FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

COMENIUS UNIVERSITY BRATISLAVA

Mathematics of Economics and Finance

Equilibrium exchange rate

Master thesis

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Bratislava 2007

Rovnovážny výmenný kurz

Diplomová práca

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UNIVERZITA KOMENSKÉHO
FAKULTA MATEMATIKY, FYZIKY A INFORMATIKY
KATEDRA APLIKOVANEJ MATEMATIKY A ŠTATISTIKY
Ekonomická a finančná matematika

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BRATISLAVA 2007

I declare this thesis was written on my own, with the only help provided by my supervisor and the reffered-to literature.

I would like to express special thanks to my supervisor RNDr. Juraj Zeman, CSc. for all support and guidance he offered throughout the elaboration of this thesis. I would like to also thank my family and friends, who were supporting me throughout my whole studies.

Abstract

In this paper a CPI and PPI based real effective equilibrium exchange rate of the Slovak koruna is investigated. The model inspired by Alberola et al. (1999, 2002) estimates the behavioral equilibrium exchange rate (BEER) by using macroeconomic fundaments, dual productivity (Balassa-Samuelson effect) and country's net foreign assets position. The resulting model is decomposed to permanent (permanent equilibrium exchange rate) and transitory component using Gonzalo-Granger decomposition and HP filter. The BEER approach enables to estimate the future path of real equilibrium exchange rate using the observed relationship. By backward transformation of real equilibrium exchange rate, the estimate of the value of equilibrium nominal exchange rate at the period of Slovakia's euro adoption can be obtainted.

Keywords: real equilibrium exchange rate, cointegration analysis, Balassa-Samuleson effect, net foreign assets, Gonzalo-Granger decomposition

Abstrakt

V tejto diplomovej práci skúmame reálny efektívny rovnovážny výmenný kurz Slovenskej koruny založený na indexe spotrebiteľských cien a cien výrobcov. Model na určenie behaviorálneho rovnovážneho kurzu (BEER) použitý v tejto práci bol inšpirovaný Alberola et al. (1999, 2002). Správanie sa rovnovážneho kurzu je popísané pomocou produktivity a čistých zahraničných aktív. Výsledný časový rad je následne rozložený na permanentnú zložku (PEER) a krátkodobé výkyvy, pričom použité sú dve metódy: Gonzalo-Grangerov rozklad a HP filter. Metóda BEER na základe odhadnutých vzťahov umožňuje predikovať vývoj rovnovážneho výmenného kurzu. Spätnou transformáciou prognózovaného reálneho rovnovážneho kurzu môžeme získať predikciu konverzného kurzu voči euru.

Kľúčové slová: reálny rovnovážny výmenný kurz, kointegrácia, Balassa-Samuelsonov efekt, čisté zahraničné aktíva, Gonzalo-Grangerov rozklad

Contents

List of Figures

List of Tables

Introduction

After entering the European Union in May 2004, Slovakia is facing an other great challenge in the convergence process, the adoption of the euro in 2009. Consequently, the knowledge of the equilibrium exchange rate or even more the deviation of the actual exchange rate from its equilibrium is one of the key interests of policy makers and market participants. An overvalued currency may lead to an unsustainable current account deficit, increasing external debt and the risk of speculative attacks. On the other hand, an undervalued currency has undermining effects on economic growth.

In recent years, the Slovak economy is pulled by the production start in automotive industry. Thanks to huge foreign direct investments in this sector, Slovakia became a so-called Central-European Tiger, with almost double-digit growth rates starting from the second half of 2006. Along with high economic growth, also the Slovak currency appreciates continuously. One of the aims of this thesis is to determine, whether the appreciation is in line with the economic fundaments. If not, further policy instruments are needed to ensure sustainable growth and the fulfilment of Maastricht criteria, inevitable for adopting the European currency.

Foreign capital brings new technologies and know-how, hence indirectly improving the productivity. Foreign direct investments and productivity are the fundaments, with the help of which we attempt to model the real effective equilibrium exchange rate of Slovakia. As for the relative price measures we utilize the series of CPI and PPI indexes in Slovakia and its main trading partners. For these purposes, we construct two sets of foreign countries consisting of Slovakias two (Czech Rep. and Germany) and nine (Cz, Ge, It, Fr, At, Us, Ne, Uk, Sw) main trading partners.

The paper is structured as follows. Chapter 1 provides a brief overview of the econometric methodology (unit root test, cointegration) later used in modelling the equilibrium exchange rate. Chapter 2 discusses the basic definitions and different methods the real equilibrium exchange rate can be modelled. Chapter 3 describes the construction of the data set, followed by chapter 4 providing the unit-root tests and the estimated models. It concludes with the forecasts of real and nominal equilibrium exchange rates.

Chapter 1

Econometric Methodology

"In all econometrics, a model is a set of restrictions on the joint distribution of observed variables or, in other words, a set of joint distributions satisfying a set of assumptions."

Robert M. Kunst

In this chapter, first we decribe the basic terms in econometric terminology. Econometric time series can be classified in several ways, indicating various methods for modelling time series. In this section, we will get closer with integration of time series and related Durbin-Watson test. Finally we introduce the term cointegration and the most widespread Johansen's cointegration test.

1.1 Time series and stochastic processes

A time series is a sequence of data points that show the evolution of a variable measured usually in equidistant time points.

A stochastic process $X_t(\omega)$ can be defined as a sequence of real-valued random variables $X_t : \omega \rightarrow R$, defined on a common probability space such that all joint probabilities exist. If $X_t(\omega)$ is a stochastic process, $X_1, ..., X_T$ will denote the realization of discrete-time stochastic process[1]. This sequence

[1]We talk about discrete-time series when it has finite or countable realizations.

of realization of the random variable is called time series.

1.1.1 Stationary stochastic processes

If the time series has time constant mean, time constant variance and the covariance between two observations depends just on the distance between the time points:

$$E(X_t) = \mu; \;\; \forall t \tag{1.1}$$

$$E(X_t - \mu)^2 = \sigma^2 < \infty; \;\; \forall t \tag{1.2}$$

$$E[(X_s - \mu)(X_t - \mu)] = C(s - t); \;\; \forall s, t \tag{1.3}$$

than the time series is called covariance stationary.

The most known and the so called "basic models" of time series analyses are white noise and random walk. Stochastic process is called a *white noise* when it has zero mean and constant variance, so it can be written:

$$E(X_t) = 0; \;\; \forall t \tag{1.4}$$

$$E(X_t^2 - \mu)^2 = \sigma^2 < \infty; \;\; \forall t \tag{1.5}$$

$$E(X_s X_t) = 0; \;\; \forall t \neq s \tag{1.6}$$

Further, the process is called *random walk*, when the series of its increments (first differences) are white noise:

$$X_t - X_{t-1} = \varepsilon_t \tag{1.7}$$

$$E(\varepsilon_t) = 0; \;\; E(\varepsilon_t^2) = \sigma^2; \;\; E(\varepsilon_s \varepsilon_t) = 0, \;\; s \neq t \tag{1.8}$$

Many econometric and financial time series show similar pattern as random walk processes like share prices, interest rates, exchange rates. Relating to the time path of the process is often compared with a path followed by a "drunken seaman"[2].

[2] Dixon, R. (2004)

1.1.2 Autoregressive and moving average processes

The most frequently used and probably the most relevant class of time series models are *autoregressive processes*. The general p-th order autoregression process, *AR(p)* is expressed as follows:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \varepsilon_t \tag{1.9}$$

where ε_t is a while noise and $\phi_1, ..., \phi_p$ are fixed parameters.

Another popular class of time series models is *moving-average* processes, *MA(q)*. This process is a weighted sum of white noise processes, which are always stationary:

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} +, ..., +\theta_q \varepsilon_{t-q} \tag{1.10}$$

1.2 Integrated processes and tests for Unit Root

The time series is said to be integrated of order d (denoted by *I(d)*), if it becomes stationary after differencing d-times. It follows that the random walk process is $I(1)$.

If the time series are stationary, for estimating coefficients we can use standard methods like ordinary least squares. If all time series are integrated of the same order, the standard methods (e.g. OLS) could be misleading. On the other hand, if the series are not integrated of the same order, there could occur the so called "spurious regression", which was firstly described by Granger and Newbold (1974). Equations estimated by standard methods could show statistically significant realationships with R^2 close to 1. However, in most cases, Durbin-Watson test indicates autocorrelation of the residuals.

If the series are integrated of the same order and there is also a cointegration relationship between them, the parameters can be estimated by vector autoregression model or vector error correction model. Therefore, it was important to establish simple statistical test for determining the order of integration[3] and the cointegration realationship.

1.2.1 Dickey-Fuller test for Unit Root

The most widely used statistical test for testing order of integration is Dickey-Fuller test[4] and the later version Augmented Dickey-Fuller test.

The processes can be classified by characteristic polynomials, which are made as follows:

- From an underlying process one should remove all deterministic terms such as trends or constants.
- The primary process should be formed to autoregressive (difference-equation) process, which mainly consists of lagged variables and only a contemporaneous noise, which should be replaced by 0.
- All lags X_{t-j} should be replaced by z^j, therefore X_t by $1 = z^0$ (X_{t-j}=Z^j).

We call the solutions of characteristic polynomial equation as roots and the processes can be classified by these roots:

- If all roots of characteristic polynomial is greater than 1, the process is stationary.
- If one root is around 1, the variable may be $I(1)$.
- If at least 1 root is less than 1, we talk about explosive process.

This feature has been used by Dickey and Fuller for their well-known test, a "milestone" in integration testing. The test is built as follows:

1. The lag order is increased, until the residual is close to white noise. This is tested by one of these criteria: Ljung-Box statistics, LM statistics, likelihood-ratio statistic or by information criteria.

[3]Very effective concept is visual examination of the processes. This is based on examination of the time constancy of the mean and the variance. If the process is stationary, it fluctuates around a constant mean and the variance is finite constant.

[4]Based on assumption, that the process is AR(1)

2. Second step is to estimate the regression by OLS[5]:

$$\Delta X_t = a[+bt] + \phi X_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta X_{t-j} + \varepsilon_t \tag{1.11}$$

By transforming this regression to characteristic polynomial form we get:

$$\Phi(z) = (1-z)(1-\phi_1 z-, ..., -\phi_{p-1} z^{p-1}) - \phi z \tag{1.12}$$

The Null-hypothesis is that Φ(z) has a unit root, therefore ϕ must equal to zero. If null-hypothesis is rejected, the examined time series is stationary.

3. In third step, the t-statistics of ϕ is calculated, the distribution of which differs from usual t-distribution and was made specially for this test by Dickey and Fuller.

1.2.2 Alternative approaches for testing Unit Root

Although this test is the most popular one, there are some other alternative approaches. Phillis and Perron (1988) formed a new test based on the basic version of Dickey-Fuller test. Instead of lagged differences they used more general non-parametric correction term. Another very often used unit root test is Kwiatowski-Phillips-Schmidt-Shin (KPSS) which is based on differencing of the data and therefore, controversial to the Dickey-Fuller test. Here, the null hypothesis is stationarity and the alternative hypothesis is integratedness.

1.3 Cointegration

If two series are integrated to different order, intuitively one can say that linear combination of these series will be integrated to the higher of the

[5]The basic version of the test estimates regression: $\Delta X_t = a[+bt] + \phi X_{t+1}$, while the augmented one is expanded by lagged differentials.

two orders. Engle&Granger considered two first order integrated processes, the linear combination of which was stationary. As a generalized case, they introduced the term *cointegration* for n-vector variable x_t:

The components of the n-vector variable x_t are said to be cointegrated of order (d,b) denoted by I(d,b) iff *a)* all components of x_t are I(d), *b)* there exits a vector $\beta \neq 0$ such that $z_t = \beta' x_t$ is $I(d-b)$ where β is called a cointegrating vector.

There can exist n-1 linearly independent cointegrating vectors (n is a number of variables), which define the cointegrating rank of the system. Cointegration between variables indicates a long-run relationship between variables. It also determines a short-run relationship between variables and its long-run trend, which is called the error correction. Intuitively one can suggest that from cointegration of order I(d,d) (hence the aggregated series is stationary) some information has to be missed out. This can be illustrated by the following example, where we consider two I(1) variables:

$$y_{1,t} = \alpha + \beta t + \varepsilon_{1,t} \tag{1.13}$$

$$y_{2,t} = \gamma + \delta t + \varepsilon_{2,t} \tag{1.14}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are white noise processes.

A linear combination $z_t = y_{1,t} + \theta y_{2,t} = (\alpha + \theta\gamma) + (\beta + \theta\delta)t + \varepsilon_{1,t} + \varepsilon_{2,t}$ will be evidently stationary if the common linear trend is removed. Therefore the cointegration will occur if the trend in one variable is a linear combination of the trends in cointegration equation.

1.3.1 Vector error correction (VEC)

Cointegration describes the long-run equilibrium of given variables. There exists a mechanism called error correction, which describes the speed of adjustment of the variable to return to the equilibrium when a shock occurs in the evolution of the variable. Let us assume that the vector of variable X_t contains only I(1) variables, than it can be written in the following error correction form:

$$\Delta X_t = \mu + \pi X_{t-1} + \sum_{j=1}^{p-1} \pi_j \Delta X_{t-j} + \varepsilon_t \tag{1.15}$$

where μ is a vector of constants, π_i's give the measure of the influence of first differenced lags of the variable, describing short-run variation. For economists the most important item is matrix π, which defines the long-run relationship between the variables. The rank of the matrix π designates the number of independent cointegrating vectors. Let us consider some special cases:

1. $rank(\pi) = 0$: There is no cointegration relationship, so no long-run equilibrium between variables.

2. $rank(\pi) = 1$: This case is the most special in practice, because indicates only one cointegrating relationship.

3. $rank(\pi) = n$: The matrix π has full rank and indicates stationarity of X_t.

It is very important that all variables were I(1). If there existed some I(0) variables, then such a linear combination where all I(1)'s coefficients are zeros and at least one I(0) has non-zero coefficient, would be stationary. For determination of the number of cointegration relationships, it is essential that the matrix π was estimated.

1.3.2 Testing cointegration

For determination of the number of cointegration relationships, there exist two main approaches. Engle & Granger (1987) test the stationarity of equilibrium errors of estimated equations. The Johansen method (1988) is more widespread and comes out from VAR:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} +, ..., + \phi_p Y_{t-p} + \varepsilon_t \tag{1.16}$$

where Y is a vector of variables and ϕ_j; $j = 1, ..., p$ are matrices of dimension $n \times n$ assuming n is the dimension of Y_t. Furthermore, let Z_t denote the vector of $(n \times p - 1)$ variables:

$$Z_t = (\Delta Y_{t-1}, \Delta Y_{t-2}, ..., \Delta Y_{t-p+1})$$

and let denote D, E matrices of least squares residuals in the regression of ΔY_t on Z_t and Y_{t-p} on Z_t respectively. Then canonical correlations are computed between columns in D and those in E. Canonical correlations give the strength of the linear relationship of two vectors[6]. Squared canonical correlation is defined as ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r \geq ... \geq \lambda_n$ of the matrix

$$R^* = R_{DD}^{-\frac{1}{2}} R_{DE} R_{EE}^{-1} R_{ED} R_{DD}^{-\frac{1}{2}} \tag{1.17}$$

where $R_{i,j}$ is the correlation matrix between variables in set i and set j for $i, j = D, E$. Using trace statistics, the following null hypothesis is tested: there are r or fewer cointegrating vectors in the system. That is, the following statistics is calculated:

$$J(r) = -T \sum_{j=r+1}^{n} log(1 - \lambda_j^2) \tag{1.18}$$

If J(r) does not imply rejection, there are r or *less than* r cointegration relationships.

According to Granger theorem, we can decompose the $n \times n$ matrix π into two $n \times r$ matrices α and β of rank r:

$$\pi = \alpha\beta^{'} \tag{1.19}$$

The columns of the matrix β constitute the cointegrating vector, so the long-term equilibrium and the matrix α gives the speed of adjustment of the variables to the equilibrium.

[6] Green(2000), p. 796

1.3.3 The econometric decomposition - Gonzalo and Granger

Let us consider a vector of variables X_t with one cointegration vector (X_t is a $n \times 1$ matrix and the matrix π is of rank 1). We can define the orthogonal comlements $\alpha_\perp$ and $\beta_\perp$ as complements of α and β respectively (α and β are the matrices of the Granger decomposition of the matrix π):

$$\alpha_\perp = (I - \alpha(\alpha^{'}\alpha)^{-1}\alpha^{'}) \tag{1.20}$$

$$\beta_\perp = (I - \beta(\beta^{'}\beta)^{-1}\beta^{'}) \tag{1.21}$$

It holds that $\alpha^{'}_\perp\alpha = \beta^{'}_\perp\beta = 0$. Furthermore, it can be written:

$$X_t = \beta_\perp(\alpha^{'}_\perp\beta_\perp)^{-1}\alpha^{'}_\perp X_t + \alpha(\beta^{'}\alpha)^{-1}\beta^{'}X_t \tag{1.22}$$

Gonzalo and Granger denominated the first component of equation (1.22), $P_t = \beta_\perp(\alpha^{'}_\perp\beta_\perp)^{-1}\alpha^{'}_\perp X_t$ as permanent and the second, $T_t = \alpha(\beta^{'}\alpha)^{-1}\beta^{'}X_t$ as transitory component and showed that the transitory component has no effect on permanent (long-term) component.

Chapter 2

The Econometric Model

"When thinking about the meaning of equilibrium it quickly becomes apparent that it is a difficult concept to pin down. ... The debate over what constitutes equilibrium has ranged over issues as diverse as its existence, uniqueness, optimality, determination, evolution over time and indeed whether it is even valid to talk about disequilibrium. All of these points are important, as is the question of whether the concept of equilibrium can be separated from the models which are used to measure it."

Rebecca L. Driver and Peter F. Westaway

2.1 Nominal and real exchange rate

The nominal exchange rate between two currencies specifies how much one currency is worth in terms of the other. It can be given by two ways:

1. American - home currency per unit of foreign currency
2. English - foreign currency per unit of home currency

Throughout the paper the nominal exchange rate will be given by the English definition, so increase of nominal exchange rate indicates appreciation and oppositely decrease indicates depreciation:

$$S = \frac{\#unit\ \ of\ \ foreign\ \ currency}{1\ \ SKK} \tag{2.1}$$

Appreciation of home currency means that home products get more expensive abroad and foreign products cheaper in home country. Accordingly, if there are lot of foreign products in consumption basket, appreciation of home currency is favourable for consumer. On the contrary, disadvantage is that home products loose their competitiveness, so the appreciation has a negative impact on trade balance[1]. In literature we can find numerous definitions of the real exchange rate. According to Égert, Halpern and MacDonald (2004), *internal exchange rate* is the price ratio of non-tradables to tradables[2]:

$$Q^I = \frac{P^{NT}}{P^T} \tag{2.2}$$

where P^{NT}, P^T is the price level in non-tradable and tradable sector respectively.

According to the macroeconomic definition, real exchange rate or *external real exchange rate* is the nominal exchange rate multiplied by the ratio of domestic and foreign price levels:

$$q = s + p - p^* \tag{2.3}$$

where s is the logarithm of nominal exchange rate, p and p^* represent the logarithms of the relative price of domestic and foreign consumption basket respectively[3].

[1] This is the consequence of Marshall-Lerner condition, which tells us that the sum of price elasticity of exports and imports exceeds one (in absolute value).

[2] Tradable goods are those that have export or import potential. As a consequence of different transportation costs or life expectancy, there are some goods, which are non-tradable. These are mainly electricity and water (very high transport cost). Services are considered as non-tradables.

[3] Throughout the paper, lower case will denote natural logarithms transformation of the variables.

2.2 The real effective exchange rate

Nominal effective exchange rate (NEER) is a geometrically weighted average of bilateral nominal exchange rates, where the weights are mostly given in terms of the volume of bilateral trade:

$$S^{ef} = \prod_{i=1}^{n} (S_i)^{w_i} \tag{2.4}$$

$$s^{ef} = \sum_{i=1}^{n} w_i s_i \tag{2.5}$$

where S_i is the bilateral nominal exchange rate of home country to country i, n is the number of foreign countries to which the NEER is calculated. Further, w_i is the proportionally calculated weight, based on the ratio of foreign trade between trading partner i and home country to the total amonunt of home country's foreign trade. Equation (2.5) is the logarithmic transformation of (2.4).

The real effective exchange rate differs from the NEER by the fact, that the ratio of the home price level and the geometrically weighted average of foreign price levels are included:

$$Q^{ef} = S^{ef} \frac{P}{P^*} = S^{ef} \frac{P}{\prod_{i=1}^{n} (P_i^*)^{w_i}} \tag{2.6}$$

where S^{ef} is the NEER, P is the price level in home country, P_i^* is the price level in country i and w_i is the weight of country i (the same used in NEER). There exist several ways how to measure the country's price level. Consumer price index (CPI) and producer price index (PPI) are ranked among the most widely known indexes. However, in literature the wholesale price index (WPI) and the unit labour costs (ULC) are also mentioned as a good proxies for price measure.

2.3 Equilibrium exchange rate modelling

There exist several methods for modelling equilibrium exchange rate. Each method is a normative concept, which defines the equilibrium exchange rate in different way. Other viewpoint is the time horizon of the equilibrium exchange rate. We can distinguish short-term, medium-term and log-term equilibrium. The resulting misaligned exchange rate is not the consequence of the "inactive" market forces, but rather suggests the future development of the exchange rate.

2.3.1 Purchasing power parity (PPP)

The first and maybe the mostly criticised method is purchasing power parity[4], which supposes that the Law of one price[5]holds. As a result, the nominal exchange rate is defined as a ratio of home and foreign prices. This definition indicates a constant real exchange rate and ensures that goods cost the same in all countries. This theory defines the equilibrium in very long-term. The main criticisms of PPP are:

1. The weights of the goods in consumption basket are not the same across countries - there are cultural differences.
2. There are differences in quality of goods - different technological equipment used.
3. There exist non-tradable goods in consumption basket - mainly services.

The observable phenomenon is that in less developed countries, so as in transition countries, non-tradable goods are cheaper, than in more developed countries. This implies the failure of the PPP theory[6]. The failure

[4]Cassel (1916 and 1918)

[5]Is an economic rule stating that in an efficient market, all identical goods must have only one price.

[6]This is a direct consequence of Balassa-Samuelson effect stated below.

of PPP led to new approaches in measuring equlibrium exchange rate. The most commonly used are Fundamental equlibrium exchange rate (FEER), Behavioral equilibrium exchange rate (BEER) and Permanent equilibrium exchange rate (PEER).

2.3.2 Fundamental equilibrium exchange rate (FEER)

The notion of FEER has been formalized in papers issued by Williamson (1985, 1994). This approach defines the equilibrium exchange rate in medium-term, adjusting the country's internal and external balance to equilibrium simultaneously.

Internal balance is defined as a stage when GDP equals to potential GDP[7], so it is consistent with the NAIRU[8]. There are several ways to determinate the output gap. The simplest-statistical-method is the Hodrick-Prescott filter, which decomposes historical GDP into trend and cyclical component. There exist a couple of approaches different from ststistical filters, based on economic theory.

The external balance is interpreted as a normative stage, a sustainable position over a medium-term of current account. There is no general approach to compute the sustainable position of current account. It is usually modeled by export and import equations.

Another less frequently used method for estimating equilibrium exchange rate is Natural real exchange rate[9] (NATREX), evolved from FEER. This approach comes out from internal-external balance achieved similarly as in FEER, but the equilibrium is given in medium- and long-term. The difference between NATREX and FEER is in external balance, where the current account is modeled as a difference between savings and investments.

[7]Implying zero output gap
[8]NAIRU - non-accelerating inflation rate of unemloyment
[9]developed by Stein (1994, 1995, 2002)

2.3.3 Behavioral equilibrium exchange rate (BEER)

This approach is free of any normative elements of theoretical background. The equilibrium exchange rate is described by economical fundamentals, influencing the exchange rate in long- and medium-term. Hence, this measure is a statistical approach, linking the real exchange rate and the fundaments in a single equation. Clark and MacDonald (1999) started from equation of uncovered interest parity which relates interest rates and exchange rates in home and foreign country:

$$s^e_{t,t+k} - s_t = -(i_t - i^*_t) + \sigma_t \tag{2.7}$$

where s_t is the logarithm of nominal exchange rate, i_t and i^*_t is the domestic and foreign nominal interest rate in respectively, σ_t is the risk premium of the country and $s^e_{t,t+k}$ is the expected nominal exchange rate in time t for the period $t+k$. If we include home and foreign expected inflation rate, the equation changes to:

$$q_t = q^e_{t,t+k} - (r - r^*) + \sigma_t \tag{2.8}$$

where q_t is the real exchange rate, $q^e_{t,t+k}$ is the expected real exchange rate in time t for the period $t+k$. The variable r and r^* represent the domestic and foreign real interest rate which is the difference between nominal interest rate and the expected inflation in t for $t+k$.

$$r = i - \pi^e_{t+k} \tag{2.9}$$

For the expectation of real exchange rate in t for $t+k$ we can assume, that it is equal to long term real exchange rate, which can be interpreted by economic fundamentals:

$$q_t = \beta(fund)'_t - (r_t - r^*_t); \tag{2.10}$$

$$\beta = (\beta_1, ..., \beta_n); \quad (fund)_t = (fund_1, ..., fund_n)$$

This equation indicates that the real exchange rate is a function of the long- and medium-term fundamentals:

$$q_t = q_t(l_t, m_t) \tag{2.11}$$

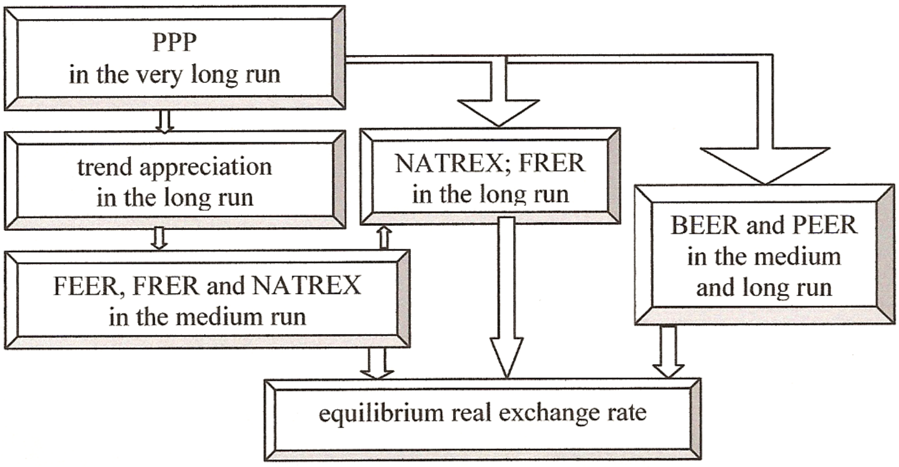
2.3.4 Permanent equilibrium exchange rate (PEER)

Permanent equilibrium exchange rate can be obtained by decomposing the real exchange rate into permanent q_t^P and transitory q_t^T components. The permanent component is than defined as the equilibrium.

$$q_t = q_t^P + q_t^T \tag{2.12}$$

Clark and MacDonald (2000) described the PEER as a decomposition of BEER using Gonzalo-Granger method. This method ensures smoother equilibrium exchange rate than BEER, and is better interpretable.

Finally, the following chart provides a birds-eye view summary of particular methods' time-dependent effectiveness.



Source: Égert, Halpern and MacDonald(2004).

Figure 2.1: The comparison of different methods.

2.4 Which fundamentals use for modelling?

We can distinguish two kinds of products - tradable (T) and non-tradable (NT). The home and foreign prices can be than decomposed as follows:

$$p = \alpha p^T + (1-\alpha)^{NT}; \;\; p^* = \alpha^* p^{T*}(1-\alpha)p^{NT*} \tag{2.13}$$

where p^T and p^{NT} are the prices of tradable and non-tradable goods respectively and α and $1-\alpha$ are their weights in consumption basket[10]. Following Alberola and Tyvainen (1998) who measured these weights for EMU countries, we can suppose that α equals to α^*. Using equation (2.3) and (2.13), the real exchange rate can be split into two main components - real exchange rate for tradable goods q_X and the relative price of non-tradable to tradable goods across countries q_I:

$$q = q_X + (1-\alpha)q_I \tag{2.14}$$

where equation $q_X = s + p^T - p^{T*}$, called *external prices equation*, contains price ratio of home and foreign tradable goods and represents how much the price of home tradable goods is worth in terms of foreign tradable prices. Hence, q_X represents the competitiveness of the economy and affects the balance of payments of the country. The second component of equation (2.14), $q_I = [(p^{NT} - p^T) - (p^{NT*} - p^{T*})]$ is called the *equation of internal relative prices*, which captures the excess demand across sectors. Obviously, equation (2.14) implies that the external and internal prices determine the equilibrium exchange rate.

2.4.1 External balance

Nurske (1944) and Mussa (1984) argue that foreign asset position of the country has a large effect on equilibrium exchange rate in the long-run. The divergence from equilibrium is adjusted by current account[11], which on the other hand leads to accumulation of net foreign assets (NFA). The change of net foreign assets it can be written as:

$$\Delta nfa = ca = xn + (i^* - g)nfa \tag{2.15}$$

[10] The variables with asterisk refer to foreign country.

[11] Current account together with capital account, financial account and change in official reserves create balance of payments. Current account is defined as a sum of balance of trade (the difference between export and import of goods and services), balance of income (e.g. dividends) and current transfers (e.g. EU-funds).

where nfa is net foreign assets, ca is current account balance and xn is the trade balance, all expressed relative to GDP. The variable g is the real GDP growth and i^* is the international interest rate.

The country's export and import is the same as trading partners's import and export respectively, therefore the trade balance is the same in absolute value but differs only in sign. The same can be written for the net income, therefore the home country's current account balance position is the same as its tradeing partners' but with the opposite sign:

$$ca = -ca^* = xn + i^* f \tag{2.16}$$

Let us assume that the above-mentioned Marshall-Lerner condition holds, therefore, the change in relative prices (real exchange rate) negatively influences the trade balance:

$$xn = -\gamma Q_x; \gamma > 0 \tag{2.17}$$

The external balance in medium-term is characterized by convergence of net foreign assets towards their optimal (desired) level:

$$\Delta nfa = a(nfa - \overline{nfa}) \tag{2.18}$$

The equilibrium level of NFA is given exogenously and is defined mainly by savings, demographics and the stage of development. If the country's net foreign assets position is below its equilibrium value, the country will starting to accumulate assest. On the other hand, if it is above the equilibrium, the country will reduce the assets. Using equations (2.15), (2.17), (2.18), Q_x can be expressed as follows:

$$Q_x = \frac{a}{\gamma}(nfa - \overline{nfa}) + \frac{i^* - g}{\gamma} nfa \tag{2.19}$$

According to equation (2.19), the current stock of net foreign assets positively influences the exchange rate. This means that if the amount of assets (debt) accumulates, the exchange rate will appreciate (depreciate).

2.4.2 Internal Balance - The Balassa-Samuelson effect

The recently known Balassa-Samuelson effect has been firstly formalized by Roy Forbes Harrold in 1939. Bela Balassa and Paul Samuelson have separately completed and extended this theory[12], and issued in 1964. They come out from two main observations:

1. Consumer prices in wealthier countries are higher than in poorer countries - this is the consequence of Penn effect[13].

2. The productivity growth rate in tradable sector is higher than in non-tradable sector - there are new technical processes and the competition between tradable goods is higher than between non-tradable goods.

The B-S effect is based on 3 assumptions. The first one says that the economy is divisible into two sectors, open sector (contains tradable goods) and sheltered sector (contains non-tradable goods). The second condition is that the wages depend on productivity in both sectors. In open sector, rising productivity boosts the output and hence the company's benefits. After all, this will be reflected in higher wages, because the company can afford to pay bonuses or pay higher salaries. The third assumption is the perfect mobility of labour between sectors, which has a consequence that nominal wages equalize between sectors. Let us assume that the production function is defined by Cobb-Douglas definition, so the production is a function of labour (L) and capital (C) in a following way:

$$Y_{NT} = A_{NT} L_{NT}^{\delta} C_{NT}^{1-\delta}; \;\; Y_T = A_T L_T^{\theta} C_T^{1-\theta} \tag{2.20}$$

where A is a trend component of total factor productivity interpreted also as technological progress, δ and θ are the elasticity of labour in non-tradable and tradable sectors respectively. According to the second and third condition,

[12]Hence the phenomenon is sometimes called as Harold-Balassa-Samuelson effect.

[13]The Penn effect tells that the real income ratios between high and low income countries are systematically exaggerated by GDP conversion at market exchange rates

wages equal to marginal product and these wages equalize in tradable and non-tradable sector. It can be written:

$$\frac{\delta Y_{NT}}{\delta L_{NT}} = \frac{W}{P_{NT}}; \quad \frac{\delta Y_T}{\delta L_T} = \frac{W}{P_T} \tag{2.21}$$

Using equations (2.20) and (2.21):

$$\frac{P_{NT}}{P_T} = \frac{\theta \frac{Y_T}{L_T}}{\delta \frac{Y_{NT}}{L_{NT}}} = \frac{\theta PRO_T}{\delta PRO_{NT}} \tag{2.22}$$

where PRO_T and PRO_{NT} denote the productivity in tradable and non-tradable sector respectively. As a result, it follows from Balassa-Samuelson effect that higher productivity growth in open sector causes increase in prices in sheltered sector. By substituting equation (2.22) into internal price equation (second term of equation (2.14)) and according to third assumption we can write:

$$Q_I = [(p_N - p_T) - (p_N^* - p_T^*)] = (pro_T - pro_N) - (pro_T^* - pro_N^*) = (pro - pro^*) \tag{2.23}$$

Therefore, if the home country has larger productivity growth than foreign countries, this will positively influences home country's real exchange rate and negatively the inflation.

2.5 Gonzalo and Granger decomposition

The decomposition of the observed real exchange rate into two components is based on the cointegrating information contained in the data. The first component presents the equilibrium exchange rate (the fundamentals are at their equilibrium level) and the second component expresses the deviations from this equilibrium. In our work we follow Gonzalo and Granger (1995) decomposition into permanent and transitory components. The permanent component characterizes the long-run equilibrium expressed by cointegration relationship. This component is I(1). On the other hand, transitory component is stationary and demonstrates the deviation from the permanent component. Therefore, the deviation has no effect on the permanent component. In previous section we have described the econometric methodology of the Gonzalo and Granger decomposition. Relating to this description, the first and the second component of equation (1.22) denote the permanent and the transitory component respectively.

Chapter 3

Data Sources and the Construction of Time Series

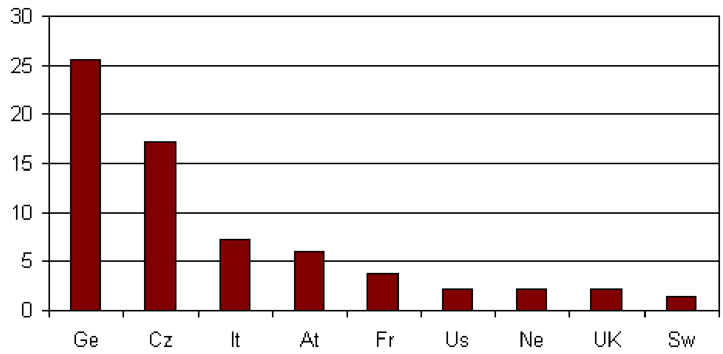
"The high external disequilibria accumulated by the new EU members coupled with the appreciation trend of their currencies questions the sustainability of such situation before and after the euro is adopted. Indeed, large net net capital inflows and persistent current account deficit have been observed in most transition countries as they increased their income levels in the last decade."

Enrique Alberola

The purpose of this paper is to derive an effective real exchange rate using BEER approach described in previous chapter. There are two main methodological questions: the price measure and the structure of weights, which have to be determined for calculating real exchange rate. In our analysis, we use consumer price index (CPI) and producer price index (PPI) as price measures. The real exchange rate expressed in different price measures show different evolution.

3.1 Volume of bilateral trade

In the process of generating time series as nominal and real exchange rate and productivity differential, the weights of foreign countries are determined by bilateral trade between Slovakia and its trading partners.



Source: SOSR, 1997-2003 average.

Figure 3.1: The percentage of Slovak trade volume with its main trading partners.

42,83 percent of trade balance consist of two countries, Germany and Czech Republic (see Figure 3.1). In order to distinct between different levels of detail of the analysis, consider two scenarios [1]:

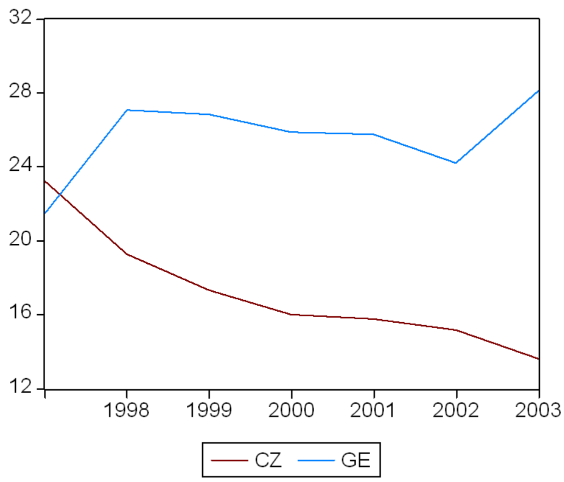
1. In basic scenario, the two main trading partners, Germany and Czech Republic are encompassed only.

2. The extended scenario involves 9 trading partners: Germany, Czech Republic, Italy, Austria, France, the Netherlands, Great Britain, USA and Switzerland. Including 9 countries instead of 2 seemingly provides more accurate measure of the effective exchange rate. On the other hand, the weights of Germany and Czech Republic constitutes almost two thirds of all 9 countries total weight. Therefore, we can expect

[1]The transformations of trade volume weight coefficients for both scenarios are presented in Appendix A.

similar real effective exchange rate and its equilibrium level to those in the basic scenario.

The Slovak trade has undergone several changes throughout 1997 and 2003. The trade volume with two main trading partners changed in time significantly, but in contrary, the trade volume with other trading partners shows a smooth evolution.



Source: SOSR.

Figure 3.2: The evolution of the trade of goods with Czech Republic and Germany relative to total trade of Slovak Republic (in %).

The bilateral trade volume with Czech Republic relative to Slovakia's total trade volume decreased gradually in time period 1997-2003 by 9.6 percentage points from 23.2% to 13.6%. The progress of trade volume between Slovakia and Germany relative to the total trade volume of Slovakia changed over time as well. It grew about 5.6 percentage points in 1997, but after this period it mildly decreased until the end of 2001. In 2002, the trade volume increased again, by 3.92%. Overall, the trade volume of Slovakia with Germany increased by 6.6 p.p. over the period 1997-2003. This probably relates to increasing FDI inflow and boosting automobile industry in Slovakia.

Data for creation of real effective exchange rate time series and fundaments were obtained from IMF IFS and EUROSTAT database. Since data availability, for the extended scenario, quarterly data covering the period from the first quarter of 1993 to the second quarter of 2006 (54 observations) are used. The time period for the basic scenario is longer, covering the period 1993:1 - 2006:4 (56 observations).

3.2 The real effective exchange rate in Slovakia

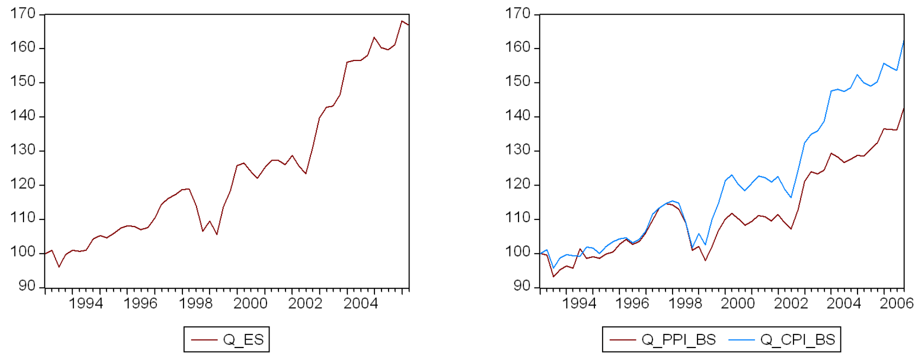
In chapter 2 we have mentioned the definition of the real effective exchange rate. According to the definition of nominal and real effective exchange rate (equations (2.4) and (2.6)), for generation of time series we specify the following form:

$$Q_t^E = \prod_{i=1}^{n} (S_{i,t})^{w_i} \frac{P_t^{SR}}{\prod_{i=1}^{n} (P_{i,t}^*)^{w_i}} \tag{3.1}$$

where $S_{i,t}$ is the nominal bilateral exchange rate with country i, P^{SR} is price level in Slovakia in time t, $P_{i,t}^*$ is the price level in foreign country i in time t. Further, w_i is the weight coefficient of country i, computed on the basis of bilateral trade volume, n is the number of countries ($n = 2$ in basic scenario (BS) and $n = 9$ in extended scenario (ES)). For modelling purposes we transform this variable into a natural logarithmic form: $q_t^E = ln(Q_t^E)$.

Figure 3.3 illustrates the real effective exchange rate for both scenarios. On the left chart, the CPI based RER for the extended model is depicted, and the right chart shows the CPI and PPI based RER for the basic scenario. The shape of all curves are similar, displaying appreciation trend over the time period.

All the RER indexes show a calm development in first years. Let us consider the basic scenario's CPI based RER index. It started to strongly



Source: IMF IFS, EUROSTAT.

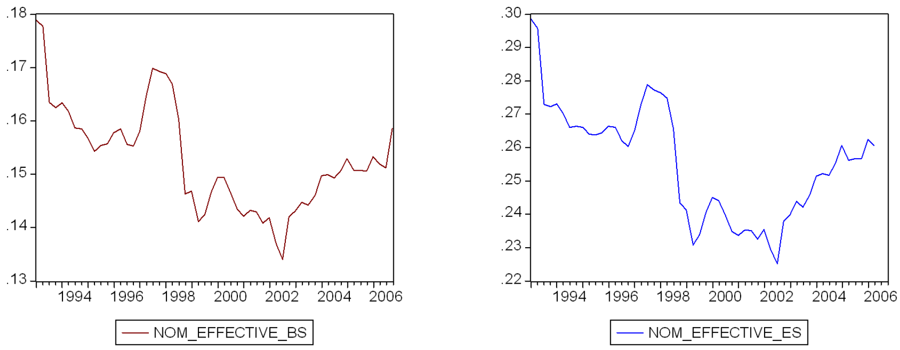
Figure 3.3: The CPI and PPI based real exchange rate.

appreciate at the beginning of 1997. To the end of the year 1997 it appreciated about 15.3% compared to starting value in 1993, but after depreciating a year, it almost came back to 93's level. As of the beginning of 1999, the RER started to strongly appreciate and in the middle of 2000, it reached 23% above the 93's level. In the next two and half years period, the RER stabilized and fluctuated around this value with small deviations. From mid-2002, the RER strongly appreciates thanks mainly to the nominal exchange rate appreciation. By end 2006, the CPI-based RER has been 62.4% stronger and the PPI-based RER 42.6% stronger, than the 93' value. The CPI based RER for extended scenario shows the biggest appreciation. The index reached 67.1% in second quarter of 2006.

Until end-2002, the bilateral nominal exchange rates depreciated with respect to all other currencies[2]. However, from 2003 it started to strongly appreciate, therefore the nominal effective exchange rate showes a similar development. This means that until the end of 2002, the appreciation of RER passed through the inflation differential. That is, the prices in Slovakia must have grown faster than prices in its trading partners. Indeed, CPI index

[2]See appendix A.

in Slovakia grew about 173% and the PPI index 120%, while the weighted average of trading partners's CPI and PPI indexes grew just by 48% and 36% respectively. This implies that the inflation differential played a crucial role in RER appreciation in period before 2002. After 2002, the RER appreciated more robustly, although due to appreciation of nominal effective exchange rate (Figure 3.4).



Source: IMF IFS.

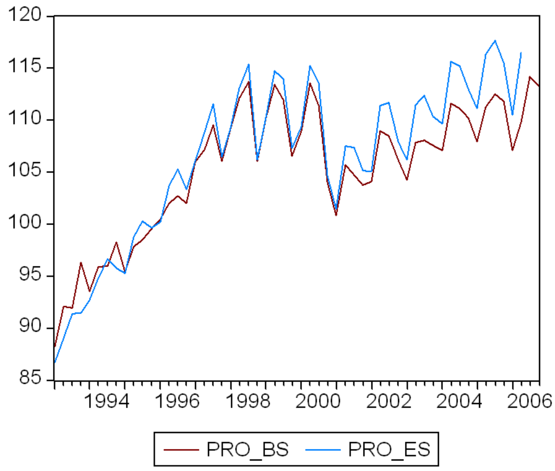
Figure 3.4: The nominal effective exchange rate for basic and extended scenario.

3.3 The dual productivity differential

Relative productivity is computed as a ratio of labour productivity in Slovakia relative to geometrically weighted foreign labour productivity. Labour productivity is definied as a ratio of GDP to the total employment.

$$PRO_t = \frac{Labour \;\; productivity_i}{\prod_i^n (Labour \;\; productivity_{i,t})^{w_i}} = \frac{\frac{GDP_t^{sr}}{EMP_t^{sr}}}{\prod_{i=1}^n (\frac{GDP_{i,t}^*}{EMP_{i,t}^*})^{w_i}} \tag{3.2}$$

where *GDP* is the index of gross domestic product in constant prices and *EMP* is the index of employment. Further, *SR* and the asterisk denote the home country and foreign countries respectively. Again, in the models we use the natural logarithm of this variable: $pro_t = ln(PRO_t)$.



Source: IMF IFS, EUROSTAT.

Figure 3.5: Dual productivity differential for both scenarios.

According to Balassa-Samuelson effect, the productivity differential has positive effect to the RER. In the examined time period, the productivity differential increased by 28.3% and 34,3% in basic and extended scenario respectively. For both scenarios, the main increase has been attained in the first 5 years of the whole time span, so in 1993-1998. In this time period the series do not show seasonality, but after 1997 the seasonality is significant. Due to unsustainable economic growth before 1999 and a consecutive need for structural changes, the dual productivity differential decreased significantly in 2000 and started gaining repeatedly together with the beginning of economic sanitation (Figure 3.5).

From Taylor's expansion, the percentage change of productivity differential (in time) equals the intertemporal difference of its natural logarithm. Therefore, we should become aware what is behind the percentual change of the productivity differential.

Productivity differential is nothing else than the ratio of GDP and number of employed (or hours worked). We can analyze the whole influence of the growth of these variables in Slovakia and in its trading partners to the productivity differential. Consider the percentage change of GDP and employment in one time period ($t \rightarrow t+1$) individually:

Let g_{SR} and g_i^* denote the real GDP growth rate in Slovakia and its trading partner i respectively. Further, let e^{SR} and e_i^* denote employment growth in Slovakia and its trading partner i respectively.

Then the expected percentual change of variable PRO:

$$\%\Delta PRO = \frac{PRO_{t+1} - PRO_t}{PRO_t} = \frac{\frac{1+g^{SR}}{1+e^{SR}}}{\prod_{i=1}^{n}\left(\frac{1+g_i^*}{1+e_i^*}\right)^{w_i}} - 1$$

3.4 Net foreign assets

The influence of the fundament net foreign assets to real exchange rate is questionable for different economies. For transition countries like the Slovakia, inflow of the foreign direct investments (FDI) is necessary to strength their economic situation. The higher growth potential is financed by the "inflowing foreign money", accumulating foreign liabilities, so positively influencing the real exchange rate (leading to appreciation of RER) in the medium run. On the other hand in the long-run, the current account deficit, which occurs when the country starts paying factor payments on previous FDI, would lead to depreciation of the real exchange rate.

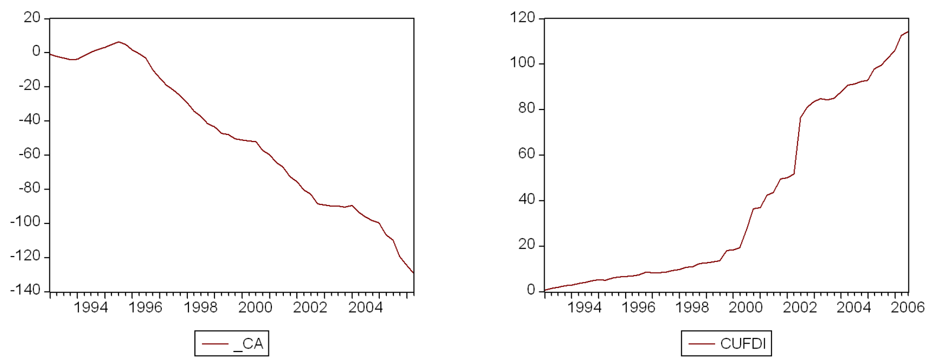
Empirical results showed that in transition economies the NFA have positive link to the real exchange rate. In contrast, in emerging market economies, NFA influence RER negatively. Égert, Revil and Lommatzsch (2004) showed the same in their analysis. Though, Alberola (2005) comes to different conclusions for transition countries. In his paper, NFA positively influence the real exchange rate in case of Hungary and Poland, but negatively for Czeech Republic.

In literature we can find many different concepts for modelling NFA. Mostly it is proxied by net foreign assets of the banking sector or by the cumulated current account balance. Égert and Babetskii (2005) used these two approximations and productivity differential for estimating BEER for the Czech Republic. There are large FDI inflows in the background of the overall net foreign assets decrease, which mainly reflect the interventions of national banks. Therefore, in our work we decided to use two proxies of NFA: are cumulative sum of current account deficit (CUCA) and cumulative sum of net foreign direct investments (CUFDI)[3]. Both proxies are set to a zero level at the beginning of the sample and normalized by GDP in constant prices:

$$CUCA = \sum_{j=1}^{t} \frac{ca_j}{GDP_j} \tag{3.3}$$

$$CUFDI = \sum_{j=1}^{t} \frac{fdi_j}{GDP_j} \tag{3.4}$$

where ca_j is current account balance in time j in millions of euro, fdi_j is net foreign direct investments in millions of euro and GDP_j is the index of GDP in constant prices in time j.



Source: IMF IFS, EUROSTAT.

Figure 3.6: The two proxies of net foreign assets.

The right chart of the Figure (3.6) shows that the development of net

[3]Alberola (2002, 2005) uses both cumulative sum of current accout and foreign direct investment inflow relative to GDP.

foreign direct investments was very calm until 2000. This means that the investments inflow was just a little bit higher than the investments outflow. Following structural reforms in Slovakia, a huge inflow of foreign direct investments started in 2000, apparently supporting the economy. The most important FDI inflows were the automobile factories, naturally "pulling" suppliers for automotive and metal industry. These companies play a crucial role in current economic growth.

These variables were not transformed into logarithm, hence we are focusing on the absolute change of the variable:

$$\Delta CUCA = CUCA_t - CUCA_{t-1} = \sum_{i=1}^{t} \frac{ca_i}{GDP_i} - \sum_{i=1}^{t-1} \frac{ca_i}{GDP_i} = \frac{ca_t}{GDP_t}$$

We can see that the absolute change of the variable CUCA depends only on net current account balance and GDP in time t. Let us examine the percentage change of two consecutive period's absloute changes if the net current account balance increase by c % and the GDP by g%:

$$\%|CA| = \frac{\frac{ca_t}{GDP_t} - \frac{ca_{t-1}}{GDP_{t-1}}}{\frac{ca_{t-1}}{GDP_{t-1}}} = \frac{c-g}{1+g}$$

$\Delta CUFDI$ and $\%|FDI|$ is calculated similarly.

Chapter 4

Estimation Results

"The future behaviour of nominal and real exchange rates in new EU members is an issue of lively discussion. The election of the most suitable exchange rate regime and monetary strategy in the run-up to EMU and the scope for the persistence of inflation differentials within EMU have placed exchange rates issues at the centre of the debate in recent times."

Enrique Alberola

In this chapter, we present some selected relations between the real effective exchange rate and macroeconomic fundamentals described above. The models are estimated by two different methods. The long-run relationships estimated by Johansen cointegration method are denoted with *a)* and equations estimated by standard OLS method are denoted with *b)*.

For Johansen's method is indispensable that all time series were integrated of the same order. Therefore, first we test stationarity of time series, using Augmented Dickey-Fuller (ADF) test.

Tables 4.1 and 4.2 imply that all series are I(1) on 1% confidence level (first differences are I(0)). A couple of models of the equilibrium exchange rate have been estimated, using fundamentals or their seasonally adjusted values, if the seasonal adjustment was necessary. Choosing the best model among them is difficult, beacause of the short time span and inconsistency of

Time series	*RER-bs (CPI)*	*RER-bs (PPI)*	*RER-es*	*PRO-bs*	*PRO-es*	*CUCA*	*CUFDI*
t-statistic	-0.7632	0.2	-0.7754	-1.753	-2.2335	0.956	1.75
probability	0.8211	0.97	0.817	0.71	0.461	0.3354	0.9996

Source: own calculations.

Table 4.1: ADF test results for time series in levels.

Time series	*RER-bs (CPI)*	*RER-bs (PPI)*	*RER-es*	*PRO-bs*	*PRO-es*	*CUCA*	*CUFDI*
t-statistic	-6.146	-5.8047	-6.53	-7.645	-7.3663	-3.364	-5.925
probability	0	0	0	0	0	0.0087	0

Source: own calculations.

Table 4.2: ADF test results for first differences of the time series.

the time series. Furthermore, the history of Slovak economy is characterized by several structural changes, for example change in consumer preferencies. While chosing the best model we have proceeded as follows:

1. The optimal number of lags was determined.

2. The cointegration equation was established.

3. The independence of the variables, the normality of the residuals (Jarque-Bera statistics) and the autocorrelation of the residuals (LM statistics) were tested.

4. The equations estimated by OLS have been compared with the equations using the same variables, estimated by Johansen method.

Another very important criterion is the sign and the value of the estimated coefficients, which have to be consistent with economic theory. The results of outperformed tests and of the estimations are presented in Appendix B.

4.1 Estimation of behavioral equilibrium exchange rate

The estimations of real equilibrium exchange rate were performed using variables pro and $CUCA$ or $CUFDI$. For all scenarios in both CPI and PPI based estimations of equilibrium exchange rate, the fundament $CUCA$ showed statistically insignificant or economically uninterpretable results. In most of the models, the productivity differential negatively influenced the RER, so the sign was the opposite of expected, which would correspond economic theory. Henceforward, we use the variable $CUFDI$ as a proxy for NFA .

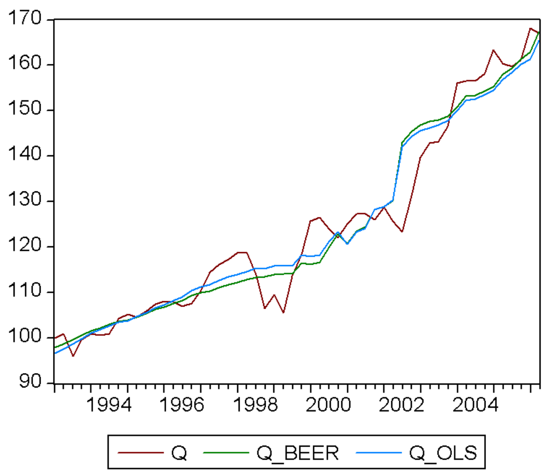
We present the long-run relationship between RER and fundamentals estimated by Johansen cointegration and OLS method for both scenarios consecutively.

4.1.1 The CPI based real effective exchange rate

The long-run relationship for basic scenario:
Model 1

$$\begin{aligned} &\text{(1a)}\ q^{BEER} = 0.5254pro + 0.0035CUFDI + 2.1883 \\ &\text{(1b)}\quad q^{OLS} = 0.5663pro + 0.0032CUFDI + 2.0122 \end{aligned}$$

In this model, the coefficients are consistent with the economic theory, both productivity differential and cumulative net foreign direct investment has positive influence on the BEER. Since the variable *pro* is in natural logarithm form, the percentual increase of this variable will cause a 0.53% appreciation of the real equilibrium exchange rate. The interpretation of the variable *CUFDI* differs from interpretation of *pro*, since logarithmic transformation has not been realized. The unit absolute growth of this variable appreciates the BEER by 0.35%. The model encompasses lagged variables up to time $t-4$. Model (1b) has been estimated by OLS, including the same variables as model (1a).



Source: EUROSTAT, own calculations.

Figure 4.1: CPI based BEER for extended scenario.

The long-run relationship for extended scenario:

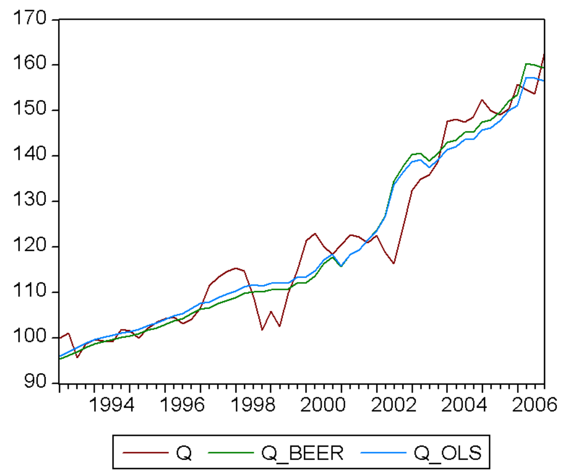
Model 2

$$(2a)\ q^{BEER} = 0.4447pro + 0.0037CUFDI + 2.5921$$
$$(2b)\quad q^{OLS} = 0.5853pro + 0.0033CUFDI + 1.9497$$

In this model, the percentage growth of the productivity differential appreciates the real equilibrium exchange rate by 0.44% and an absolute unit growth of the parameter CUFDI appreciates the BEER by 0.37%. As before, the model includes lagged variables up to time $t-4$.

In equation estimated by OLS method, the percentage change of productivity has bigger influence on equilibrium exchange rate than in the case of Johansen method. The difference between estimated parameters of the fundament $CUFDI$ is not significant much.

Comparing basic and extended scenario's CPI based real exchange rate, the estimations show a tiny bias. In both models, the productivity and the



Source: EUROSTAT, own calculations.

Figure 4.2: CPI based BEER for basic scenario.

NFA proxy appreciate the BEER.

4.1.2 The PPI based equilibrium exchange rate

In this case we expect the same sign of fundaments as in CPI based models. However, the estimated values of parameters are expected to differ, because of the differentness of CPI and PPI based real exchange rate. We decided to examine the basic scenario only, because of the similarity of two CPI based BEER.

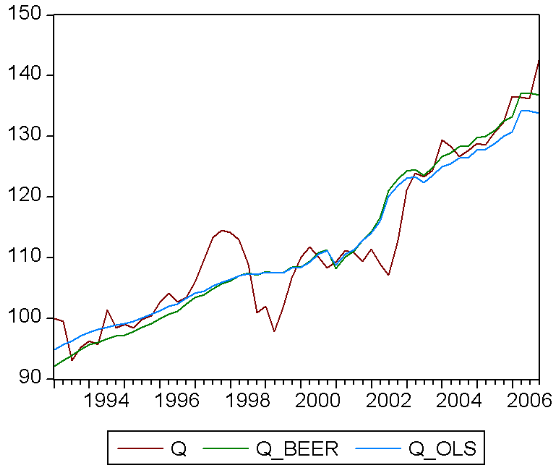
The long run relationship between variables:

Model 3

$$\text{(3a)}\ q^{BEER} = 0.6282pro + 0.0022CUFDI + 1.6924$$

$$\text{(3b)}\quad q^{OLS} = 0.4958pro + 0.0021CUFDI + 2.3179$$

The percentage growth of productivity differential appreciates the real equilibrium exchange rate by 0.63% and an absolute unit growth of the fundament $CUFDI$ appreciates it by 0.22%.



Source: EUROSTAT, own calculations.

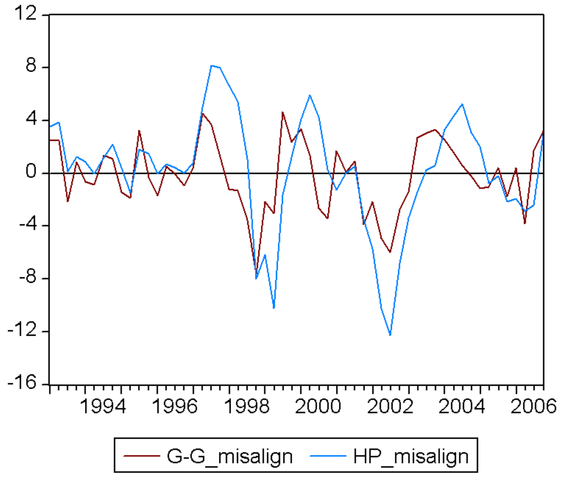
Figure 4.3: PPI based BEER for basic scenario.

Comparing *model 3* with CPI based *model 1*, a percentage change in productivity differential has higher effect on equilibrium exchange rate in the case of PPI based BEER. On the other hand, the absolute change of $CUFDI$ indicates smaller impact on BEER. Comparing q^{BEER} and q^{OLS} (based on PPI), the difference between methods is similar as in case of *model 2*.

All parameters of equations estimated by OLS method are significant and the R-squared is near to one. Durbin-Watson test and Breusch-Godfrey LM test indicate positive correlation between residuals, therefore we consider hereinafter the equations estimated by Johansen method.

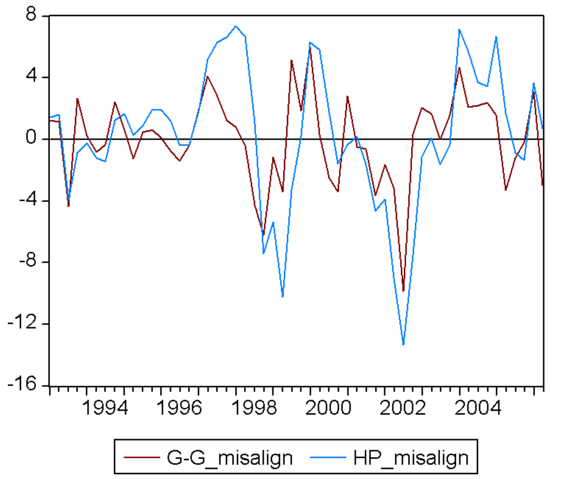
4.2 Real misalignment

As already stated, two methods of decomposition of BEER have been used: Hodrick-Prescott filter and Gonzalo-Granger decomposition. The misalignments indicated by these two methods are depicted in Figure 4.4, 4.5 and 4.6, where positive bias expresses overvalued exchange rate and on the contrary, the negative bias indicates undervalued exchange rate.



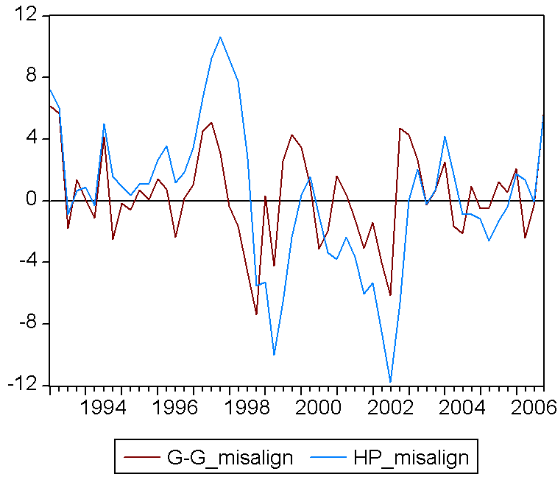
Source: own calculations.

Figure 4.4: Real misalignment indicated by model 1.



Source: own calculations.

Figure 4.5: Real misalignment indicated by model 2.



Source: own calculations.

Figure 4.6: Real misalignment indicated by model 3.

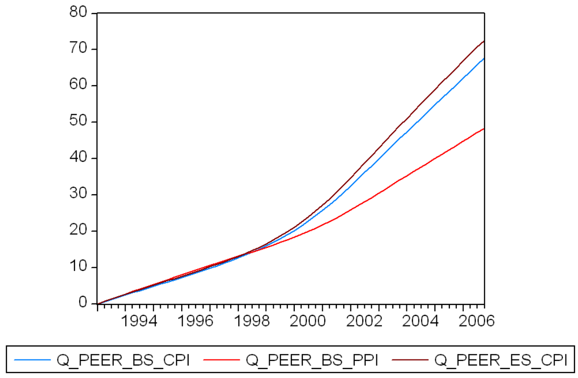
All models display periods of undervaluation and overvaluation ranging from 8.2% to -12.3% in *model 1*, 7.3% to -13.4% in *model 2* and 10.6% to -11.8% in *model 3*. From visual comparison of figures, which show the deviation of the actual exchange rate from its long-run equilibrium exchange rate is noticeable, that they have similar shape in all cases. Moreover, there is small difference between CPI and PPI based misalignments.

The CPI based equilibrium exchange rate shows smaller real misalignment in extreme situations, in 1997, 1998, 2002 and 2004. In period 2000-2001 the CPI based exchange rate was overvalued, but in contrast the PPI based was fluctuated around its equilibrium.

The over- and undervaluation of the real exchange rate is extensively connected with country's macroeconomic evolution. In first 5 years of its introduction, the Slovak koruna was fixed to a basket of currencies and the real exchange rate fluctuated around its equilibrium. The exchange rate was overvalued in 1997-1998, because of keeping nominal exchange rate on its overvalued level (koruna had managed float exchange rate regime), which led to a strong depreciation after introduction of new, floating exchange rate regime. The nominal exchange rate stabilized after reforms of the new government and consequently the real exchange rate returned to its equilibrium to at the end of 1999. Because of the effect of the market uncertainty stemming from government change, the nominal exchange rate depreciated in 2002 and the real exchange rate was undervalued. After new reforms of the government, the nominal exchange rate started to appreciate and together with high inflation (brought by the price deregulating process) it caused the overvaluation in 2003-2004. In recent years, the inflation has stabilized and as figures show, at the end of 2006 the real exchange rate started to be mildly overvalued. This could be because of enormous appreciation of the nominal exchange rate, underpinned by high economic growth and high productivity differential, however influenced by speculative attacks as well.

In all cases, the PEER appreciates across the whole time spam. They have almost the same development until 1998, which is the date of the nominal exchange rate regime change. This is the reason, why the real equilibrium exchange rate appreciation (13% until 1998) passed through the inflation differential. All models show bigger appreciation after the introduction of the floating exchange rate regime. Figure 4.7 depicts the permanent equilibrium exchange rate for all 3 models. The average appreciation of two CPI based models is 70%, which corresponds to a 3.9% appreciation per year. The third, the PPI based model presents a smaller whole period increase 48.3%, corresponding to an average 2.9% yearly appreciation.

In examined time period, the deviation from equilibrium established by Gonzalo and Granger decomposition shows a similar pattern as the deviation estimated by Hodrick-Prescott filter, but smaller biases from the equilibrium. The misalignments range from -7.7% to 4.7% in *model 1*, -9.9% to 6.0% in *model 2* and -7.4% to 6.2% in *model 3*. In the second quarter of 2006, *model 2* indicates an undervalued exchange rate (by -3%), in consistency with *model 1* and *model 3*. On the other hand at the end of 2006, *model 1* and *model 3* indicate an overvalued exchange rate, by 3.2% and 5.5% respectively.



Source: own calculations.

Figure 4.7: The percentage appreciations of the models

4.3 Forecasting real equilibrium exchange rate

The obtained models can be used for forecasting the future development of equilibrium exchange rate. We decided to predict the REER until the end of the year 2008, which is the expected date of the euro adoption. We also decided to forecast the real equilibrium exchange rate for basic scenario due to the similarity of two scenarios. The forecast of REER is obtained through the forecasts of used fundamentals. For these purposes, the Slovak macroeconomic indicators' forecasts of the Ministry of finance of the Slovak Republic and the European Commission's Autumn 2006 Economic Forecasts (for Slovakia's trading partners) are exploited. Table 4.3 shows the weighted average of the forecasts of productivity growth rate and of the CPI growth of trading partners for basic scenario[1]. Since the forecasts of the PPI index are not available, we construct them using CPI inflation forecasts.

The forecasts for Slovakia are presented in Table 4.4. Since it is difficult to forecast net foreign direct investments, we distinguish 3 cases. In the

[1]The forecasts of the real GDP growth, the employment growth rate, from which the productivity is calculated and the CPI growth rates for Germany and Czech republic are stated in Appendix A.

	2006	*2007*	*2008*
Productivity growth	2.8	1.3	2.0
CPI growth	2.0	2.2	1.7

Source: EC Autumn 2006 Economic Forecasts.

Table 4.3: The weighted average of the forecasts for basic scenarios.

first "realistic" case, we prolong the recent trend of the fundamental and the average level[2] of period 2004-2006 is taken. In the second "optimistic" case, a rapid increase of foreign direct investments inflow is supposed, so as the predicted value of net foreign direct investments is 20 precent higher than in the first case. The third case, in contrast, assumes a less favourable picture: a 20% slowdown of net foreign direct investments by compared with the first case.

Fundament	*2007*	*2008*
Real GDP growth	8.1	5.6
Employment growth	1.6	0.8
Productivity growth	6.3	4.8
CPI growth	2.2	1.9
FDI_1	1335	1335
FDI_2	1602	1602
FDI_3	1068	1068

Source: MF SR.

Table 4.4: The forecasts of macroeconomic fundamentals for Slovak Republic.

Since the positive coefficients of the particular fundaments in the modelled equations, their predicted positive growths imply appreciation of the REER in the future[3]. In Table 4.5, the forecasts of REER appreciation for years 2007, 2008 and the average appreciation for these years are presented.

Assuming realistic evolution of net foreign direct investments, the CPI

[2]in millions of euro

[3]In appendix B, the predicted appreciations of two fundamentals can be find. The sum of the appreciations of these fundaments gives the total appreciation of REER.

		2007	*2008*	*Average 07-08*
CPI based REER, basic scenario	FDI_1	5.3	4.0	4.6
	FDI_2	5.8	4.5	5.1
	FDI_3	4.7	3.5	4.1
PPI based REER, basic scenario	FDI_1	4.8	3.3	4.0
	FDI_2	5.1	3.6	4.4
	FDI_3	4.4	3.0	3.7

Source: own calculations.

Table 4.5: The forecasts of REER.

based and the PPI based predictions of yearly percentage growth rate of equilibrium exchange rate are 4.6% and 4.0% respectively.

4.4 The forecast of the nominal exchange rate

It is possible to compute the nominal effective equilibrium exchange rate from the forecasts of REER. According to above-mentioned sources, the predicted inflation differential is 0.1%, so we can neglect it. Therefore, the evolution of nominal effective equilibrium exchange rate will "copy" the development of real equilibrium exchange rate. Further, by additional transformation through nominal exchange rates, the equilibrium nominal exchange rate for euro at the period of Slovakia's entrance to the Eurozone can be obtained. To do this, the knowledge of the nominal exchange rate between Slovakia and other non-euro using trading partners is also necessary.

It is extremely difficult to forecast the bilateral nominal exchange rate and in most cases, the predictions are imprecise. For forecasting in basic scenario we need to know only the predictions of the bilateral exchange rate between Czech Republic and Slovakia[4]. The development of SKK/CZK nominal exchange rate was very calm in last 2 years. Therefore we assume that

[4]Hence we can say, that the forecast of nominal SKK/EUR exchange rate in basic scenario is more accurate, because in extended scenario 4 bilateral nominal exchange rates have to be predicted (SKK vs. CZK, GBP, CHF, USD).

its value in the end of the year 2006 remains unchanged until the end of 2008. However, with respect to the weight of the Czech koruna in effective nominal exchange rate (40%), the prediction of the euro could be misleading if its value markedly changes.

As a final evaluation of our modelling, in Table 4.6 we introduce the optimal nominal exchange rates between SKK and euro indicated by different models. The predictions range from 30.48 to 31.9 SKK/EUR. The central parity of the Slovak koruna has been determined to 35.44 SKK/EUR and the fluctuation bands are (±15%) 30.13 and 40.76 SKK/EUR. Therefore, the equilibrium nominal exchange rate has to remain between these intervals. Based on our calculations, the optimal value of nominal exchange rate assuming "realistic" evolution of net foreign direct investments should fall into the interval 30.99 - 31.57.

		4Q 2008
CPI based REER, basic scenario	FDI_1	30.99
	FDI_2	30.48
	FDI_3	31.51
PPI based REER, basic scenario	FDI_1	31.57
	FDI_2	31.24
	FDI_3	31.9

Source: own calculations.

Table 4.6: The forecast of the SKK/EUR exchange rate at the end of 2008.

Conclusion

The primary aim of this thesis was to analyze the misalignments of the real effective exchange rate (RER) of the Slovak koruna from its equilibrium level, calculated on the basis of CPI and PPI, using the behavioral equilibrium exchange rate model proposed by Alberola et al. (1999, 2002). In this model, the fundaments influencing the REER are productivity differential and net foreign assets.

Our estimation results for Slovakia show a phenomenon typical for transition economies: the increase of net foreign assets or productivity differential leads to an appreciation of real exchange rate. Applying Johansen method, 3 models were estimated. Model 1 estimates the REER, where the RER is calculated on the basis of CPI and 2 main trading partners (Germany and Czech Republic). Model 2 and model 3 differ from model 1 in calculating RER. Model 2's RER is based on 9 main trading partners, while the price measure for model 3 is PPI. Furthermore in all models, the behavioral effective exchange rate was decomposed into permanent and transitory component by two different approaches: Gonzalo-Granger decomposition and Hodrick-Prescott filter.

According to model 1, the REER appreciated by an annual average of 3.8 % in the period between 1993-2006. The average appreciation in this period estimated by model 2 is 4.0 %. That means, it is unnecessary to extend the RER to the basis of 9 main trading partners, because of the similar estimation results and higher level of forecast uncertainty. So we do not perform forecasts based on model 2. While model 1 forecasts 4.6 % annual appreciation of real equilibrium exchange rate, model 3 predicts a 4.0 % appreciation per a year.

Comparing different decomposition methods across all models, the estimated misalignments show a similar pattern. Generally we can say, that the Gonzalo-Granger decomposition gives lower values of misalignments than the HP filter. From the beginning of 1993 until the end of 1996, the real effective exchange rate fluctuated around its equilibrium with smooth deviations from is. We found that the Slovak koruna was undervalued in the beginning of 1999 and in 2002, periods of overvaluation appeared in 1997, 2000 and 2004. Since 2005, the REER fluctuated around its equilibrium, but it started to slightly overvalued at the end of 2006.

Based on our calculations, we can conclude that the equilibrium nominal SKK/EUR exchange rate at the period of Slovakia's euro adoption should be around 30.99-31.57 SKK/EUR.

Appendix A

Indicators for Trading Partners

Country	*The percentage of trade volume*	*The weight coefficient*
Czech republic	17.2	0.401
Germany	25.6	0.599

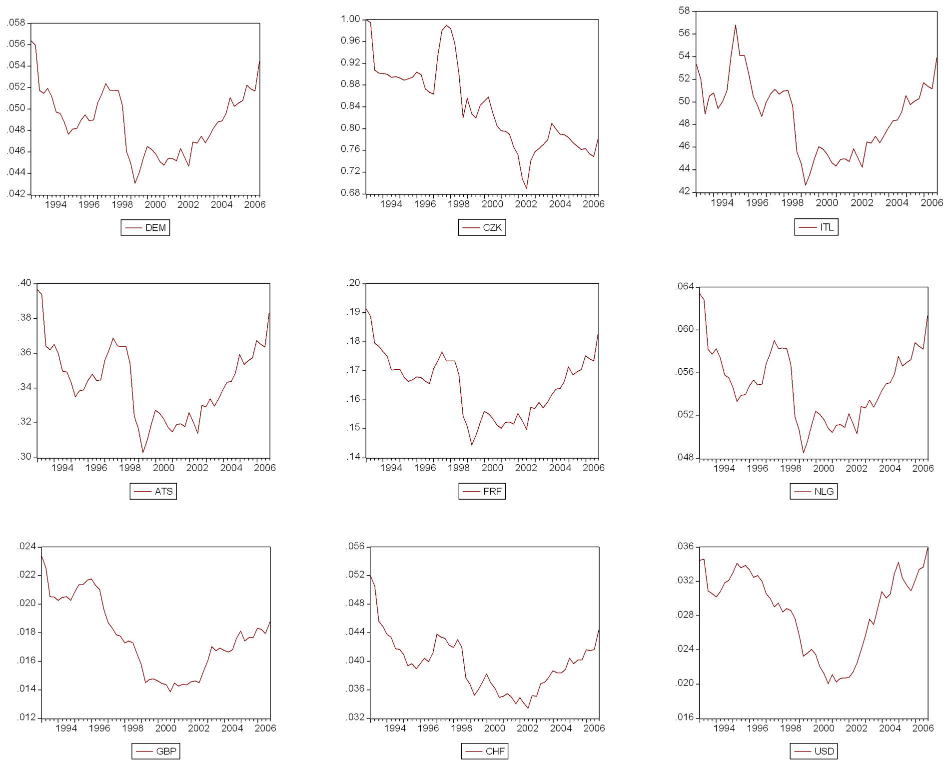
Source: SO SR, 1997-2003 average.

Table A.1: The weight coefficients of trade balance for basic scenario

Country	*The percentage of trade volume*	*The weight coefficient*
Czech republic	17.2	0.253
Germany	25.6	0.377
Italy	7.3	0.108
France	3.9	0.057
Austria	6.0	0.088
USA	2.2	0.033
The Netherland	2.2	0.032
The United Kingdom	2.2	0.032
Switzerland	1.3	0.020

Source: SO SR, 1997-2003 average.

Table A.2: The weight coefficients of trade balance for extended scenario



Source: IMF IFS

Figure A.1: The nominal exchange rates (SKK vs. trading partners)

Country	*Indicator*	*2006*	*2007*	*2008*
Czech Republic	Real GDP growth	6.0	5.1	4.7
	Employment growth	0.8	0.7	0.5
	Productivity growth	5.1	4.3	4.1
	CPI growth	2.5	2.4	2.9
Germany	Real GDP growth	2.4	1.2	2.0
	Employment growth	0.4	0.5	0.6
	Productivity growth	1.9	0.6	1.3
	CPI growth	1.8	2.2	1.2

Source: EC Autumn 2006 Economic Forecasts.

Table A.3: Forecasts of trading partners' macroeconomic indicators

Appendix B

REER Forecasts and Estimation Results

	Fundament	*2007*	*2008*
CPI based REER, basic scenario	Prod. differential	2.6	1.4
	FDI_1	2.7	2.5
	FDI_2	3.2	3.0
	FDI_3	2.1	2.0
PPI based REER, basic scenario	Prod. differential	3.1	1.7
	FDI_1	1.7	1.6
	FDI_2	2.0	1.9
	FDI_3	1.3	1.3

Source: own calculations.

Table B.1: Contributions of particular fundaments to the appreciation of REER (in percentage points)

		1q1993-4q2008
CPI based REER, basic scenario	FDI_1	83.5
	FDI_2	85.3
	FDI_3	81.6
PPI based REER, basic scenario	FDI_1	60.5
	FDI_2	61.5
	FDI_3	59.5

Source: own calculations.

Table B.2: The percentage appreciation of the REER in the period of 1993-2008

	Model 1		*Model 2*		*Model 3*	
	LM-statistics	Probabitlity	LM-statistics	Probabitlity	LM-statistics	Probabitlity
LM(1)	7.0	0.64	3.2	0.97	5.0	0.83
LM(2)	12.0	0.21	13.4	0.07	12.6	0.8
LM(3)	7.5	0.58	9.0	0.43	7.0	0.64
LM(4)	6.8	0.66	3.5	0.94	11.9	0.21

Source: own calculations.

Table B.3: LM test results

The H_0 hypothesis of LM test is that there is no serial correlation in residuals at lag order h (argument $= h$). In all models, the LM test showes no serial correlation on 5 % confidence level.

Vector Error Correction Estimates
Date: 04/16/07 Time: 17:57
Sample(adjusted): 1994:2 2006:4
Included observations: 51 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
Q_E(-1)	1.000000		
PRO(-1)	-0.525401 (0.11715) [-4.48470]		
CUFDI(-1)	-0.003527 (0.00017) [-20.8768]		
C	-2.188332		
Error Correction:	**D(Q_E)**	**D(PRO)**	**D(CUFDI)**
CointEq1	-0.626100 (0.14190) [-4.41235]	0.026547 (0.06778) [0.39168]	-10.31567 (21.3881) [-0.48231]

Source: own calculations.

Figure B.1: Johansen method estimation results for model 1

Vector Error Correction Estimates
Date: 04/16/07 Time: 18:07
Sample(adjusted): 1994:2 2006:2
Included observations: 49 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
Q_E(-1)	1.000000		
PRO(-1)	-0.444746 (0.10344) [-4.29937]		
CUFDI(-1)	-0.003745 (0.00018) [-20.6890]		
C	-2.592125		
Error Correction:	**D(Q_E)**	**D(PRO)**	**D(CUFDI)**
CointEq1	-0.718807 (0.13306) [-5.40229]	-0.018198 (0.05602) [-0.32488]	16.44733 (27.4390) [0.59941]

Source: own calculations.

Figure B.2: Johansen method estimation results for model 2

Vector Error Correction Estimates
Date: 04/16/07 Time: 18:03
Sample(adjusted): 1994:2 2006:4
Included observations: 51 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
Q_E(-1)	1.000000		
PRO(-1)	-0.628233 (0.12723) [-4.93782]		
CUFDI(-1)	-0.002273 (0.00018) [-12.3986]		
C	-1.692455		
Error Correction:	D(Q_E)	D(PRO)	D(CUFDI)
CointEq1	-0.404722 (0.16716) [-2.42124]	0.095107 (0.06748) [1.40938]	-42.05238 (20.7622) [-2.02543]

Source: own calculations.

Figure B.3: Johansen method estimation results for model 3

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