platí, že:

$$\mathrm{P}\left(\mathrm{X}_{t_n} = i \,\middle|\, X_{t_{n-1}} = j, X_{t_{n-2}} = a_{n-2}, \ldots, X_{t_1} = a_1\right) = \mathrm{P}\left(\mathrm{X}_{t_n} = i \,\middle|\, X_{t_{n-1}} = j\right)$$

Definícia: (Wienerov proces)

Nech W_t je Markovov proces, *t* je čas, *dt* je malý časový interval a dW_t zmena hodnoty W_t za interval *dt*. W_t sa nazýva Wienerov proces, ak platí:

- $dW_t = \varepsilon\sqrt{dt}$, kde ε je náhodná premenná so štandardizovaným normálnym rozdelením N(0, 1)
- hodnoty dW_t sú pre ľubovoľné rôzne intervaly *dt* nezávislé.
- trajektórie realizácií sú spojité.

Z prvej vlastnosti vyplýva, že aj dW_t má normálne rozdelenie so strednou hodnotou 0 a varianciou *dt*. Z druhej vlastnosti vyplýva, že ide o Markovov proces.

Lema: (Itôova lema)

Nech *x* je Itôov proces pre ktorý platí $dx = \mu(x,t)dt + \sigma(x,t)dW_t$ a $f(x,t) \in C^1$ je hladká funkcia. Potom prvý diferenciál funkcie *f* je daný vzťahom:

$$df = \frac{\partial f}{\partial x}dx + \left(\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2(x,t)\frac{\partial^2 f}{\partial x^2}\right)dt$$

Definícia: (Diracova δ-funkcia)

Nech $\Phi(x)$ je ľubovoľná spojite diferencovateľná funkcia. Potom Diracova δ-funkcia je daná vzťahom:

$$\int_{-\infty}^{\infty} \delta(x)\Phi(x)dx = \Phi(0) \quad \forall \Phi(x) \in C^1(R)$$

Definícia: (Heavisidova funkcia)

Heavisidova funkcia H(x) je definovaná vzťahom:

$$\mathrm{H}(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}.$$

Z definície vidno, že platí: $\int_{-\infty}^{x} \delta(\xi) d\xi = \mathrm{H}(x)$, alebo inverzne $\mathrm{H}'(x) = \delta(x)$.

III. PRÍLOHA B

Odvodenie ukazovateľov (tzv. greeks) európskej call opcie na akciu vyplácajúcu spojitý dividendový úrok. Odvodenie pre put opciu je analogické.

Explicitné riešenie Black-Scholesovho modelu:

$$V_{ecd}(S,t) = e^{-D(T-t)} S\Phi(d_1) - Ee^{-r(T-t)}\Phi(d_2), \text{ kde}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy,$$

$$d_1 = \frac{\ln\frac{S}{E} + \left(r - D + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln\frac{S}{E} + \left(r - D - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Odvoďme si najskôr pomocný vzorec:

$$Ee^{-(r-D)(T-t)}\Phi'(d_2) = Ee^{-(r-D)(T-t)} \frac{1}{\sqrt{2\pi}} e^{-\frac{\left[\ln\frac{S}{E} + \left(r-D-\frac{1}{2}\sigma^2\right)(T-t)\right]^2}{2\sigma^2(T-t)}} =$$

$$= Ee^{-(r-D)(T-t)} \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\ln\frac{S}{E}\right)^2 + 2\ln\frac{S}{E}\left(r-D-\frac{\sigma^2}{2}\right)(T-t) + \left(r-D-\frac{\sigma^2}{2}\right)^2 (T-t)^2}{2\sigma^2(T-t)}} =$$

$$= E\frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\ln\frac{S}{E}\right)^2 + 2\ln\frac{S}{E}\left(r-D-\frac{\sigma^2}{2}\right)(T-t) + \left(r-D-\frac{\sigma^2}{2}\right)^2 (T-t)^2 + (r-D)(T-t)^2 2\sigma^2}{2\sigma^2(T-t)}} =$$

$$= E\frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\ln\frac{S}{E}\right)^2 + 2\ln\frac{S}{E}\left(r-D-\frac{\sigma^2}{2}\right)(T-t) + \left(r-D+\frac{\sigma^2}{2}\right)^2 (T-t)^2}{2\sigma^2(T-t)}} =$$

$$= E\Phi'(d_1) e^{\frac{\sigma^2(T-t)2\ln\frac{S}{E}}{2\sigma^2(T-t)}} = E\Phi'(d_1)\frac{S}{E} = S\Phi'(d_1)$$

Odvodenie delta:

$$\Delta = \frac{\partial V_{ecd}}{\partial S} = e^{-D(T-t)}\left[\Phi(d_1) + S\Phi'(d_1)\frac{\partial d_1}{\partial S}\right] - Ee^{-r(T-t)}\Phi'(d_2)\frac{\partial d_2}{\partial S} =$$

$$= e^{-D(T-t)}\Phi(d_1) + e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial S} - S\Phi'(d_1)e^{-D(T-t)}\Phi'(d_2)\frac{\partial d_2}{\partial S} =$$

$$= e^{-D(T-t)}\Phi(d_1)$$

Odvodenie theta:

$$\Theta = \frac{\partial V_{ecd}}{\partial t} = e^{-D(T-t)}D.S\Phi(d_1) + e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial t} - Ee^{-r(T-t)}r\Phi(d_2) -$$

$$- Ee^{-r(T-t)}\Phi'(d_2)\left(\frac{\partial d_1}{\partial t} - \sigma\frac{1}{2\sqrt{T-t}}\right) =$$

$$= e^{-D(T-t)}D.S\Phi(d_1) + e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial t} - Ee^{-r(T-t)}r\Phi(d_2) -$$

$$- S\Phi'(d_1)e^{-D(T-t)}\frac{\partial d_1}{\partial t} - S\Phi'(d_1)e^{-D(T-t)}\sigma\frac{1}{2\sqrt{T-t}} =$$

$$= -\frac{S\Phi'(d_1)\sigma e^{-D(T-t)}}{2\sqrt{T-t}} + e^{-D(T-t)}D.S\Phi(d_1) - rE\Phi(d_2)e^{-r(T-t)}$$

Odvodenie gama:

$$\Gamma = \frac{\partial^2 V_{ecd}}{\partial S^2} = \frac{\partial \Delta}{\partial S} = \frac{\partial}{\partial S}\left(e^{-D(T-t)}\Phi(d_1)\right) = e^{-D(T-t)}\Phi'(d_1)\frac{\partial d_1}{\partial S} =$$

$$= \frac{e^{-D(T-t)}\Phi'(d_1)}{S\sigma\sqrt{T-t}}$$

Odvodenie vega:

$$\Lambda = \frac{\partial V_{ecd}}{\partial \sigma} = e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial \sigma} - Ee^{-r(T-t)}\Phi'(d_2)\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{T-t}\right) =$$

$$= e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial \sigma} - e^{-D(T-t)}S\Phi'(d_1)\frac{\partial d_1}{\partial \sigma} + S\Phi'(d_1)e^{-D(T-t)}\sqrt{T-t} =$$

$$= e^{-D(T-t)}S\Phi'(d_1)\sqrt{T-t}$$

Odvodenie rho:

$$\rho = \frac{\partial V_{ecd}}{\partial r} = e^{-D(T-t)} S\Phi'(d_1)\frac{\partial d_1}{\partial r} + Ee^{-r(T-t)}(T-t)\Phi(d_2) - Ee^{-r(T-t)}\Phi'(d_2)\frac{\partial d_2}{\partial r} =$$

$$= e^{-D(T-t)} S\Phi'(d_1)\frac{\partial d_1}{\partial r} + Ee^{-r(T-t)}(T-t)\Phi(d_2) - S\Phi'(d_1)e^{-D(T-t)}\frac{\partial d_1}{\partial r} =$$

$$\underline{\underline{= Ee^{-r(T-t)}(T-t)\Phi(d_2)}}$$