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Ekonomická a finančná matematika

Diplomová práca

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The Welfare Cost of Inflation in an Endogenous Growth Economy with Costly Credit.

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Týmto prehlasujem, že predkladanú diplomovú prácu som vypracoval samostatne, len s pomocou konzultácií a literatúry uvedenej v zozname.

Zároveň ďakujem svojmu školiteľovi Maxovi Gillmanovi, PhD. za odbornú pomoc, cenné pripomienky a podnety, ako aj za ochotu a podporu prejavenú pri vedení práce.

Peter Brnčík

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1. Introduction

Early efforts to examine the effects inflation taxes in monetary general equilibrium models concluded, that under moderate inflations, these taxes have modest effects on steady-state employment and output and this way on welfare. Money is assumed to be valued as a medium of exchange and enters the models via a cash-in-advance constraint on consumption. The mechanism that operates in these models is that inflation taxes consumption expenditures and hence the benefits to the household of market activity. As a consequence, this tax reduces consumption and distorts households' resource allocations, so they lower labor supply in order to increase leisure, and thus offset the utility loss associated with lower consumption. The standard references in this literature are to Cooley and Hansen (1991), who find, for example, that an increase in inflation from zero to ten percent reduces steady-state employment by 2.3 percent, reduces steady-state output by 2.4 percent, and lowers welfare by 0.376 percent, where the welfare loss is measured as the percent increase in steady-state consumption under a ten percent inflation rate that would be required to make the household indifferent between the two inflation regimes. The welfare losses from inflation can be substantially larger than those measured by Cooley and Hansen if inflation also distorts the required return on capital or labor. This distortion can occur if firms are required to finance their working capital expenses with short-term nominal debt that must be repaid with current sales revenues. In this case, higher nominal interest rates increase the required productivity of capital and labor and thereby reduce firms' factor demands. However, the nature of the payment system can also affect those costs. If the borrowed funds arise from intermediated loans, where a portion of the funds are supplied to financial intermediaries by households, say, in return for interest-bearing deposit accounts with high liquidity value, then an increase in inflation would cause households to shift the composition of their media of exchange away from currency and toward bank deposits to insulate themselves partially from inflation with the interest income that they receive on deposits.

As a consequence, financial intermediaries can reduce the welfare costs of inflation by providing valued liquidity services in exchange for deposit funds. There is a substantial literature that has emerged on the welfare costs of inflation in an endogenous growth context. In a Lucas (1988) model, Gomme finds the welfare effects of inflation taxes on consumption to be very small when comparing balanced growth paths. Einarsson and Marquis (1999) find that the transitional dynamics enhance the benefits of disinflation in this model as households reduce employment in order to build up their human capital stock, while the attendant decline in output in the short run is absorbed by lower physical capital investment, thus insulating consumption to some extent. In a Romer style model with technology spillovers into the payment system, Marquis (2001) finds much larger welfare costs of inflation as households allocate excessive resources to the payment system at the expense of output. Ireland finds a similar overinvestment in “financial capital” in an AK-model. Other papers in this literature, not all of which explicitly compute welfare costs, but rather attempt to identify effects of inflation on growth include Gillman and Kejak (2000, 2002), Gillman, Harris and Matyas (2001), who also provide some cross-country empirical evidence on the negative correlation between growth and inflation, and Gillman and Nakov (2002).

Model used here is an endogenous growth model with Lucas type human capital investment within a cash-in-advance economy that includes credit sector using Hicksian “banking time”. Physical and human capital are used as production inputs in human capital and goods producing sectors, while only human capital is used as input for credit production.

In Section 2, model of endogenous growth economy is designed. Section 3 calibrates this model. In Section 4 behavior of selected variables and steady-state comparisons are made. Presented model’s results are also compared to Cash-only model’s results. Section 5 takes into account also transition paths into new steady-state and Section 6 concludes.

2. Endogenous Growth Monetary Model

The representative agent works in a constant-returns-to-scale (CRS) goods producing sector that employs physical capital and effective labor. Effective labor is defined as raw labor factored by the human capital. The agent also devotes resources to two additional, implicit price, sectors. These are the CRS human capital production that involves the investment of physical capital and effective labor, and a credit services sector that involves only effective labor in a diminishing returns technology. The agent faces four constraints on the maximization of his lifetime utility over goods consumption and leisure, in terms of the flow of human capital, the flow of financial capital that consists of money and physical capital, the stock of financial capital, and the cash-in-advance constraint. The technology of the credit services sector is built into the cash-in-advance constraint.

At time t, denote the real quantities of output and consumption goods by y_t and c_t , and the fraction of time devoted to leisure, to credit services production, and to goods production by x_t, l_{Ft} and l_{Gt}. The rest of the time, $(1- x_t - l_{Ft} - l_{Gt})$, is used in human capital production. The share of physical capital in goods production is given by s_{Gt}. The rest of physical capital, $(1-s_{Gt})$, is used in human capital production. The stocks of physical and human capital and their depreciation rates are given by k_t, h_t, δ_k, and δ_h respectively. Denote the input prices of capital and effective labor by r_t, the real interest rate, and w_t, the real wage. The positive shift parameters of the production functions of goods, credit services, and human capital are A_G, A_F, and A_H. Nominal variables are the price of goods P_t, the stock of nominal financial capital Q_t, the stock of money M_t, and the lump sum government transfer of cash V_t. In addition denote by d_t the amount of real credit used in making purchases. Given parameters γ, β, δ, capital intensities, and ρ, rate of time preference, are in the (0,1) interval, and α, utility parameter for leisure, and θ, intertemporal elasticity of substitution, are positive parameters.

2.1. The Goods Producing Firm

The output of goods is produced by the Cobb-Douglas CRS function:

$$y_t = A_G\,(s_{Gt}\,k_t)^{1-\beta}\,(l_{Gt}\,h_t)^{\beta} \tag{1}$$

The firm's first-order conditions set the market's real interest rate and real wage equal to the marginal products of effective capital and effective labour:

$$r_t = (1 - \beta)\, A_G\, (s_{Gt}\, k_t)^{-\beta}\, (l_{Gt}\, h_t)^{\beta} \tag{2}$$

$$w_t = \beta A_G\, (s_{Gt}\, k_t)^{1-\beta}\, (l_{Gt}\, h_t)^{\beta-1} \tag{3}$$

2.2. The Consumer Problem

The consumer's lifetime utility function is given by:

$$U = \int_0^{\infty} e^{-\rho t} \left[\frac{c_t^{\,1-\theta}\, x_t^{\,\alpha(1-\theta)}}{(1 - \theta)} \right] dt \tag{4}$$

2.3. Income and Human Capital Constraints

The nominal financial capital stock is:

$$Q_t = M_t + P_t\, k_t \tag{5}$$

The money supply progresses through the government transfer V_t:

$$\dot{M}_t = V_t \tag{6}$$

With goods production defined by (1) and assumption that output of goods can be costlessly converted into physical capital, goods output is divided between consumption of goods and investment:

$$c_t + \dot{k}_t + \delta_k\, k_t = y_t = A_G\, (s_{Gt}\, k_t)^{1-\beta}\, (l_{Gt}\, h_t)^{\beta} \tag{7}$$

The nominal capital and labour income from goods production is the nominal value of marginal products factored by the effective capital and effective labour used in output production.

The change over time in financial capital equals the income from capital rental plus labour income, government transfer and the change of the nominal value of physical capital, minus consumption expenditures and depreciation:

$$\dot{Q}_t = \dot{M}_t + P_t\,\dot{k}_t + \dot{P}_t\,k_t = r_t\,P_t\,s_{Gt}\,k_t + w_t\,P_t\,l_{Gt}\,h_t + V_t - P_t\,c_t - \delta_k\,P_t\,k_t + \dot{P}_t\,k_t \qquad (8)$$

Human capital is produced by Cobb-Douglas fashion CRS function, with capital not used in goods production (1-s_{Gt}) and time not used in leisure, credit services production, or goods production. The human capital flow constraint is given by:

$$\dot{h}_t = A_H\,([1 - s_{Gt}]\,k_t)^{1-\delta}\,([1 - x_t - l_{Ft} - l_{Gt}]\,h_t)^{\delta} - \delta_h\,h_t \qquad (9)$$

2.4. Exchange Technology

Money and credit are perfect substitutes in purchasing the consumption goods. This can be expressed by equating the sum of real money balances and total real credit to the aggregate consumption:

$$(M_t / P_t) + d_t = c_t \qquad (10)$$

Define by $a_t \in (0,1)$ the fraction of purchases made with cash, so that:

$$(M_t / P_t) + d_t = (a_t \,/\, c_t) + d_t = c_t \qquad (11)$$

This makes the so-called cash-in-advance, “Clower constraint”,a part of the description of the perfect substitutability of money and credit:

$$M_t = a_t\,P_t\,c_t \qquad (12)$$

From equation (11), it is clear that the share of purchases made by credit is given by (1-a_t). Or the total amount of credit used can be expressed as:

$$d_t = (1 - a_t)\,c_t \qquad (13)$$

Consider specifying the production of this credit using an effective-labor only technology, with diminishing returns, and dependent on the level of consumption c_t, so the credit production function is Cobb-Douglas in $l_{Ft}h_t$ and c_t:

$$d_t = A_F \,(l_{Ft}\, h_t)^{\gamma}\, c_t^{\,1-\gamma} \tag{14}$$

Which can be written using equation (13) as:

$$(1 - a_t)\, c_t = A_F \,(l_{Ft}\, h_t)^{\gamma}\, c_t^{\,1-\gamma} \tag{15}$$

The rationale of the introduction of c_t into the total productivity factor is that the credit supplier, which in a decentralized framework can be thought of as a hypothetical firm similar to American Express, would maximize its profits while taking as given how much is spent on goods for consumption. American Express would not try to change this goods expenditure but must consider it in making its optimal credit supply available to the consumer. By making its inputs grow as the consumption of goods grows, it can maintain its share of supplying credit. This simply means that if the aggregate consumption increases, and the credit sector does not increase its effective labor proportionally, then it will lose its share of output for which it provides the service.[1]

Solving for a_t from equation (15), and substituting into equation (12):

$$M_t = [1 - A_F \,(l_{Ft}\, h_t / c_t)^{\gamma}]\, P_t\, c_t \tag{16}$$

This is the CIA constraint that enters the consumer maximization problem.

Note that equation (16) can be solved for l_{Ft}, the total time devoted by the consumer in the role as credit producer. This “banking time” solution of the constraint presents an exchange constraint that can be viewed as being equivalent to a special case of the shopping-time economy. Here the time spent in exchange activity is only that time that enters into the credit production function.[2] The advantage of this over the shopping time models is that those models are typically calibrated so as to yield a constant interest elasticity of money. Here the interest elasticity rises in magnitude as the inflation rate goes up, as consistent with evidence (see Mark and Sul, 2002).

[1] Gillman and Yerokhin (2003) detail how this model is equivalent to an interpretation of an Beckerian household production economy with a production of exchange using the intermediate goods of money and credit; the exchange is itself also and intermediate good that is then combined with the goods output to yield the Beckerian household consumption good.

[2] See Gillman and Yerokhin (2003) for proof of the shopping-time/banking-time equivalence and for further discussion.

2.5. Balanced Growth Path Equilibrium

The agent maximizes his lifetime utility defined in equation (4) subject to the stock constraints (5) and (16), and the flow constraints (8) and (9), with respect to the control variables $c_t, x_t, s_{Gt}, l_{Ft}, l_{Gt}$, and stock variables Q_t, M_t, k_t, h_t.
The present value Hamiltonian for this problem is:

$$\begin{aligned} \mathbb{H} = {} & e^{-\rho t} c_t^{1-\theta} x_t^{\alpha(1-\theta)} \Big/ (1-\theta) \\ & + \phi_t \left(M_t - [1 - A_F (l_{Ft} h_t / c_t)^{\gamma}] P_t c_t \right) \\ & + \varphi_t (Q_t - M_t - P_t k_t) \\ & + \lambda_t [r_t P_t s_{Gt} k_t + w_t P_t l_{Gt} h_t + V_t - P_t c_t - \delta_k P_t k_t + \dot{P}_t k_t] \\ & + \mu_t [A_H ([1 - s_{Gt}] k_t)^{1-\delta} ([1 - x_t - l_{Ft} - l_{Gt}] h_t)^{\delta} - \delta_h h_t] \end{aligned} \qquad (17)$$

First-order equilibrium conditions that characterize the balanced growth path behavior (independent on time), can be written as:

$$-\frac{\dot{\lambda}}{\lambda} \equiv R = r + \frac{\dot{P}}{P} - \delta_k \qquad (18)$$

$$R = \left(\frac{w}{A_F \; \gamma (l_{Ft} h_t / c_t)^{\gamma-1}} \right) \qquad (19)$$

$$\frac{u_c}{u_x} = \frac{x}{\alpha c} = \frac{1 + aR + w l_F \; h/c}{wh} \qquad (20)$$

$$\frac{w}{r} = \left(\frac{s_G k}{l_G h} \right) \left(\frac{\beta}{(1-\beta)} \right) = \left(\frac{(1-s_G) k}{(1 - l_G - l_F - x) h} \right) \left(\frac{\delta}{(1-\delta)} \right) \qquad (21)$$

$$\begin{aligned} g \equiv {} & \dot{c} \big/ c = \dot{k} \big/ k = \dot{h} \big/ h = [r - \rho - \delta_k] / \theta \\ & = [(1-x) A_H \, \delta \left([(1 - s_{Gt}) k_t] / [(1 - x_t - l_{Ft} - l_{Gt}) h_t] \right)^{1-\delta} - \rho - \delta_h] / \theta \end{aligned} \qquad (22)$$

Equation (18) sets the nominal interest rate R equal to the sum of the marginal product of effective capital in goods production and inflation, minus physical capital depreciation.

In this model, consumer optimally chooses between two exchange mechanisms, money and credit, according to the cost of each relative to the other. In equilibrium, the marginal cost of money, nominal interest rate R, has to be equal to the marginal cost of credit. Gillman and Kejak (2000) show that the nominal interest rate, R, represents the marginal cost of credit services. Equation (19) shows this with the marginal cost of credit set equal to the marginal factor cost of effective labor in the credit sector, w, divided by the marginal product of labor in the credit sector. This is a standard input price condition for market equilibrium.

The marginal rate of substitution of goods consumption relative to the leisure is given by equation (20). It sets this rate equal to the ratio of the shadow price of the goods consumption to the shadow price of the leisure. The shadow price of goods consumption is one, the goods cost, plus the exchange cost of $aR + wl_F\, h/c$ per unit. If only money were used in exchange, this would be just the nominal interest rate R. With credit also used, this exchange cost is less than R and, using equation (19), can be expressed as a weighted average of money and used credit costs, $1+aR+(1-a)\gamma R$. The shadow price of leisure is wh, lost real wage per unit of raw labour.

In equlibrium, the ratio of the return on human capital to the return on physical capital has to be equal in both goods and human capital production sectors, as states equation (21). Equation (21) also implies that the effective capital to the effective labour ratios in both production sectors are constantly related by Cobb-Douglas coefficients of these sectors.

Finally equation (22) sets the growth rate of the economy, same growth for consumption, physical and human capital, equal to the marginal product of effective capital minus capital depreciation and rate of time preference all divided by intertemporal elasticity θ. It also implies relation:

$$r - \delta_k = (1-x)\, A_H\, \delta \left([(1-s_{Gt})\, k_t] / [(1-x_t - l_{Ft} - l_{Gt})\, h_t] \right)^{1-\delta} - \delta_h \qquad (23)$$

This says that return on all capital net of capital depreciation is equal to the return on all employed labour net of human capital depreciation.

2.6. Model Predicted Inflation Effects

Equation (18) implies that an increase in inflation makes nominal interest rate to increase.

From equation (19), a rise of the nominal interest rate causes the marginal costs of credit services to increase and the quantity of real money demanded to decrease. Rise of the marginal costs of credit sector is achieved through real wage increase and increase of time devoted into credit production. Agent devotes more time into credit sector to lessen inflation tax impact on consumption.

Other implications of higher nominal interetest rate result from equation (20). It implies that if nominal interetest rate, R, rises as a result of higher inflation, and real wage, w, also rises but by less than R (equation (19)), leisure, x, has to rise relative to the normalized consumption, c/h. There are second-order effects of opposite direction, through the decrease of money purchased fraction of goods a, but rise of R overcomes these. The relative increase of time devoted to leisure, x, to the normalized consumption, c/h, gets smaller as inflation rises (the second-order effects rise in magnitude relative to the increase of R).

In taxing goods consumption relative to leisure, inflation reduces the return on both the physical and human capital used in goods production. This is reflected in a lower real interest rate r (the marginal product of physical capital in goods production).[3] Thus the input price ratio, of the real wage to the real interest rate, rises and from equation (21), the capital to effective labor ratio rises across both sectors. The relative Cobb-Douglas coefficients of production β and δ matter because they determine whether there will be an additional effect to the size of output in these sectors due to their relative capital intensities. I consider the case when $\beta < \delta$, human capital production sector is less capital intensive, to be more realistic. In this case, the goods production sector expands relative to the human capital production sector as inflation rises. The total effective physical capital to the effective labor employed in production sectors ratio, $k/(1-x-l_F)h$, rises. Including the labor in the credit services sector l_F , the ratio $k/(1-x)h$ should also rise but by less.

From equation (22), inflation has negative effect on the growth rate through lowering the real interest rate, or equally, through increasing the time devoted into leisure.

[3] Long run evidence presented in Ahmed and Rogers (2000) supports a decrease in the real interest rate as a result of an increase in the inflation rate.

3. Model Calibrations

The strategy in the calibration is to fit the evidence as well as possible, while using standard parameter values based on US data for the commonly calibrated parameters.

The baseline calibration starts with setting a growth rate of 2%, as in Chari et al (1996), for inflation rate of 5%, and a rate of time preference of ρ=4%. Next the value for leisure is set at x=0.7, similar to the 0.69 in Jones et al (1993). The utility parameter for leisure, α, is set at 5.289, within the range of estimates in the literature. The depreciation rates of both physical and human capital are δ_k=δ_h=0.1, as in King and Rebelo (1990). The Cobb-Douglas parameters for the effective labor intensity in the goods and human capital sectors are set at β = 0.64 and δ = 0.8. The intertemporal elasticity of substitution θ is set equal to 1.5. This ranges usually between 1 and 2 in the literature. The shift parameters of the sectoral production functions are given at A_G = 1, A_H = 0.659, and A_F = 0.801.

The hours used in banking l_F, are set to 0.0009 for the baseline inflation rate. This is less than the aggregate hours in the US Finance sector, but that sector includes more than only the time spent offering exchange credit as a means to avoid the inflation tax, such as the time used for supplying intertemporal credit. In a related McCallum and Goodfriend (1987) economy, Dotsey and Ireland (1994) use a value of 0.0028 at a 4% inflation rate for a value analogous to the labor time in the exchange credit sector. For other parameters relating to the exchange technology, the share of purchases made with cash at the baseline, we have that a=0.7. Money in the model is non-interest bearing money, which could be measured as currency plus non-interest bearing demand deposits.. The degree of diminishing returns in the credit services sector is set at γ = 0.2. Values of 0.21 and 0.265 are found by Gillman and Otto (2002) when estimating money demand for the US and Australia using the last quarter of the century quarterly data, based on the same money demand model as in the economy of this model.

With the addition of the still-novel credit sector, parameters of this sector are not given by well-established previous calibration work. Therefore, model's results for the different technology parametersof the credit services sector, γ, are reported.

4. Steady-state Comparisons

4.1 Comovements of Selected Variables with Inflation

Baseline calibrated model is used to examine the steady-state values of variables for inflation levels ranging from 0% to 55%.

With increasing inflation, *k/h* ratio initially rises, but with further inflation rise, it decreases. The situation is shown in Figure 1 for γ=0.2. The peak of *k/h* ratio is achieved at approximately 19% inflation. For γ=0.5 situation looks similar, the peak is achieved at 32% (see Figure A1 in Appendix). Nonlinearity of steady-state *k/h* ratio, as a function of inflation, is a result of different capital intensities in human capital and goods production sector, and of form of credit sector production function. Human capital production sector used here, is with lower physical capital intensity than goods producing sector, which makes *k/h* ratio to rise as inflation rises, as in Einarsson and Marquis (1999). Form of credit sector used, using human capital only, forces k/h ratio in opposite direction. With rising inflation is credit sector expanding, and its need of human capital overcomes the effect of realignment in goods production and human capital production sectors. Here the rate of diminishing returns in credit sector production, γ, determines the strenght of this effect. Except for very low γ values, the higher γ is, the higher inflation is the *k/h* peak achieved at.

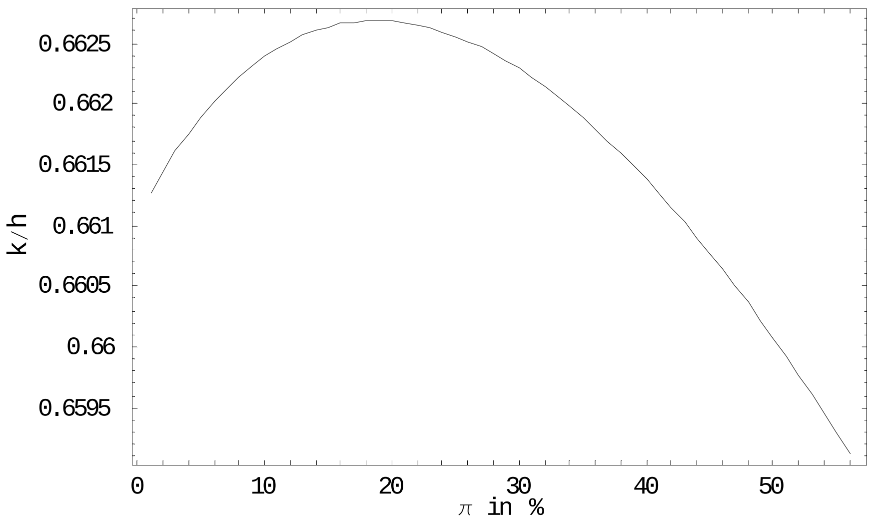

Figure 1 - The k/h ratio for γ=0.2.

Leisure time x is increasing as inflation rises. The magnitude of this increase is slightly getting smaller (see Figure A2 in Appendix). For higher γ values, is x rising by more. Since inflation represents tax on consumption, agent realigns output that enters utility function from consumption towards leisure (result of equation (20)).

The normalized consumption c/h is falling approximately linearly for all γ values (see Figure A3 in Appendix). As mentioned before, inflation represents tax on consumption and agent realigns output that enters utility function from consumption towards leisure.

Time devoted into credit production l_F, is increasing with inflation increase. For γ=0.2 this is approximately linear (see Figure A4 in Appendix). For higher γ values, l_F is rising less for lower inflation rates and the speed of its rise increases (for γ=0.5 see Figure A5 in Appendix). Time in credit services rises with rising inflation, as agent economizes his money holdings. The nonlineaity is caused by higher effectivity of effective labour in credit production for lower l_F values and higher rate of diminishing returns, affecting as l_F increases.

The value of l_G, time devoted to goods production is decreasing as inflation increases. For all γ values this decrease is almost linear (see Figure A6 in Appendix). As inflation increases it makes consumption level and goods production to decrease. Lower goods production requires less labour invested.

Time devoted into human capital production l_H, is decreasing with inflation increase. The magnitude of this decrease is slightly getting smaller (see Figure A7 in Appendix). For higher γ values, is l_H falling by more. Since inflation causes the output to shrink, the growth of economy is decreasing, there is less human capital growth, and less labour in this sector necessary.

The steady-state growth of the economy is decreasing with inflation increase (see Figure A8 in Appendix). Higher γ values tend to make the magnitude of this decrease higher. For γ=0.2 the growth of the economy becomes negative at inflation rate of 167%. The growth of the economy is falling since agent realigns resources from production sectors into inflation tax avoidance, credit sector, and towards leisure.

As was theoreticly predicted, real interest rate[4] is decreasing and real wage is increasing (see Figures A9 and A10 in Appendix).

The numerical values of these and more variables for selected inflation rates are shown in Table A1 (for γ=0.2) and Table A2 (for γ=0.5) in Appendix.

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[4] real interest rate here means marginal product of goods producing function, to get the market real interest rate, depreciation of physical capital, δ_k, has to be subtracted.

4.2 Welfare Costs Based on Steady-state Comparisons

The welfare costs of inflation change are computed using the method of Cooley and Hansen (1991). The lifetime utility under two different inflation levels is calculated. These numbers are converted into consumption levels given for the leisure value in the new inflation level steady-state. The consumption loss is expressed and normalized by dividing by output level given for new inflation level steady-state. Thus are welfare costs expressed as a consumption loss in a percentage of GDP.

Welfare costs for inflation changes among inflation levels of 0%, 5%, 15%, 25%, 35%, 45% and 55% are shown in Table 1 (for γ=0.2). The results for γ=0.5 are shown in Table A4 in Appendix.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs	0,3472310	0,9030920	1,0792800	1,1852200	1,2481200	1,2826300

Table 1 - Welfare costs of inflation change for γ=0.2.

Welfare costs of 10% increase of inflation are about 1% (for γ=0.5 it's mostly between 1% and 2%) of GDP. These are rising as initial (before change) inflation rises, with peak of 1.299% for inflation change 65%->75% (for γ=0.5 it's 2.11% for 55%->65%) and decreasing as inflation rises further. These results are in line with Dotsey and Ireland (1996), who report 0.9% welfare costs for change of inflation 0%->10% with money defined as currency. Welfare costs here are also higher then estimates in Aiyagari, Braun and Eckstein (1998), but they use exogenous growth model, where growth is not affected by inflation. Einarsson and Marquis (1999) found also lower welfare costs in endogenous growth model, but without credit sector and different calibrated.

Welfare costs can be decomposed into two components. The first is represented by resources invested into credit sector. This is a nonproductive sector and resources invested here are social vaste. This part of welfare costs of inflation therefore equals costs of credit services. Table 2 shows change in costs of credit services as percentage of GDP (Table A5 in Appendix for γ=0.5). The second component of welfare costs of inflation is a result of realignments in production sectors and rise of leisure time, both decreasing growth rate of the economy, this way also future consumption. The utility function shows inefficient rate of substitution from goods consumption to leisure which enables this component to arise.

Comparing Tables 1 and 2 (resp. A4 and A5) shows that this effects are of almost the same magnitude, forming each almost half of total welfare costs.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs/GDP	0,2011110	0,4608950	0,5144310	0,5532200	0,5836700	0,6086500

Table 2 – Change in costs of credit sector as percentage of GDP.

4.3 Comparison to Cash only Economy

This is a special case of model used before, l_F is fixed at 0. Behavior of selected variables is shown in Table A3 in Appendix. This is similar to previous model except for *k/h* ratio is only increasing, as in Einarsson and Marquis (1999), who use similar cash only model, although with different calibration. This is because there is no force affecting in opposite direction. There is no inflation tax avoidance mechanism, so inflation taxes goods consumption more than in previous model. Inflation induced realignments from consumption to leisure are therefore stronger and growth rate falls faster (zero growth is achieved by approximately 53% inflation).

Welfare costs are computed the same way as for the previous model. Results are shown in Table 3. These are higher than in model using credit sector and increase faster. The peak of about 2.5% is achieved for inflation change 115%->125% and welfare costs are decreasing as inflation rises further.

Here, welfare costs consist only of one component, caused by realignments in production sectors and rise of leisure time.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
WelfCost%	0,2671010	0,9437970	1,3796300	1,7114400	1,9631600	2,1527600

Table 3 - Welfare costs of inflation change for cash only economy.

5. Including Transition Path

For modeling transitional behavior of the model, the Brunner and Strulik (2002) introduced method of backward integration is used. Backward integration consists of two central ideas. The first idea has been introduced with the method of time elimination (Mulligan and Sala-i-Martin, 1991, 1993). It is the transformation of an inherently unstable boundary value problem into an inherently stable initial value problem, which can be solved easily using standard numerical methods. In contrast to Mulligan and Sala-i-Martin backward integration method doesn’t eliminate but reverse time and exploit the numerical stability of the backward looking system. The second idea is that an approximation of the infinite time horizon is endogenously determined. The time horizon depends on the initial deviation of the backward looking system from its steady-state and is derived by the ordinary differential equation solution algorithm.

Inflation change is achieved with change of government money supply. At time 0 government decides to change inflation rate and changes money supply accordingly. I am assuming, that is keeps new inflation from this point, which means money supply is changing during transition period. Discretisation of time into 1/50 of year is used here. Since first 1/50 of year new money supply rule is imposed, and so new inflation rate is given.

Behaviour of *k/h* in transition has to be continous. It is the only stock variable in this model. Control variables can behave uncontinously, since decisions can be changed anytime and with any magnitude. But stock variables have to change continously (capital can not appear or disappear). Figure 1 (and for γ=0.5 Figure A1 in Appendix) shows that for some inflation changes, *k/h* has to rise, for some it stays unchanged and for some it falls.

Behaviour of control variables depends on the necessary change of *k/h* ratio. If *k/h* stays unchanged control variables change by shock into new steady state values. There is no transition necessary, since transition is present only because of necessity of continous change of *k/h* ratio.

If *k/h* has to rise, l_G falls by a shock to a value higher than new steady-state and transitionally decreases to this value (see Figure A11 in Appendix). This is because buildup of *k* relative to *h* is necessary. Time devoted into human capital production, l_H, falls by a shock to a value lower than new steady-state and transitionally rises to new steady-state value (see Figure A12 in Appendix). The reason for this is again necessarity of buildup of *k* relative to *h*.

Time devoted into credit services, l_F, rises by a shock to a value higher than new steady-state (but almost identical) and transitionally decreases to this value (see Figures A13 in Appendix). Normalized consumption, *c/h*, falls by a shock to a value lower than new steady-state and transitionally increases to new value (see Figure A15 in Appendix). This is also a result of necessary buildup of *k* relative to *h*. Agent consumes temporarily less to help to accumulate more capital. Leisure, *x*, rises by a shock to a value higher than new steady-state and transitionally decreases to it's value (see Figure A14 in Appendix). This happens to substitute the lost utility caused by temporary low consumption.

If *k/h* falls, l_G falls by a shock to a value lower than new steady-state and transitionally increases to this value (see Figure A16 in Appendix). The reason for this is necessary decrease of *k* relative to *h*. Time devoted into human capital production, l_H, falls by a shock to a value higher than new steady-state and transitionally decreases to new steady-state value (see Figure A17 in Appendix). This is because the buildup of *h* relative to *k* is necessary. Time devoted into credit services, l_F, rises by a shock to a value lower than new steady-state (but almost identical) and transitionally increases to it's new value (see Figure A18 in Appendix). Leisure, *x*, rises by a shock to a value lower than new steady-state and transitionally increases to this value (see Figure A19 in Appendix). Normalized consumption, *c/h*, falls by a shock to a value higher than new steady-state and transitionally decreases to new steady-state value (see Figure A20 in Appendix). Agent consumes temporarily more to help to lower *k* relative to *h*.

Welfare costs are computed the same way as in previous section. Results are shown in Table 4 (Table A6 in Appendix for γ=0.5). Differences against values computed without considering transition are shown in Table 5 (Table A7 in Appendix for γ=0.5).

change	**0% -> 5%**	**5% -> 15%**	**15% -> 25%**	**25% -> 35%**	**35% -> 45%**	**45% -> 55%**
Costs	0,3519360	0,9073740	1,0783500	1,1799100	1,2391200	1,2706500

Table 4 - Welfare costs of inflation change including transition.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs	0,0047050	0,0042820	-0,0009300	-0,0053100	-0,0090000	-0,0119800

Table 5 – Difference of welfare costs of inflation change including and excluding transition.

If *k/h* ratio rises, welfare costs with transition are higher than without considering transition period, as in Einarsson and Marquis (1999). The difference is much lower than that found by Einarsson and Marquis which is the result of credit sector used, since this makes *k/h* ratio rise much less and so there is less transition necessary.
If *k/h* ratio falls welfare costs with transition are lower than without considering transition period as in Aiyagari, Braun and Eckstein (1998). The difference is again much lower than that found by Aiyagari, Braun and Eckstein.

6. Conclusions

The presented model of economy appears to be a good extension of the standard Lucas (1988) endogenous growth model. Introduced credit sector helps in modeling effects of inflation more realistically, since it gives reasonable possibility of inflation tax avoidance mechanism.
All variables react to inflation consistent with theoretical expectations.
Transition dynamics, used to observe the changes in this model of the economy, help to model the impact of inflation on the economy more precisely, despite of the fact that for this calibration of presented model is the difference between welfare costs with taking transition period into account and without it very small. The method of construction of transitional dynamics appears to be proper, since all variables behave in the simply economic explainable way.
The form of credit producing function remains still an open issue. Higher rate of diminishing returns in credit producing function tends to make the magnitude of the inflation induced changes higher. Including physical capital into credit producing function would probably change the behavior of *k/h* ratio, which heavily affects inflation induced changes in the economy of this model. The need of physical capital would probably make *k/h* rise faster and probably not decrease at all (if,than much slower). This would make the difference in welfare costs computed including and excluding transition path higher.

References

Ahmed S., and J.H.Rogers, 2000, "Inflation and the Great Ratios: Long term evidence from the US," *Journal of Monetary Economics*, 45 (1) February: 3-36.

Aiyagari, S. Rao, R. Anton Braun, and Zvi Eckstein, 1998, "Transaction Services, Inflation, and Welfare," *Journal of Political Economy*, vol. 106, no. 6, 1274-1301.

Blackburn, Keith, and V.T.Y Hung, 1996, "Money, Inflation and Growth," *Seoul Journal of Economics*, Summer, 9(2): 145-61.

Brunner M., and H. Strulik, H., 2002, "Solution of Perfect Foresight Saddle-point Problems: A Simple Method and Applications", *Journal of Economic Dynamics and Control* 26/5: 737-753.

Chari, V.V., Larry E. Jones, and Rodolfo E. Manuelli, 1996, "Inflation, Growth, and Financial Intermediation", *Review*, Federal Reserve Bank of St. Louis, May/June.

Cooley, Thomas F.and Hansen, Gary D., 1991, "The Welfare Costs of Moderate Inflations", *Journal of Money, Credit, and Banking*, 23, 483-513.

Dotsey, Michael and Peter N. Ireland, 1996, The Welfare Cost of Inflation in General Equilibrium, *Journal of Monetary Economics*; 37(1), February 1996, pages 29-47.

Einarsson, Tor, and Milton Marquis, 1999, "Transitional and Steady-state Costs of Disinflation when Growth is Endogenous", *Economica*, vol. 66, issue 264: 489-508.

Gillman, M. and Yerokhin, 2003, "Ramsey-Friedman optimality with money as an intermeditate good". Manuscript, Central European University.

Gillman, M. and Michal Kejak, 2002, "Modeling the Effect of Inflation: Growth, Levels, and Tobin" in Proceedings of the 2002 North American Summer Meetings of the Econometric Society: Money, edited by David K. Levine, William Zame, Lawrence Ausubel, Pierre-Andre Chiappori, Bryan Ellickson, Ariel Rubinstein and Larry Samuelson, *http://www.dklevine.com/proceedings/money.htm*.

Gillman, Max, and Anton Nakov, 2002, "A Revised Tobin Effect from Inflation: Relative Input Price and Capital Ratio Realignments, US and UK, 1959-1999", *Economica*, 2003, vol. 70, issue 279, pages 439-450.

Gillman, Max, Mark Harris, and Laszlo Matyas, 2001, “Inflation and Growth: Some Theory and Evidence”, Central European University Department of Economics Working Paper, WP1/2001.

Gillman, Max, and Michal Kejak, 2000, “A Non-Linearity in the Inflation-Growth Effect”, Central European University Department of Economics Working Paper 14/2000.

Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi, 1993, “Optimal Taxation in Models of Endogenous Growth”, *Journal of Political Economy,* (June), 485-517.

King, Robert G., and S Rebelo, 1990, “Public Policy and Economic Growth: Deriving Neoclassical Implications”, *Journal of Political Economy*, 98: S126-S150.

Lucas, Robert E., Jr., 1988, “On the mechanics of economic development”, *Journal of Monetary Economics,* 22 (July):3-42.

Mark, N.C. and Sul, D., 2002 “Cointegration vector estimation by panel DOLS and long run money demand”, Technical Working Paper 287, National Bureau of Economic Research, Cambridge, MA.

Marty, Alvin L., 1999, “The Welfare Cost of Inflation: A Critique of Bailey and Lucas;” *Federal Reserve Bank of St. Louis Review,* Jan.-Feb. v. 81, iss. 1, pp. 41-46.

McCallum, Bennett T., and Marvin S. Goodfriend, 1987, “Demand for Money: Theoretical Studies”, in *New Palgrave Money*, J. Eatwell, M.Millgate and P.Newman, eds., Macmillan Press, New York.

Mulligan, C.B. and X. Sala-i-Martin, 1991, “A Note on the Time Elimination Method for Solving Recursive Dynamic Economic Models”, NBER Technical Working Paper No. 116, 1991.

Mulligan, C.B. and X. Sala-i-Martin, 1993, “Transitional Dynamics in Two Sector Models of Endogenous Growth”, *Quarterly Journal of Economics*, 108, 739-773.

van der Ploeg, Frederick and George Alogoskoufis, 1994, “Money and Endogenous Growth”, *Journal of Money, Credit, and Banking,* November, 26(4): 771-91.

Appendix

A.1 Figures

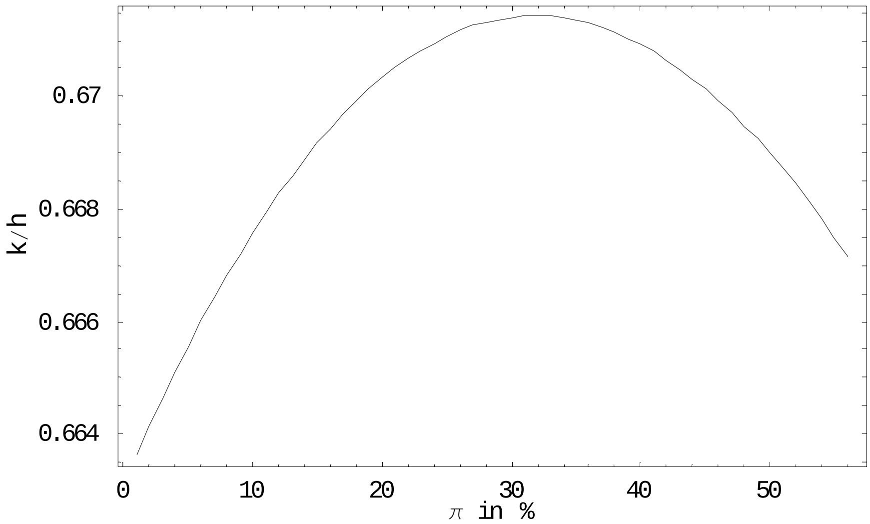

Figure A1 – k/h ratio for γ=0.5.

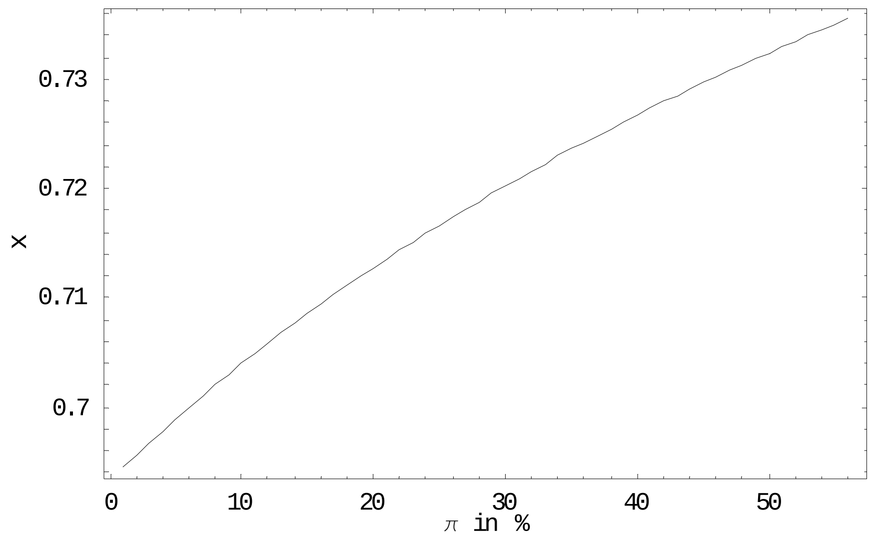

Figure A2 – x for γ=0.2.

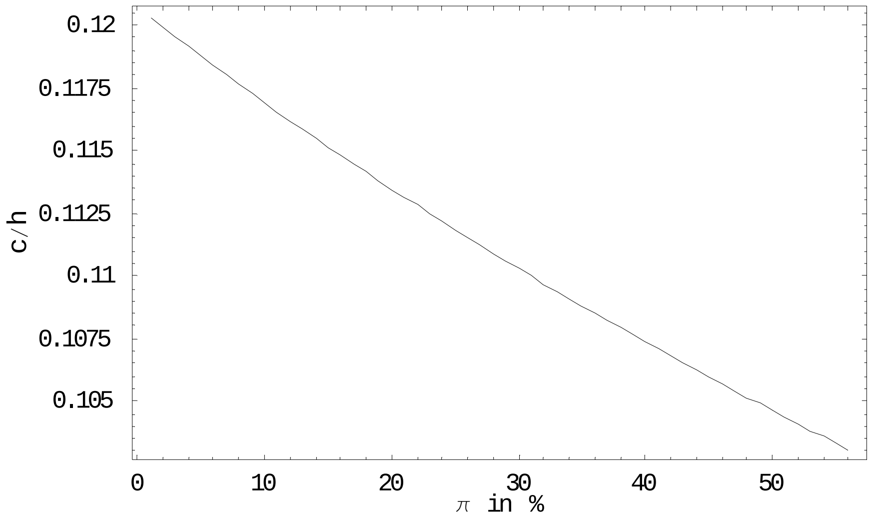

Figure A3 – c/h for γ=0.2.

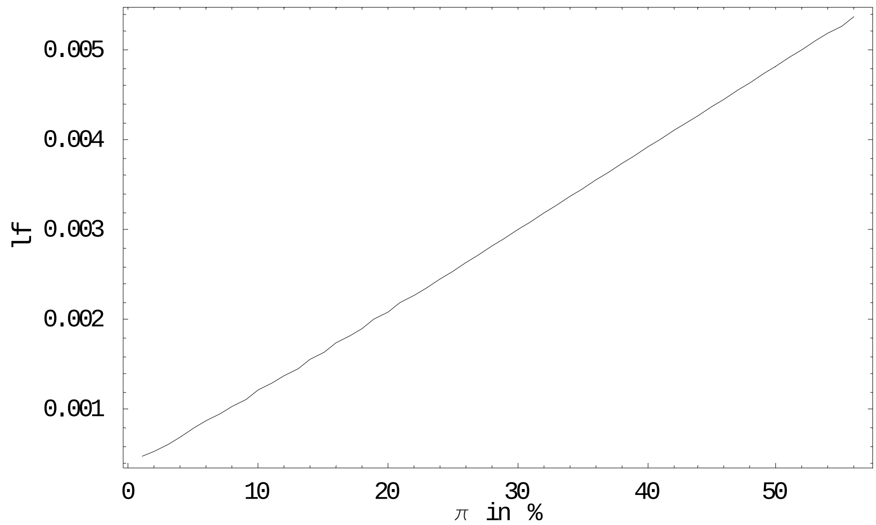

Figure A4 – l_F for γ=0.2.

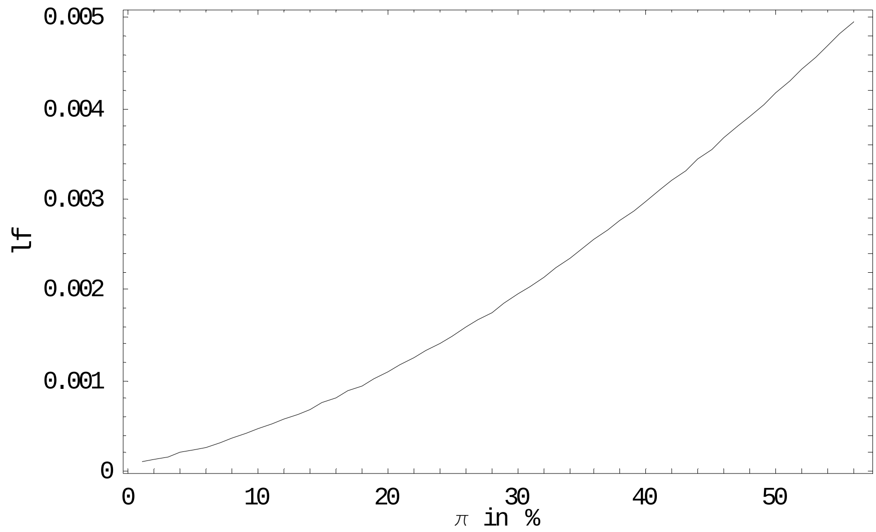

Figure A5 – l_F for γ=0.5.

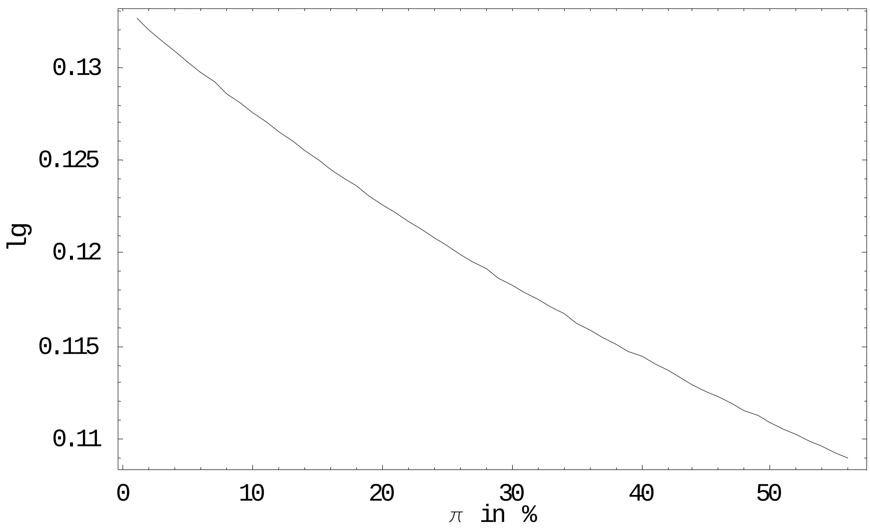

Figure A6 – l_G for γ=0.2.

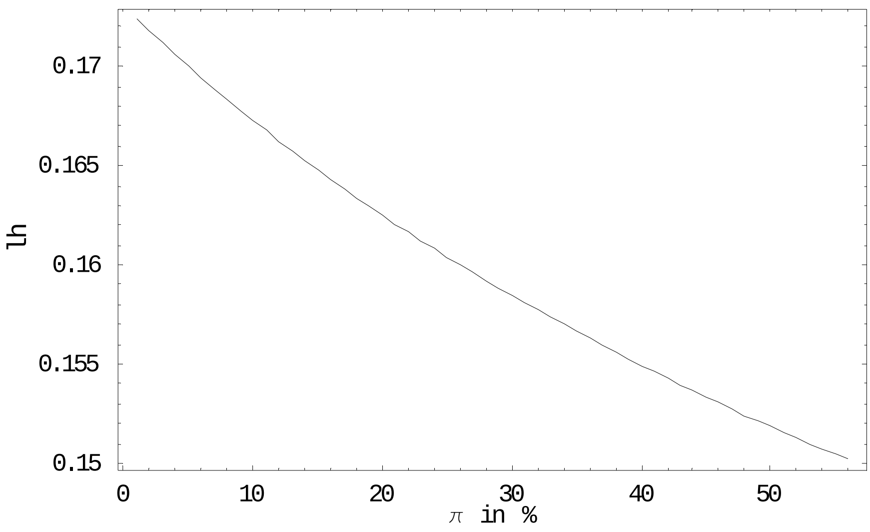

Figure A7 – l_H for γ=0.2.

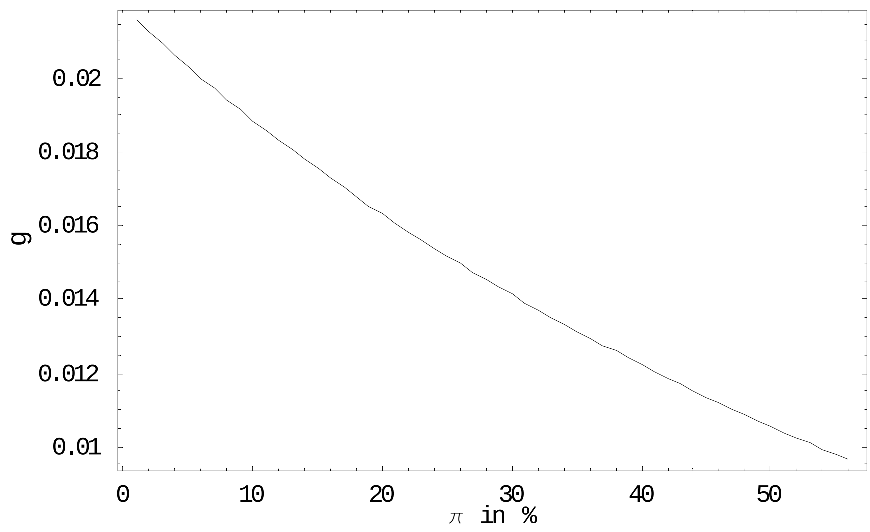

Figure A8 – g for γ=0.2.

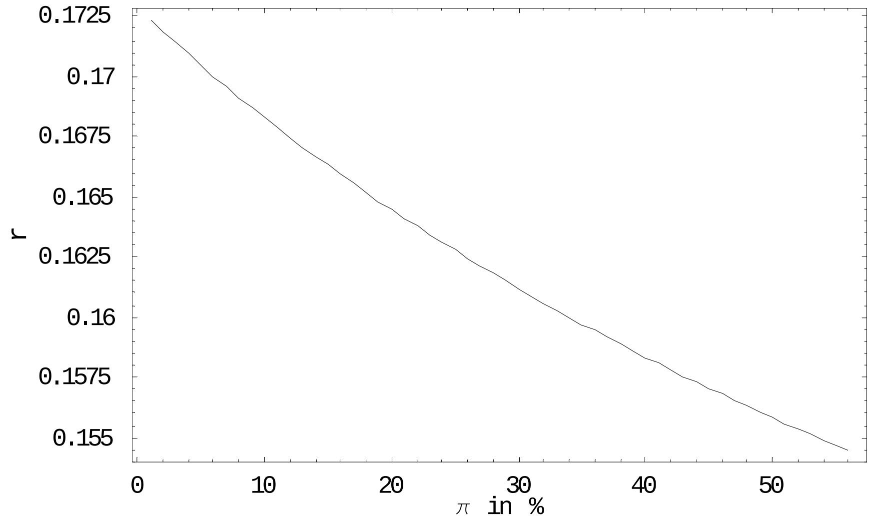

Figure A9 – r for γ=0.2.

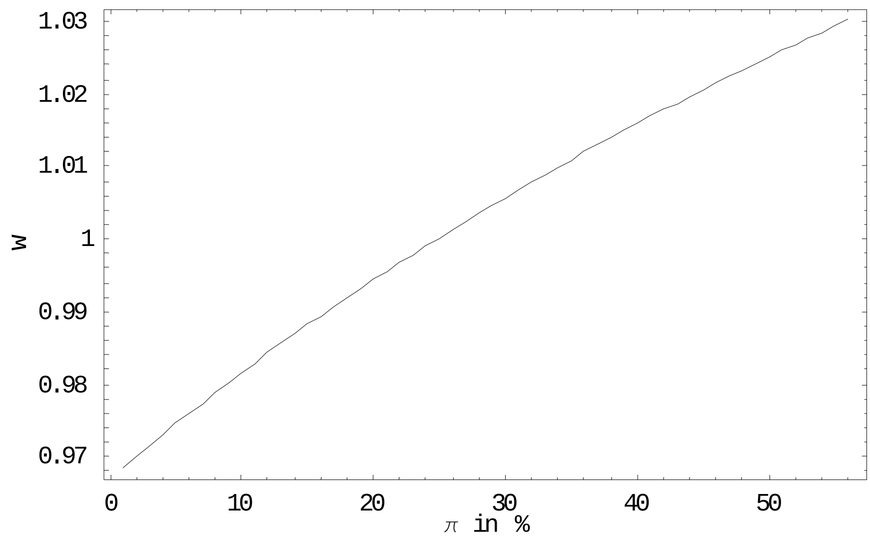

Figure A10 – w for γ=0.2.

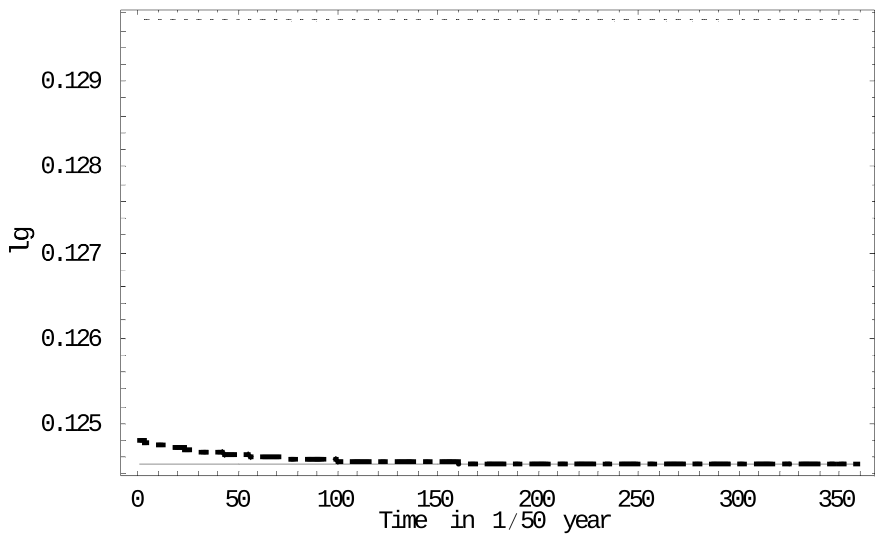

Figure A11 – lg for inflation change 5% -> 15%.

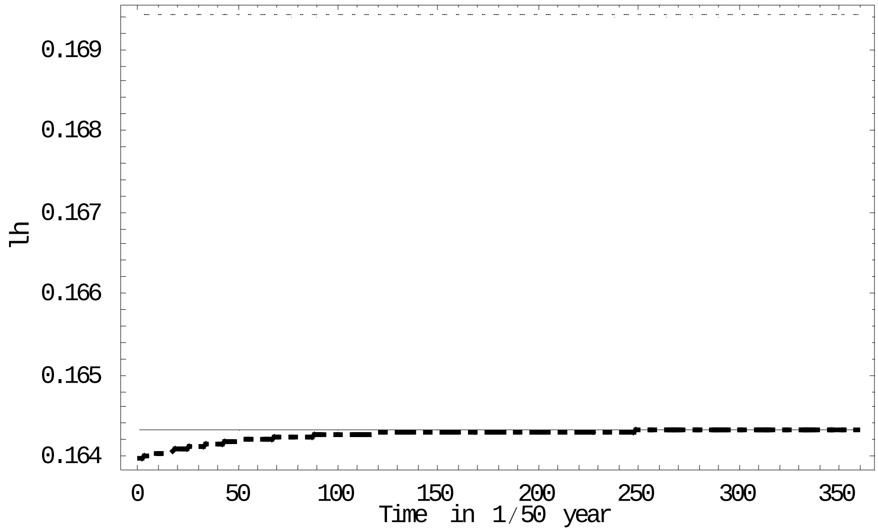

Figure A12 – lh for inflation change 5% -> 15%.

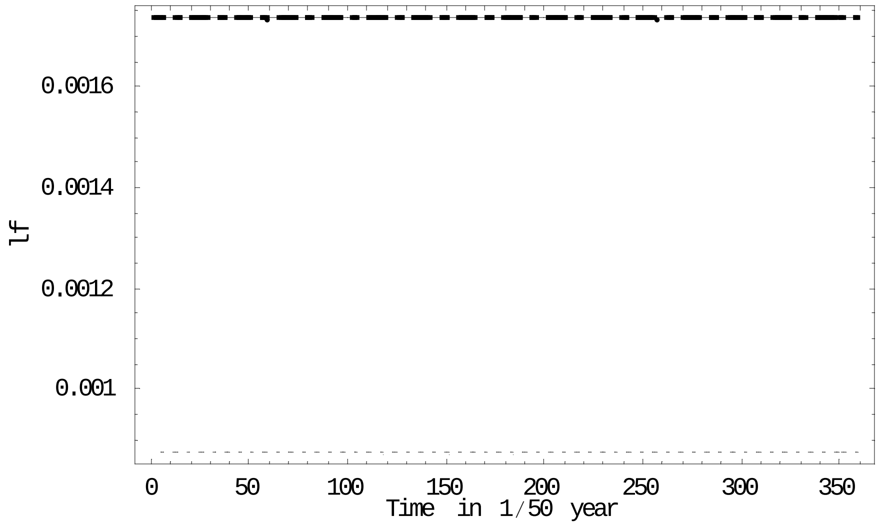

Figure A13 – lf for inflation change 5% -> 15%.

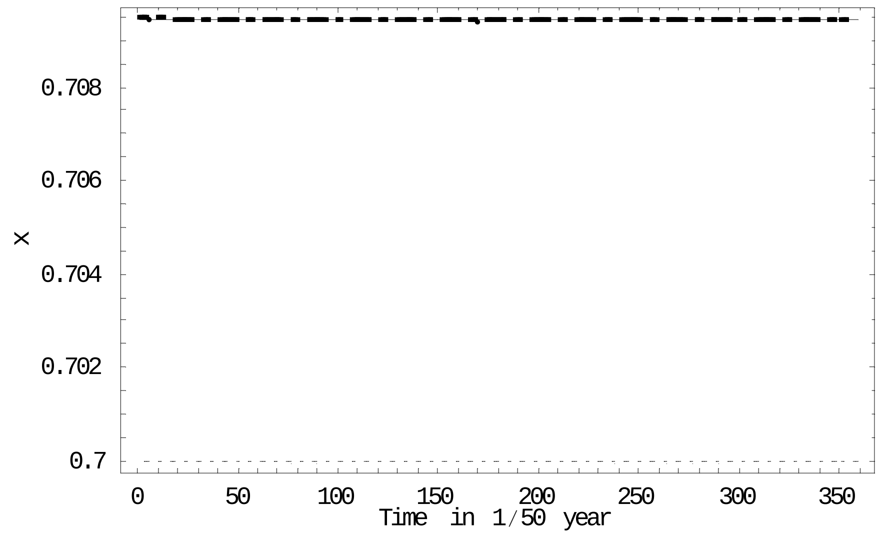

Figure A14 – x for inflation change 5% -> 15%.

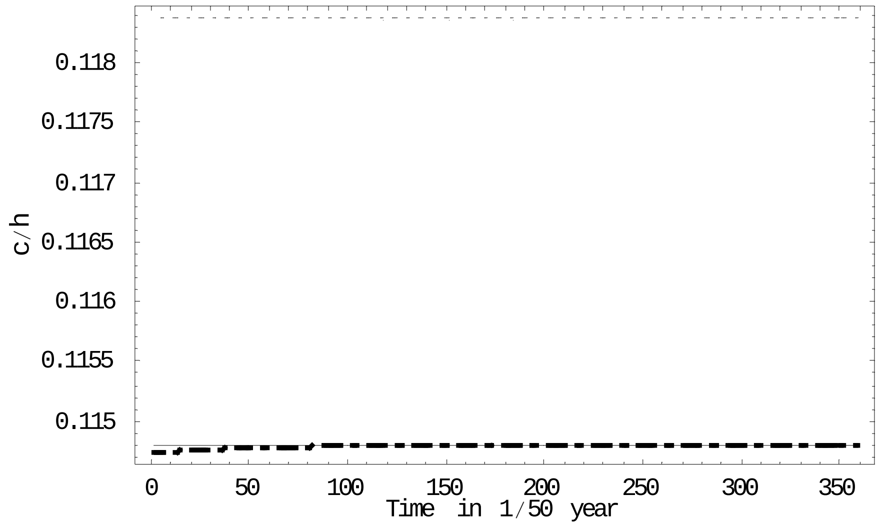

Figure A15 – c/h for inflation change 5% -> 15%.

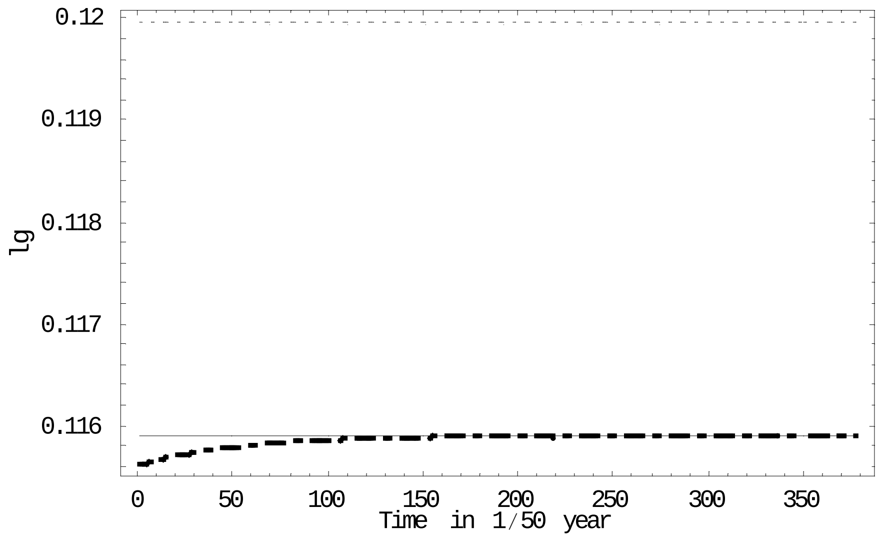

Figure A16 – lg for inflation change 25% -> 35%.

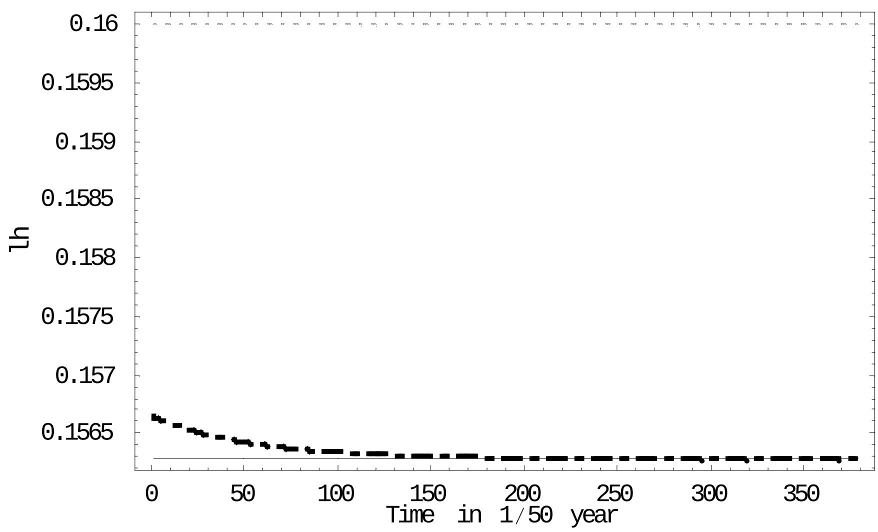

Figure A17 – lh for inflation change 25% -> 35%.

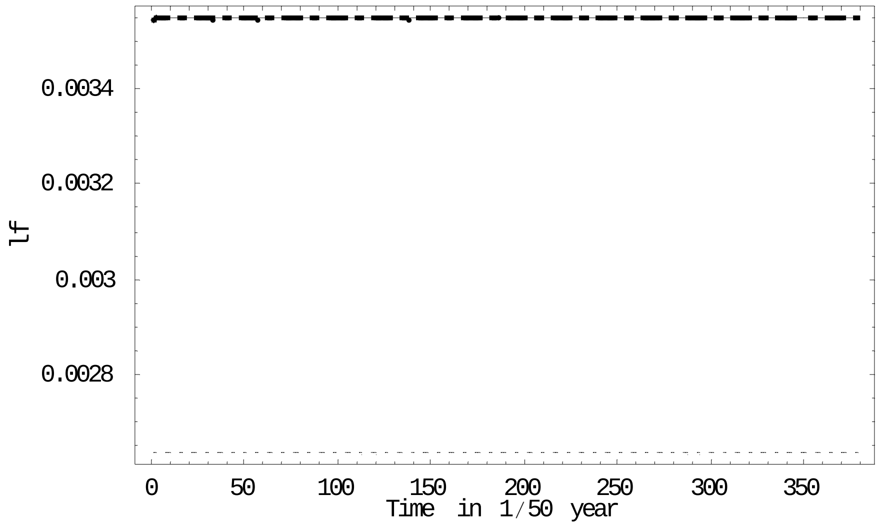

Figure A18 – lf for inflation change 25% -> 35%.

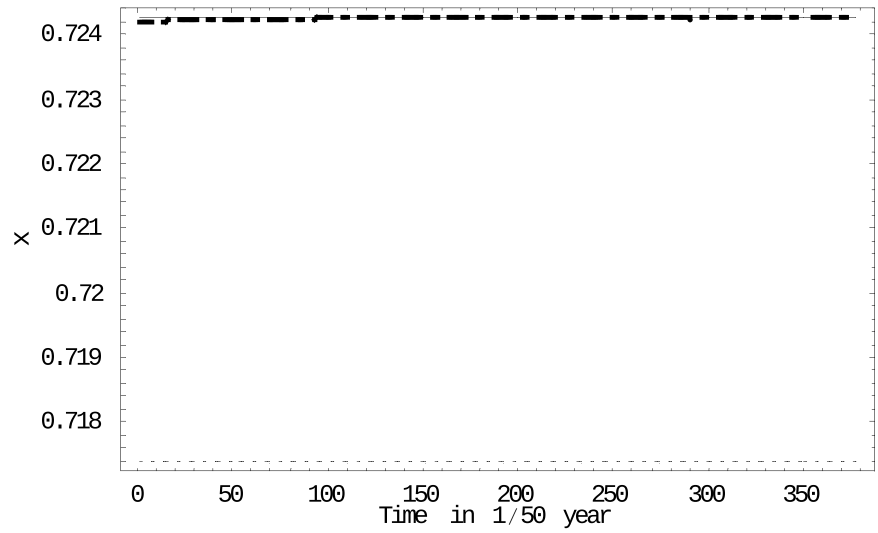

Figure A19 – x for inflation change 25% -> 35%.

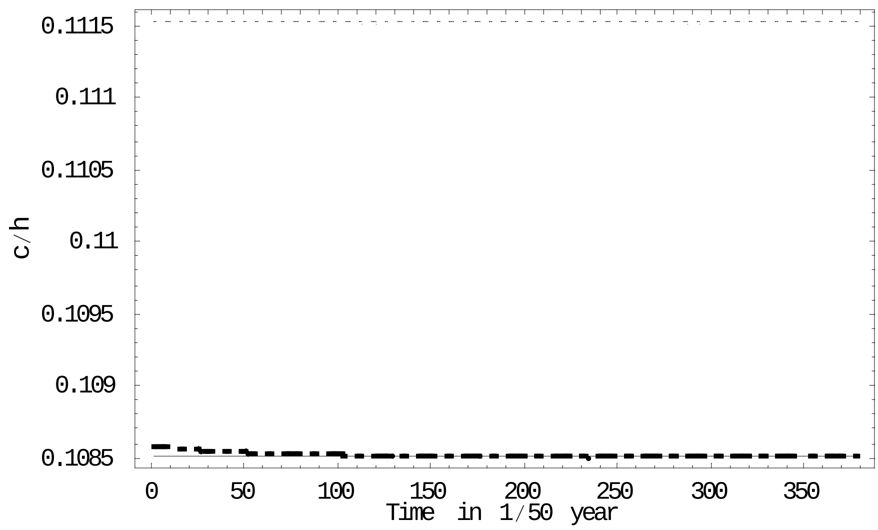

Figure A20 – c/h for inflation change 25% -> 35%.

A.2 Tables

inflation	0%	5%	15%	25%	35%	45%	55%
lg	0,132638	0,129721	0,124524	0,119967	0,115905	0,112245	0,108923
lh	0,172385	0,169418	0,164310	0,159999	0,156284	0,153041	0,150185
lf	0,000476	0,000874	0,001735	0,002636	0,003549	0,004460	0,005364
x	0,694501	0,699987	0,709430	0,717398	0,724262	0,730253	0,735528
k/h	0,661270	0,662015	0,662658	0,662514	0,661798	0,660640	0,659126
c/h	0,120305	0,118374	0,114800	0,111531	0,108509	0,105698	0,103070
g	0,021588	0,020012	0,017283	0,014965	0,012954	0,011190	0,009630
c/y	0,599404	0,598382	0,596306	0,594207	0,592098	0,589982	0,587864
r	0,172381	0,170018	0,165925	0,162447	0,159432	0,156786	0,154444
w	0,968443	0,975994	0,989464	1,001320	1,011930	1,021500	1,030190
k/(h(1-x))	2,164554	2,206619	2,280547	2,344335	2,400103	2,449110	2,492236
k/(h(lg+lh))	2,167936	2,213064	2,294249	2,366409	2,431395	2,490289	2,543833
R	0,072382	0,120018	0,215925	0,312447	0,409432	0,506786	0,604444

Table A1 – behavior of selected variables for γ=0.2.

inflation	0%	5%	15%	25%	35%	45%	55%
lg	0,131576	0,127856	0,121106	0,115155	0,109885	0,105198	0,101015
lh	0,170694	0,166510	0,159188	0,153075	0,147990	0,143790	0,140357
lf	0,000103	0,000270	0,000809	0,001580	0,002545	0,003679	0,004961
x	0,697628	0,705364	0,718898	0,730190	0,739579	0,747333	0,753667
k/h	0,663617	0,666005	0,669441	0,671172	0,671300	0,669932	0,667179
c/h	0,119887	0,117601	0,113242	0,109141	0,105266	0,101595	0,098105
g	0,020690	0,018461	0,014526	0,011209	0,008426	0,006109	0,004204
c/y	0,599498	0,598489	0,596292	0,593862	0,591209	0,588339	0,585256
r	0,171035	0,167691	0,161790	0,156814	0,152639	0,149163	0,146306
w	0,972723	0,983589	1,003610	1,021400	1,037020	1,050550	1,062040
k/(h(1-x))	2,194702	2,260435	2,381486	2,487574	2,577752	2,651449	2,708440
k/(h(lg+lh))	2,195446	2,262509	2,388358	2,502223	2,603195	2,690626	2,764105
R	0,071036	0,117691	0,211790	0,306814	0,402639	0,499163	0,596306

Table A2 – behavior of selected variables for γ=0.5.

inflation	0%	5%	15%	25%	35%	45%	55%
lg	0,131532	0,127743	0,120774	0,114520	0,108881	0,103774	0,099129
lh	0,170522	0,166058	0,157842	0,150460	0,143795	0,137751	0,132247
lf	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
x	0,697945	0,706199	0,721384	0,735020	0,747324	0,758475	0,768624
k/h	0,664065	0,667219	0,673280	0,679033	0,684503	0,689712	0,694679
c/h	0,119918	0,117683	0,113495	0,109644	0,106090	0,102799	0,099743
g	0,020599	0,018219	0,013799	0,009780	0,006112	0,002750	-0,000342
c/y	0,599579	0,598710	0,596985	0,595281	0,593597	0,591933	0,590289
r	0,170899	0,167329	0,160698	0,154670	0,149168	0,144125	0,139487
w	0,973161	0,984785	1,007440	1,029340	1,050530	1,071050	1,090940
k/(h(1-x))	2,198490	2,270990	2,416510	2,562580	2,709010	2,855660	3,002380
k/(h(lg+lh))	2,198490	2,270990	2,416510	2,562580	2,709010	2,855660	3,002380
R	0,070899	0,117329	0,210698	0,304670	0,399168	0,494125	0,589487

Table A3 – behavior of selected variables for cash only economy.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs	0,3390660	1,1249500	1,5519600	1,8276500	1,9941000	2,0820000

Table A4 - Welfare costs of inflation change for γ=0.5.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs/GDP	0,0853038	0,2922850	0,4504240	0,6045840	0,7557400	0,9048100

Table A5 - Change in costs of credit sector as percentage of GDP for γ=0.5.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs	0,3584040	1,1542000	1,5655200	1,8282500	1,9838400	2,0594800

Table A6 - Welfare costs of inflation change including transition for γ=0.5.

change	0% -> 5%	5% -> 15%	15% -> 25%	25% -> 35%	35% -> 45%	45% -> 55%
Costs	0,0193380	0,0292500	0,0135600	0,0006000	-0,0102600	-0,0225200

Table A7 – Difference of welfare costs of inflation change including and excluding transition for γ=0.5.