Structural Changes in Money Demand in Selected NMS of EU

MASTER THESIS

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9.1.9 Economic and Financial Mathematics

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Štrukurálne zmeny v dopyte po peniazoch vo vybraných novopristúpených krajinách EÚ

diplomová práca

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I hereby declare that I wrote this thesis by myself, only with help of the referenced literature, under the careful supervision of my thesis advisor.

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Abstrakt

Strapková, Lucia: Štrukturálne zmeny v dopyte po peniazoch vo vybraných novopristúpených krajinách EÚ [Diplomová práca], Univerzita Komenského, Bratislava, Fakulta matematiky, fyziky a informatiky, Katedra aplikovanej matematiky a štatistiky. Školiteľ: doc. Dr. Jarko Fidrmuc. Bratislava: FMFI UK, 2009. 68 pages

V tejto práci sme modelovali dopyt po peniazoch pomocou modelov zahŕňajúcich stavy súvisiace s jednotlivými parametrami a porovnávali ich so štandardnými modelmi s konštantnými parametrami. Stavová reprezentácia modelu, ktorá sa tu odhaduje Kalmanovým filtrom, ponúka možnosť odhadovať meniaci sa vplyv parametrov počas obdobia, kedy krajina podlieha štrukturálnym zmenám, ako sa predpokladá o nových členoch ako aj o Európskej menovej únii (EMU). Pre EMU je evidentne tento prístup prínosný a pre vybrané sledované krajiny spomedzi novopristúpených do EÚ vykazuje tiež lepšie výsledky ako MNŠ, i keď nepotvrdzuje signifikantnosť všetkých sledovaných parametrov. Stavová formulácia modelov nám umožňuje porovnávať zmenu vplyvov jednotlivých faktorov a zároveň ich vplyv porovnávať s hodnotami EMU. Slovinské stavy v tomto porovnaní vychádzajú veľmi blízke stavom EMU, Slovenské a Poľské sa k nim blížia, i keď aktuálne ich nedosahujú. Česká republika a Maďarsko sa výrazne odlišujú a ich výsledky naznačujú možnosť iných signifikantných faktorov.

Kľúčové slová: Dopyt po peniazoch, Novopristúpené krajiny EU, Kalmanov filter, Stavová reprezentácia

Abstract

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In the diploma thesis we modeled demand for money by model in state-space representation that were estimated by Kalman filter. This alternative method allows demand to adjust while the country has undergone structural reforms during the sample period, what is the situation of new member states (NMS) during last years. For Euro area (EA) money demand state-space respresentation proved to be better and appropriate. For the selected NMS are results better than with OLS estimation, despite insignificance of some of the estimated determinants. State-space representation even allows us for comparison of structural changes in effects of considered determinants in selected NMS with the values of the EA. We conclude that Slovenia has the most similar structure of main money demand determinants to the EA. Slovakia and Poland are heading towards the similar weights of studies determinants. Hungary and Czech Republic result to diversify from EA and signal to have other important determinants of their money demand.

Keywords: Money demand, New Members States of EU, Kalman filter, State Space representation

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Introduction

The demand for a money was studied in the first half of the 20th century and research done about monetary policy of European Central Bank (ECB) stimulated the interest for it recently again. Concern of this work is to study the money demand for some of the new member states that have recently entered the European Union, because of what they are expected to soon or later join the Economic and Monetary Union (EMU) as well. The single currency requires monetary union, that will enforce common monetary policy, targets and strategies to achieve for all member states, that have decided to adopt that common currency.

The primary goal of the European Central Bank is to maintain a price stability. The stability of money demand function is crucial to weather ECB should target the interest rate or the money supply. The strategy to achieve the settled target includes inevitably the monetary aggregates. The reference value for a monetary growth is taken as a benchmark for assessing monetary developments. Target for a money growth under this and related assumptions has been set at 4.5 percent per annum. Even though, Euro area has exceeded its target continuously since 2001. In fact, many researchers have detected instabilities especially when data after 2001 were included in the analysis [1]. Such a result casts serious doubts concerning the rationale of monetary aggregates in the monetary strategy of ECB. The candidates for EMU have also introduced direct inflation targeting in order to unify with union. Despite enormous research made on estimating stable aggregate money demand for Euro area prior to adopting a common currency in EMU, for example [2], the recent empirical studies have not find a stable money demand function for Euro area yet. Without stable money demand function it correctness of the policy of ECB doubtable.

Lately, a search for a stable long-run money demand function takes world-widely two directions. First of them looks for a new variables that will make the function stable in long-run. This attempt of Hamburger (1977), for example, found that average dividend to price ratio on common stocks as a measure of opportunity cost [7] resulted in a stable money demand function. Similarly, Heller and Khan [8], included entire term structure of interest rates that works comparably well.

Second direction of analysis of a money demand function focused on weather an incorrect definition of money could be the reason, why the demand function became unstable. Computer and technology development, new financial instruments, introduction of new payments mechanisms and cash management undergone rapid changes in last decades and consequently a structure of wealth portfolio changed rapidly. The financial innovation may cause the conventional definition of money assets to refer poorly in circumstances nowadays. These thoughts led economists to study rather broad definition of money M2, for example in Australia [3] or China [4], or even wide-area definition of money demand M3 [1] [6] [5]. Golinelli with Pastorello [6] show that the area-wide money demand is more smooth and less subject to shocks than the single-country ones. In addition to that evidence, Calza and Sousa [13] do argument that despite cross country differences, the Euro area money demand as a unit is more stable than other economies.

The new member states of the European Union (EU) have committed by entering EU to join the EMU, too. Different historical backgrounds, changes in political regimes as well as technical and financial developments make it hard to bring them to the equal level, therefore their reactions for impulses of the ECB monetary policy can be different in volume and sometimes even in opposite directions. Numerous studies have been made to show the impact of difference in preferences among countries within EMU, for example in papers of Fidrmuc and Korhonen [11] or Fidrmuc [10], and implications of macroeconomic discrepancies between EMU countries. Reflecting the view of Restoy and Buisan [12], countries display noticeable heterogeneity in terms of economic development, exposure to shocks and adjustment mechanisms. In spite of that, it does not seem to prevent a relatively high degree of similarity in patterns of country cycles. Therefore, they conclude, that problem is not a deficit of homogeneity so much as a possible deficit of flexibility for each economy to adjust smoothly to country specific developments. As a result, strong emphasis is given to equalization or balancing of the economic priorities before entering the EMU, to smoother their integration into the union later.

New member states (NMS) took part in transition mechanism and in The European Exchange Rate Mechanism (ERM II), which aims to stabilize and reduce the fluctuations in exchange rates, in order to get prepared for Euro. These economies introduced many reforms, which stimulated development and growth in order to be able to meet competition after entering the EMU. Despite of an extreme effort, not all of the countries perform equally well in that task. Some have adjusted for a new coming monetary policy very well, others need more time to be ready for common currency and monetary policy. Country specific factors, differences in industries structure, in national inheritance, in abilities and sources are going to be also challenge for common monetary policy to be handled.

It this work, we would like to study the determinants of money demand within selected new member states of EU. Are these countries ready to enter the EMU? Will their response to monetary policy of EMU be similar or it can be contra-productive for the monetary system of single country? Taking to consideration these questions, we will show some comparisons between selected new member states of EU and Euro area. We have chosen 5 countries, namely Czech Republic, Hungary, Poland, Slovenia and Slovakia. As far as four of those countries are rather small in comparison with other members in EMU, they need to adjust as much as possible. In the case of Poland we agree that its size may enforce some influence on the common monetary policy of ECB. The better are all NMS prepared before entering the EMU, the smoother will be their later introduction of the common currency. Specifically, we concentrated on formulation of a money demand function that would give a valuable reference of significance of its main determinants. We compare the elasticities for different macroeconomic determinants effecting money demand, to confirm or disprove their preparedness for Euro adoption. At the same time, we analyzed the structural changes during the last 14 years of preparatory period for each of these countries. We will monitor these changes in several specifications of money demand. In purpose to allow the model to cope with the adjustments that were made during previous years, we use state-space representation of the models with time-varying coefficients and Kalman filter to estimate them. At last, we make a comparison between the weight of individual determinants among the selected countries and EU as well.

This thesis is structured as follows. In the first chapter, we introduce the classical theories relevant for modeling demand for money. In the second chapter, we explain how we constructed the data set of the variables needed in for the models. Third chapter is devoted to explanation of a Kalman filter, an econometrical algorithm that will be used for estimating models formulated in chapter four. Finally, the fifth chapter examines achieved results and discusses their implications.

Chapter 1

Theory of money demand

Households' and firms' desire to hold money assets is represented by a money demand. Demand for money has been studied for about a century. Different models have been created in order to explain why do individuals and companies hold money balances which include cash as well as checkable deposits and their close substitutes. Recognizing well factors that affect this demand is necessary for formulating a reliable model that can credibly assume a level of money demand. We will start with models that include amount of money needed for transactions and therefore their solutions gives the volume of desired money balance together with the other results. Afterwards, we explain models that directly describe money demand by the means of its determinants. Hubbard's text [16] and Mishkin's work [14] provided us with a good overview of that topic and research already done in the field, for deeper explanations of the models presented we followed [17] from Blanchard and Fischer and several articles mentioned later on.

1.1 Macroeconomic foundations

Money plays three crucial roles in the economy. Money was firstly created in order to simplify transactions. It helped a market to avoid a double coincidence of wants and reduced transportation costs in comparison with barter. What is more, money introduction brought more flexibility and exactness of the prices as far as a value expressed by money was more accurate than a value measured by non-divisible goods. Beside its function as a medium of exchange, it rationally became a store of value and also a unit of account.

The studies about money demand determinants were done since beginning of the 20th century and the initiatory model was introduced by Irving Fisher. He tried to explain how much money would individuals be willing to hold relatively to their income. Later on, he replaced an income with transactions volume, because neither for income nor for output available data were good approximations of money needed for transaction purposes. In his book *Purchasing Power of Money* [15] he published the first model of money demand where he analyzed the relation of transactions and real money balances, which are the nominal money balances adjusted for price level. At the times of Fischer, there were not precise data available, but afterwards they were collected better, so subsequent economists generally used nominal income, measured by gross domestic product(GDP), to study the volume of transactions. Including replacement of transactions by income, theory of Fischer is known as a *Quantity Theory of Money*. In terms of the real balances, it is represented by formula

$$\frac{M^D}{P} = \frac{Y}{V}, \tag{1.1}$$

where M^D measures demand for money, P denotes a price level, Y stays for an output and V is a velocity of money. Velocity was a newly established variable representing the average number of times a unit of currency is used each year in the economy on purchase of goods and services. Fisher assumed that velocity was constant and proposed that demand for real money was proportional to level of transactions.

This equation can be simply turned into the *equation of exchange*, $M \times V = P \times Y$, which states that quantity of money multiplied by velocity equals nominal spendings in the economy. As it is expected, there is positive relation between money demand and output. Monetarists considered nominal amount of money as an exogenous variable that does not or just poorly depends on an interest rate, so they used only a nominal GDP as an explaining variable for the real demand for money balances. He emphasized technological factors and ruled out any possible effect of interest rates on demand for a money in the short run.

Quantity theory of money was couple of decades later deeply examined and modernized by macroeconomist Milton Friedman.

1.1.1 Clower constraint

Building a model that formally explains why and when is money used for transaction instead of credit, has proved to be a difficult task. Consequently, most of the research on macroeconomic equilibrium has simplified that problem and started with an assumption that money must be used in all or most of the transactions. "Money buys goods, goods buy money, but goods do not buy goods." This assumption is known as the *Clower* or *cash-in-advance constraint*. It can be given either explicitly or implicitly. Simple version of that constrains, in discrete time, is that all bought goods during one period have to be paid by the money available at the beginning of the same period.

Consider the following inter-temporal utility maximization problem :

$$\max \sum_{t=1}^{\infty} (1+\theta)^{-t} u(c_{1t}, \ldots, c_{nt})$$
$$\text{subject to} \tag{1.2}$$
$$\sum_{i=1}^{n} P_{it} c_{it} + M_{t+1} + B_{t+1} = Y_t + M_t + B_t(1 + i_{t-1})$$

where n is a number of goods for consumption, money M earns no interest and bond B earns nominal interest at rate i. Utility at the time t of consuming goods $c_{1t}, \ldots, c_{nt}$ is given by a utility function $u(c_{1t}, \ldots, c_{nt})$. By summing up discounted utilities with the rate θ over all times, we gain a life-time utility. To avoid negative holding of money, we add the Clower constraint

$$\sum_{i=1}^{n} P_{it} c_{it} \leq M_t, \tag{1.3}$$

that restricts individual to start each period with enough money to purchase consumer goods without debts. The optimal character of the problem certainly generates solution that will give demand for money exactly equal to the planned purchases. However, this version is too simple. One of the reasons is, that it allows cash to be used during the period only once, in other words, it restricts velocity for one. Furthermore, money demand is unrelated with an interest rate. This version can be modified in several ways into a more general model.

Lucas and Stokey [19] introduced as a mean of payment beside a cash also certain level of a credit as another form of legal tender. That separated consumer goods into two categories - those purchasable only for a cash and those available also for a credit. Disadvantage of that model was a complicated distinction between two types of goods.

Another modification included uncertainty, where an individual had to make up a decision of an amount of money to hold before the announcement of the state of the nature. This definition of the problem, including cash-in-advance constraint, forced people to be more precautions and rather have higher demand for money, in order to be able to purchase wanted goods in any case. This received appellation as *Precautionary demand for money* and has been studied by many well known economists during years. Among others, let us mention Whalen (1966), Goldman (1974), Krugman, Persson and Svenson (1985)and Woodford (1984). We will present a simple model that shows the main conclusions and implications of that approach.

Let us restrict for an individual that considers only two time periods ahead and he can purchase only one type of good. He needs to make his portfolio decision about allocation of his wealth between money M that can be used in any period, but it forgoes any earnings on interest rates, and bonds B that can not be used in first period, but in second period they can be used together with their interest gain at rate i. The individual will wish to spend his entire available sources either in the first period after decision or in the second period, with a probability q and $1-q$, respectively. Uncertainty of the individual at the time of decision about the time when will the need for sources arise, enforces his precautionary demand for cash. Let us formalize the maximization problem of such an individual. If we denote the time of the decision as t, then variables M_t and B_t respond to individual's endowment allocation between money balances and bonds at that time. In case of consume in the first period individual will be able to use only cash, so his real consumption will be $c_{t+1} = M_t/P_{t+1}$. If he consumes in later period, his consumption will be $c_{t+2} = [M_t + B_t(1+i)]/P_{t+2}$. For making optimal decision he maximizes his expected utility, which is weighted average of possible utilities given by

$$qu(c_{t+1}) + (1-q)u(c_{t+2}). \tag{1.4}$$

By replacing the consumptions according to relations given above it can be rewritten as

$$qu\left(\frac{M_t}{P_{t+1}}\right) + (1-q)u\left[\frac{[M_t + B_t(1+i)]}{P_{t+2}}\right]. \tag{1.5}$$

Partial derivation with respect to M_t is

$$\frac{\partial U}{\partial M_t} = qu'(c_{t+1})\frac{1}{P_{t+1}} + (1-q)u'(c_{t+2})\frac{1}{P_{t+2}} \tag{1.6}$$

and partial derivation with respect to B_t is

$$\frac{\partial U}{\partial B_t} = (1-q)u'(c_{t+2})\frac{1+i}{P_{t+2}}. \tag{1.7}$$

By subtracting a first-order condition with respect to B_t (1.7) from a first-order condition with respect to M_t (1.6) we receive the first-order condition for the maximal utility:

$$\frac{qu'(c_{t+1})}{P_{t+1}} = \frac{(1-q)iu'(c_{t+2})}{P_{t+2}}. \tag{1.8}$$

The questioned demand for money is obtained by solving that differential equation. Under additional assumption of relative risk aversion, that is measured by a variable γ, money demand is given by formula

$$\frac{M_t}{Y} = \frac{1+i}{(1+\pi)^{(\gamma-1)/\gamma}[(1-q)i/q]^{1/\gamma}+i}, \tag{1.9}$$

where π is an inflation rate between two periods and Y is entire endowment. If the individual is a risk-neutra, his γ is equal to 1 and we see that inflation will not effect his decision, all his sources will be hold in a form of bonds. On the other hand, if the individual is risk-averse, his γ is bigger than 1 and decision depends on both nominal interest rate and inflation. That describes precautionary demand for money balances.

Presented models already improve the approximation of demand for money, but they are unfortunately not a lot applicable for reality, as far as their assumption that bond is not negotiable during the periods at any price, is unrealistic and strongly doubtable nowadays.

1.2 Microeconomic foundations

The approach of the Cambridge economists was different from the one of monetarists. Those economists stressed the individual choice in their models. Especially, Keynes paid more attention to the motives of holding the money. He distinguished between three different sources of reasons for holding money. First of all, according to his decomposition, it is *transaction motive*, that considers time inconsistency between payments and transactions made by individuals.

Rest of the motives originated in treating money as a particular type of a financial asset among many others. This approach focuses beside transaction function of money on potential return of assets as an additional motivation. Second motive considers risk aversion of the individual. His tendency to avoid risk leads him into allocation of his wealth in mixed combination of low risk-low return assets (here it was money form) and high risk-high return assets (represented by bonds or equity). This motive is because of that called *portfolio allocation motive* or *precautionary motive*.

Third reason is *speculative motive*.I t is formed on the individuals' ambition to earn on money by holding a liquidity, because it bears interest rate. This motive is proportional to interest rate, the higher potential changes in interest rate, the higher endeavor to speculate.

Due to the contributions of James Tobin from Yale University and William Baumol from Princeton University, interest rate was introduced into the models of demand for money. They worked independently, even though both concentrated on transactional motive and developed similar models for transactions demand. Couple of years later, David Romer from Harvard University, consolidated their theories and expanded them into a general equilibrium [18].

1.2.1 A general equilibrium Baumol-Tobin model

Romer considered model for continuous time, where money emerged together with interest-bearing bonds. Individuals could hold either money or bonds, but they could at any time exchange bonds for a money at additional cost. Cash-in-advance constraint restricted purchases to be paid only by cash. Individuals were deciding about a consumption and dividing the rest of their wealth between liquid money and interest earning bonds.

Model is constructed for overlapping generations in continuous time. At each instant an individual living for a time T is born and civilization is equally distributed with an age from 0 to T. Firstly, we analyze the economy from the point of view of an individual at certain time point and limit it to the steady state, when the population is constant. Later we enlarge the analysis for dynamic dimension and consider it as a problem of whole economy, generating optimal aggregate demand for money.

Individual's decision problem

Each member of civilization receives at birth a wealth Y, that he can store in two ways. Either he keeps them as bonds at interest-bearing real rate r or in the form of money, where his return is negatively weighted by inflation as $-\pi$. Difference between returns is given by nominal interest rate, $r-(-\pi)=i$. If it is positive, holding bonds gives him higher return then keeping money. But Clower constraint must still hold, therefore individual needs to purchase good for cash and bonds cannot be used for that purpose. On the other hand, bonds can be sold at any time at utility cost b per transaction, no subject to the time or amount sold. Selling bond has in model same meaning as withdrawal from the bank account, and in that case the selling fee can be interpreted as a cost of the trip to the bank. Prime decision about storing form of the wealth given at birth is costless and the amount of part stored in form of money we denote by M_1. Assuming logarithmic utility of consumption, that is not subject to discounting over time, the inter-temporal utility function of an individual is given by

$$U = \int_0^T \ln(c_t)\,\mathrm{d}t - Nb. \tag{1.10}$$

While the first component of the function represents the sum of instantaneous consumptions during life-time, the second component enumerates the cost of transactions made during life, as far as variable N represents the number of made transactions, or equally trips to the bank. The first transaction (trip) takes place at time δt_1 and volume of M_2 that will be available from then on. Obviously is optimal to withdraw balances used throughout next period and leave the rest earn on interest, so we can imply, that it is the amount available for next period, till the second withdrawal does not take place. Total number of transactions is $N + 1$, where the prime decision at birth is included.

To summarize the problem of individual, in the presented model, he needs to decide on number of trips, their timing, amount of withdrawals and continuous plan for consumption during life. By solving the Utility maximization problem the following solutions are derived. For detailed derivation follow [17].

First, individual decides to withdraw at equidistant times. With regard to $N + 1$ planned trips it gives constant interval of length $\mu = T/(N + 1)$. Second, the *optimal interval between following trips* is be given by

$$\mu = \sqrt{\frac{2b}{i}}. \tag{1.11}$$

The optimal interval is increasing function of the cost of the trip and decreasing relatively to the nominal interest rate i, that constitutes the opportunity cost of holding liquidity instead of bonds. At third, the *amount of real balances withdrawn* can be jointly expressed for each transaction by formula for trip j as

$$M_{j+1} = \left(\frac{Y}{N+1}\right) \exp(r\mu j) \text{for } j = 0, \ldots, N. \tag{1.12}$$

Therefore, the real withdrawals are equal parts of initial endowment increased by time in size at rate r, the rate of return earned by their holding in a form of bonds. Withdrawals would be nominally equal only in case of zero return on bonds. Finally, the *consumption* expressed for each time point t between trip j and trip $j+1$ as a function of withdrawal M_{t+1} made at time μj is given by

$$c_t = \left(\frac{M_{j+1}}{\mu}\right) \exp[-\pi(t - j\mu)], \text{for } j = 0, \dots, N. \tag{1.13}$$

Thus, consumption of money available for the certain period is a decreasing function of the inflation. Plugging (1.12) into (1.13) formula can be rewritten as

$$c_t = \left(\frac{Y}{T}\right) \exp[r\mu j - \pi(t - j\mu)], \text{for } j = 0, \dots, N. \tag{1.14}$$

Therefore, the level of utility achieved by making those optimal choices is

$$U^\star = Tln\left(\frac{Y}{T}\right) + \frac{rT^2}{2} - \frac{(r+\pi)T^2}{2(N+1)} - Nb, \text{ where } N = T\sqrt{\frac{i}{2b}} - 1. \tag{1.15}$$

Accordingly, utility is an increasing function in real interest rate r and decreasing function in inflation π. These implications correspond to the rational expectations from economical theory.

In order to imply conclusions for money demand, we consider amount withdrawn at particular time to be a demand for money balances at the time of trip. Combining (1.11), (1.12) and (1.13) results in money demand for time μj:

$$M_{j+1} = \sqrt{\frac{2b}{\pi + r}} c_{\mu j}. \tag{1.16}$$

Apparently, the amount of money that is an individual willing to hold is decreasing in nominal interest rate i and it is a linear function of his consumption.

Aggregate demands

Turning back to the model characteristic, general equilibrium of the civilization in model is topic for this section. Considering the fact that in steady state is population constant and equally distributed, the amount of money hold by society in particular time point equals the sum of the amounts of money hold by one individual throughout his life. To derive an aggregate demand for consumption, for bond or for money we apply similar procedure. Integrating money demand of an individual for each period separately and consecutive summing them up gives us total monetary wealth of population at the time. In order to normalize that result for a population of size one is necessary to divide the result by T. By that procedure we gain for money demand formula

$$M = \left(\frac{1}{T}\right)\left(\frac{Y}{T}\right)\left[\frac{\exp(rT)-1}{\exp(r\mu)-1}\right]\left[\frac{\exp(-\mu\pi)+\mu\pi-1}{\pi^2}\right]. \tag{1.17}$$

The aggregate consumption and the aggregate demand for bonds can be derived by the same method. Despite extreme simplification of the underlying model, the solutions are complicated and not clear for interpretation. Nonetheless, some derivatives can be calculated. For a money demand is worth implication that holding Y and r constant, aggregate willing to hold a money decreases with an increasing nominal interest rate.

Formulation of Romer's model beside characterizing the *demand for money* gives also some other useful implications about *consumption-saving choice* as well as *effects of money growth*, which we will not discuss in this text widely. We will concentrate on conclusions about money demand.

To enclose the figure of the economy for society, variables that were up to that point considered to be exogenous as an inflation rate π, a real rate of interest r and a price level, needs to be now inserted into the economy and determined by its organs. Other members of the model for closed economy together with individuals are firms, banks and government. Their roles can fully simulate the actual economy. Government's only task is to issue money, that is transfered at constant rate to the newborn individuals. Banks communicate with firms and individuals. They hold savings of individuals and allow them to sell there their bonds for money and consequently sell those bonds back to the firms, which have issued them, in exchange for money, that companies earned by selling their goods. Role of firms is triple. First of all, they own technology on which they produce goods for consumption. Sales of those goods earns companies return at real interest rate r. Their capital is formed out of individuals' wealth, that they had received in exchange for selling goods. Using it, they can repurchase bonds from banks. Finally, the endowment of young generation is invested in firm in exchanges for bonds, in order to earn on the return that firm makes. This is simplified version to give a reader idea how closed economy fits into the presented model.

1.2.2 Money in the utility function

Studying exact ways of money transformation and specifying them in the Clower constraint became cumbersome and impractical within more complicated models. Therefore, another ways of including money into the economy have been investigated. The effort brought approach called *Money in the utility function (MIU)* where money directly enters agents' utility function capturing the *liquidity services* provided by money. An example of such a model is the model of Sidrauski [20]. His concept allows both the real money and the consumption to enter the utility function. It is constructed for the economy populated by infinitely living families and with constant growth of population. Each out of the equal households solves his maximization problem. Conclusions show that although in equilibrium every household receives transfer of the wealth proportional to their money holdings as they are identical, when making decisions each of them considers amount of transfer it receives as given and independent of holden money balances.

The MIU approach is offering way out of problematic setting of Clower constraint. On the other hand, from results we do not have clear idea of transactions and role played by money. What is more, in approach where money is inserted in production function, it is doubtable what restriction to impose on such an objective function. Research in that question was done by Fischer [21].

1.3 Direct models of money demand

As we showed above, forming demand function for real money balances has several different approaches. It was explained, that putting money straight into the utility function instead of using cash-in-advance constraint limits the findings on money function in economy. Even though, some economists took even more dramatic shortcut and tried to specify directly demand for money.

1.3.1 Keynes' liquidity preference theory

Models for money demand, that rely only on transaction motives do not completely explain movements in real balances. As it was suggested before, individuals have also portfolio allocation motives, that are influenced by various factors on the market and by individual's characteristics. Personal factors are income and wealth of concerned one, his attitude to the possible risk and as the last, but the least, his informedness and know-how. The market determines the expected return on different forms of money assets and their liquidity.

John Maynard Keynes stressed the choice between money and bonds. His *liquidity preference theory* believed that transaction reasons and precaution were irredeemable motives for holding money balances, but at the same time he emphasized the role of the speculative motives, that were sensitive mainly to the interest rate. He summarized it in his book *The General Theory of Employment, Interest and Money* [25].

From the point of view of the speculative motive, Keynes considered individual's choice to be strictly depending on the comparisson of the expected return on money and expected return on bonds, that is described by a nominal interest rate. Including the other reasons for holding real money balances, we can symbolically express his model of money demand as

$$\frac{M}{P} = L(Y, i), \tag{1.18}$$

where a demand for real balances is interpreted as *liquidity function* L depending on real income Y and nominal interest rate i. Liquidy function captures liquidity preferences of the assets. Whereas the desire to hold money raised with higher income, money demand is positively related to Y. On the other hand, the higher is the nominal interest rate, the better is to keep the value in the form of bonds and earn the interests, so it lowers the demand for money. Therefore, there is a negative relationship between demand for money and nominal interest rate.

Let us rewrite the Fischer's equation of exchange (1.1) into the form

$$V = \frac{PY}{M}. \tag{1.19}$$

Afterwards, we rearrange the Keynes' money demand function and multiply both sides by Y to get equation

$$\frac{PY}{M} = \frac{Y}{L(Y, i)}. \tag{1.20}$$

Subsequently, by plugging together both equations we can show that

$$V = \frac{Y}{L(Y, i)}. \tag{1.21}$$

Given formula denotes velocity's dependence on an income and on an interest rate. Velocity fluctuates conditionally to changes in income or in interest rate and does not stay constant as it was assumed by Fisher.

1.3.2 Friedman's money demand model

One more theory served famous macroeconomist Milton Friedman, who beside other contributions to the economy and statistics, analyzed in detail also Quantity theory of money. Instead of analyzing reasons for holding money as Keynes did, he reexamined the work of Fisher and others, who concentrated purely on determinants of money demand. On the other hand, he had something common with Keynes approach as well, because he treated money like any other asset.

He explained willingness of the people to hold money balances in broader measure than just a cash and checkable deposits for short period. He used theory of asset demand to derive the demand for money, that is a function of expected return on money, return on other substitutes for money and permanent income. He modeled demand for real broad money $\frac{M2}{P}$, where $M2$ stays for monetary aggregate M2. As an explaining variables he considered wealth, that should be proportional to permanent income Y_P, difference between returns on holding money i_M or financial assets (bonds) i_b, difference between returns on holding money i_M or equity i_e and difference between return on money and expected return on durable goods π^{exp} presented by an expected inflation. He published this theory in [26]. Symbolically, we can write Friedman's model for real balances as

$$\frac{M}{P} = L(Y, i_b - i_M, i_e - i_M, \pi^{exp} - i_M). \tag{1.22}$$

When a wealth rises, demand for a real money increases too, while for growing opportunity costs,either $i_b - i_M$ or $i_e - i_M$ or $\pi^{exp} - i_M$, demand reacts each time negatively, by decreasing.

In comparison with the model of Keynes, Friedman considered all kinds of assets while Keynes distinguished only between money and the other assets. Friedman's model included differences between rates of return because Friedman believed that it is not their size that matters. He allowed in model also return on money balances while Keynes assumed it to be zero.

1.3.3 Cagan's model of hyperinflation

Already Milton Friedman maintained that there is a close relation between inflation and money supply. Realizing that fact, it is not much surprising, that it was his doctoral student, who brought inflation into the money demand function. This alternative concept was introduced due to Phillip Cagan, who studied money demand in periods of hyperinflation [22]. The quotation from his work [22] explains the main idea of his theory. "*During hyperinflation the amount of real cash balances changes drastically. At first sight these changes may appear to reflect changes in individuals' preferences for real cash balances – that is, shifts in the demand function for the balances. But changes in real cash balances may reflect instead changes in the variables that affect the desired level of the balances.*" Cagan constructs his model on hypothesis, that changes in demand for real balances root in variations of expected rate of change in price level.

In previous settings we have considered the rate of growth as given. Note that in real economy creation of money implies certain level of revenue for government, namely *seigniorage*. Seigniorage is one of the government's revenue sources, that is minor in low-inflation periods, but in times of extream hyperinflations, money printing is often the only source of government's revenue. Those exceeding conditions in hyperinflation period allow us for an assumption that real variables move sufficiently slow compared to the price level in order to be considered as given.

The basic Cagan's model consists of two equations. First of them, it a real money demand function depending on an expected rate of inflation $\pi^\star$ given by formula,

$$m = \frac{M}{P} = c\exp(-a\pi^\star), \tag{1.23}$$

where c is a constant and a is amount of wealth per capita. The higher is the inflation, the lower will be demand for real money balances. It is crucial to mention, that as far as an output is given by assumption, it is included in constant term c. The real interest rate is constant as well, so it is included in a constant term, therefor function depends only on expected inflation rate. The higher will be expected inflation rate, the lower will be demand for real money balances. As far as a demand for real money balances in the equilibrium equals real money stocks, the equation 1.23 is interpreted as equilibrium.

Second equation specifies formation of expectations. While in previous model we assumed perfect foresight, following Cagan, for this model we use *adaptive expectations* about inflation. Under the adaptive expectations, understand expectations of inflation rate adjusted for the current inflation. If the current rate of inflation exceeds the expectations, new expectations about inflation decrease. This process can by captured by equation

$$\frac{\mathrm{d}\pi^\star}{\mathrm{d}t} = b(\pi - \pi^\star), \tag{1.24}$$

where b indicates the speed at which individuals revise their expectations. By integration it can be rewritten as

$$\pi^{\star} = b \int_{-\infty}^{t} \pi_s \exp[b(s-t)]\mathrm{d}s. \tag{1.25}$$

Given the dynamics of the money growth, equations 1.23 and 1.25 determine the dynamics of the inflation rate.

Cagan's conclusion is truly remarkable. In contrast with the apparent chaos typical for hyperinflations, Cagan argues that there is stability, a stable relationship between real money demand and expected inflation. Publication of Cagan's article generated a significant body of work, a number of leading macroeconomists either reexamined or extended Cagan's model, most notably Barro (1970), Sargent and Wallace (1973), Frenkel (1975, 1976a, 1976b, 1977, 1979), Sargent (1977), Abel et al. (1979), Salemi (1979), and Salemi and Sargent (1979). In addition, monetary economists today often refer to a *Cagan demand function* when modeling the real value of money. This shortcut proved to be useful in economical issues dealing the relations between seigniorage, deficits and inflation.

Chapter 2

Data Sources and Formation of Times Series

This section describes the main guidelines that were followed in order to form a database used for the estimations. Due to the availability, compatibility and appropriateness of the data, we decided to analyze period of 14 years since 1995q1 till 2008q3. By taking quarterly frequency of the data we got 55 observations for four selected countries (Czech Republic, Hungary, Poland, Slovakia) among NMS of EU and also for Euro area data. In case of Slovenia, we shortened the period till 2006, that limited number of the observations for 48, as far as Slovenia adopted Euro as currency in 2007 and became a part of the Euro area.

The main sources for the data were databases of Eurostat [27] and national banks of selected countries - Česká Narodní banka [28] in Czech Republic, Magyar nemzeti bank [29] in Hungary, Narodowy bank Polski [30] in Poland, Banka Slovenije [31] in Slovenia and Narodná banka Slovenska [32] in Slovakia. Unfortunately, not all data were available from the same time point in each selected country, therefore we had to limit our selves on the period given. In the next sections we describe each used time serie as well as the adjustments that were made because of compatibility.

2.1 Gross domestic product

To estimate demand for money balances, we need to recognize its main determinants. In accordance with overview of done research, we used *the gross domestic* *product(GDP)* as a measure of transactions made within the country. Collected data are in units of millions of national currency. The series were seasonally adjusted and the nominal values for each country are graphically displayed in the Figure 2.1. The variable is in this work denoted by Y. The descriptive characteristics of logarithm of output, which is used in log linear form of the models and denoted by lowercase y, are summarized with the others in Table 2.1.

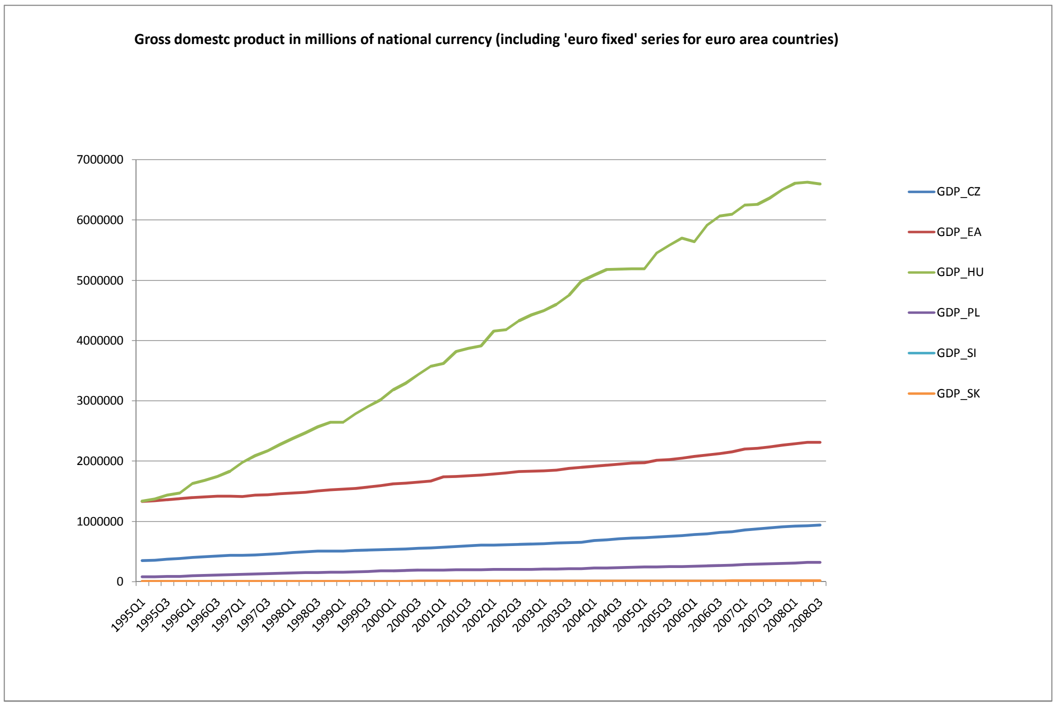

Figure 2.1: Gross domestic product in millions of national currency

2.2 Nominal interest rate

The second most important determinant of money demand among macroeconomic indicators is *the interest rate*. The literature is not unit in decision about which rate to choose. There are more applicable rates and even research done varies in their choice. Some studies even include both short-term and long-term interest rate, like Gerlach and Svenson did in [33]. We decided to consider nominal one month interest rate on deposits as a reference value to enumerate the opportunity costs, supposing it alternates the average rate earned by the assets included in monetary aggregate M2 and hold in the form different from the cash.

We could not include both type of rates as far as they were not available in satisfying long period. Problematic heterogeneity of the data may further in estimates cause wrong results. Domestic interest rate will be denoted by i, if necessary, related country will be given in brackets,i.e. $i(EA)$. Figure 2.2 presents the join graph of interest rates in the unis of percents for all the countries.

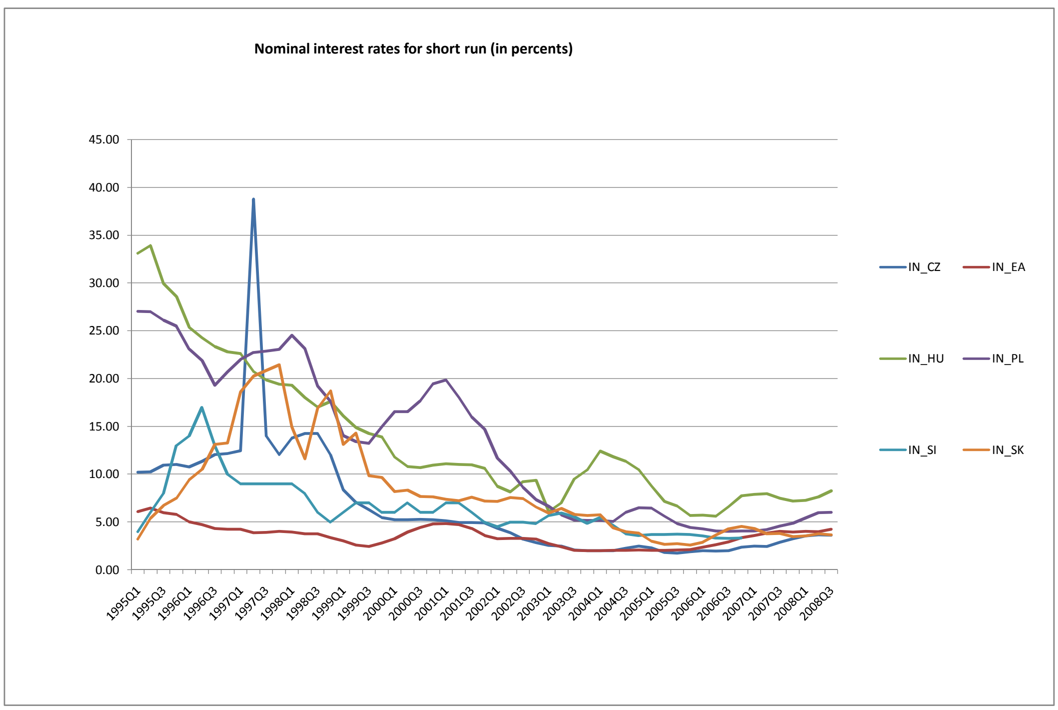

Figure 2.2: Short term interest rates in percents

2.3 Monetary aggregate M2

In order to estimate the coefficients of our models, data for monetary aggregate M2 were used as a reference value of money demand. Unfortunately, during the period that we decided to study, some of the selected countries have already enacted new definition of broad money M2 according to the methodology of ECB. The data counted according to the previous methodology were no longer available, neither new methodology was applicable for the old available data. Therefore, we had to create a substituting data set, that would be unit for a whole examined period. In all national banks, there was reported an intersecting period, when data were counted according to both methodologies, so we used it as a 'chain' in order to chain those two time series together. We chained the time series according to different methodologies.

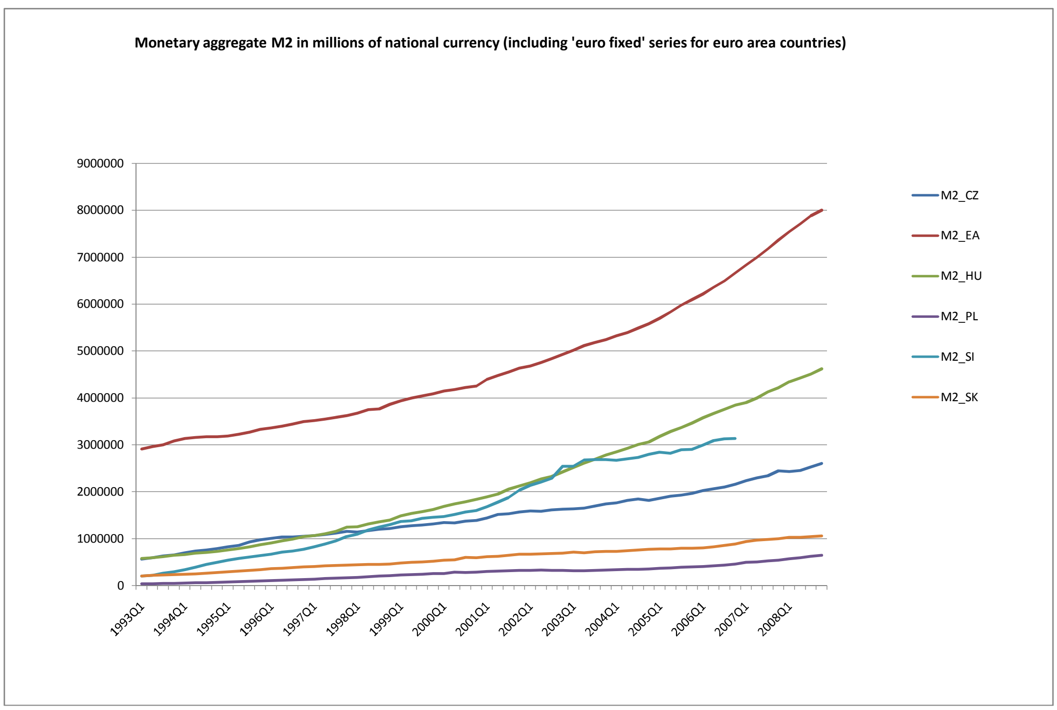

Figure 2.3: Monetary aggregate M2 in the millions of national currency

In particular, we have employed *chain ratios*. Chain ratio is a number given as a ratio of certain factor measured by different methodologies in the same time that will be used as a sample how the change of methodology changes the value of the factor. According to the chain ratio, all numbers given only by the new methodology can be shifted to the base of old methodology. We plugged these approach for money demand, in four out of five selected countries, where the changes were reported. For Slovakia, the introduction of the new methodology took place in 2005, in Slovenia in the end of 2004, Czech Republic and Poland adopted changes in 2002. To ensure correctness of this approach, we considered the sub period analysis. We did test the model on the sub period with the data serie available according to the identical methodology and we can enforce that on the common period of both filters, the values were almost the same, so the results are robust for this adjustment. Symbol M represents volume of monetary aggregate in units of million of national currency and it is graphed in Figure 2.3. Logarithm is used for log-linear model showed, therefore the descriptive statistics of the logarithmic form are available in the Table 2.1.

2.4 Price level

The last factor that we collected was *the price level* during examined period 1995q01 - 2008q03 for the selected countries and Euro area. By knowing the price levels we are able to turn the nominal demand for money into the real volume of desired money balances, which we model. This variable was recounted marked as P and is counted to the base year 2005.

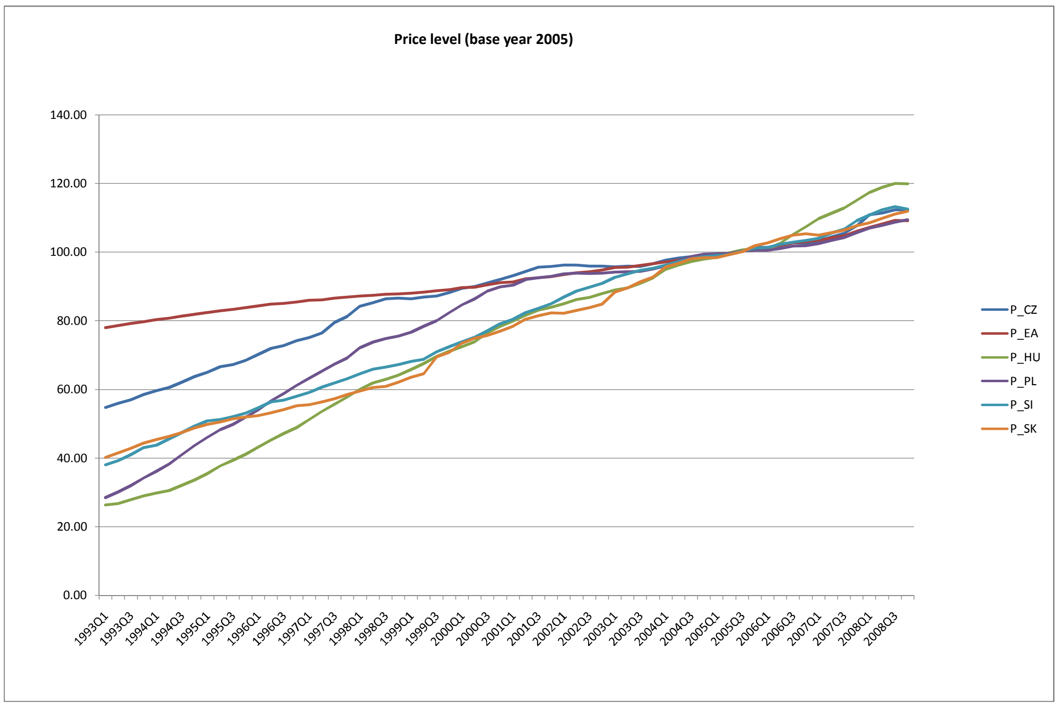

Figure 2.4: Price levels at the base of year 2005

	Mean	*Min*	*Max*	*Std.Dev.*
Czech Republic	**1995q1 - 2008q3 (55 obs.)**			
$ln(M2)$	14.23	13.63	14.74	0.30
$ln(GDP)$	13.30	12.77	13.75	0.27
$ln(P)$	4.51	4.17	4.72	0.14
i	6.52%	1.75%	38.78%	6.06%
Euro Area	**1995q1 - 2008q3 (55 obs.)**			
$ln(M2)$	15.37	14.98	15.88	0.26
$ln(GDP)$	14.37	14.10	14.65	0.17
$ln(P)$	4.54	4.41	4.69	0.08
i	3.58%	2.02%	6.46%	1.13%
Hungary	**1995q1 - 2008q3 (55 obs.)**			
$ln(M2)$	14.53	13.54	15.32	0.53
$ln(GDP)$	15.09	14.10	15.71	0.49
$ln(P)$	4.34	3.57	4.79	0.33
i	13.55%	5.60%	33.61%	7.38%
Poland	**1995q1 - 2008q3 (55 obs.)**			
$ln(M2)$	12.51	11.31	13.34	0.52
$ln(GDP)$	12.12	11.29	12.67	0.37
$ln(P)$	4.43	3.83	4.69	0.23
i	13.02%	4.02%	27.03%	7.87%
Slovakia	**1995q1 - 2008q3 (55 obs.)**			
$ln(M2)$	13.32	12.60	13.86	0.35
$ln(GDP)$	9.09	8.44	9.75	0.38
$ln(P)$	4.36	3.91	4.71	0.26
i	8.06%	2.58%	21.45%	5.12%
Slovenia	**1995q1 - 2006q4 (48 obs.)**			
$ln(M2)$	14.29	13.21	14.96	0.54
$ln(GDP)$	8.45	7.79	8.99	0.36
$ln(P)$	4.34	3.93	4.64	0.23
i	6.50%	3.29%	17.00%	2.99%

Table 2.1: Descriptive statistics of data series used in models (logarithmic forms where applicable)

Chapter 3

Econometric Methodology

In this chapter, first we describe the basic terms in econometric terminology. Econometric time series can be classified in several ways, indicating various methods for modeling time series. For estimation of the linear models with constant coefficients is usually used Ordinary least square method (OLS). On the other hand, because we intend to use mostly the models with coefficients varying over the time, we introduce the *State-space representation* of dynamic systems. For this class of models is widely, but not solely, used method named *Kalman filter* that we derive, explain and also show couple of application in economy in the end of the chapter.

3.1 Time Series and Stochastic Processes

A time series is a sequence of data taken in different, most of the times equidistant, time points. These can be values measured over the time in physics or economical statistics.

3.1.1 Stationarity of stochastic processes

If the time series has time constant mean μ, time constant variance σ^2 and time constant covariance $C(s)$, that depends just on the distance of the points,

$$E(X_t) = \mu \quad \text{for all } t \tag{3.1}$$

$$E(X_t - \mu)^2 = \sigma^2 < \infty \quad \text{for all } t \tag{3.2}$$

$$E(X_t - \mu)(X_{t+s} - \mu) = C(s) \quad \text{for all } t \text{ and any } s, \tag{3.3}$$

than we call the process generating this time series as covariance-stationary or weakly stationary.

The commonly known stochastic stationary processes are white-noise and random walk. Stochastic process is called *white noise* if $\{X_t\}_{-\infty}^{\infty}$ is a sequence, whose elements have mean zero, variance σ^2 ant these elements are uncorrelated over the time,

$$E(X_t) = 0 \text{ for all } t \tag{3.4}$$

$$E(X_t)^2 = \sigma^2 < \infty \text{ for all } t \tag{3.5}$$

$$E(X_t X_s) = C(s) \text{ for all } t \neq s. \tag{3.6}$$

Secondly, if white noise satisfies also independence of all representations across the time, then it is called an *independent white noise process.*

Thirdly, if process satisfies all the conditions for independent white noise process along with the condition of normal distribution of representations with mean zero and variance σ^2,

$$\varepsilon_t \sim N(0, \sigma^2), \tag{3.7}$$

we denote such a process *Gaussian white noise.* This process is well known also under the name *random walk* and can be equaly defined as a white noise, whose differences of consequent representations form also white noise stochastic process,

$$X_t - X_{t-1} = \varepsilon_t \tag{3.8}$$

$$E(\varepsilon_t) = 0; E(\varepsilon_t^2) = \sigma^2; E(\varepsilon_t \varepsilon_s) = 0; s \neq t \tag{3.9}$$

$$\varepsilon_t \sim N(0, \sigma^2). \tag{3.10}$$

3.2 State-Space Representation of a Dynamic System

The classical regression model, $y_t = \beta^T x_t + u_t$ where u_t is white noise, assumes the relation ship between the explanatory and explained variables to be constrant through estimated period. On the other hand, physicians and economists have found many examples were assumptions of the constantly equal influence of determinants on the processes were too restrictive for modeling dynamic nature and society. Therefore, a new phenomenon was introduced into another set of models. The base of those models can be similar to those described above, just the coefficients are allowed to vary over the time, update. Such processes are called *dynamic processes* and we will talk about some of them more in following section. For more detailed explanation read [34].

Let y_t denote a vector $(n \times 1)$ of variables observed at time t. A rich class of dynamic models can include beside visible variables also unobservable variables. Visible variables are represented in $(k \times 1)$ vector x_t while unobservable, exogenous or predetermined variables, as they are called occasionally, are represented in $(r \times 1)$ vector ξ_t. This vector is known as *state vector*. The *state-space representation* of the dynamics in the model of y_t can by given by following system of equations:

$$\begin{aligned} \xi_{t+1} &= F\xi_t + v_{t+1} && (3.11) \\ y_t &= A'x_t + H'\xi_t + w_t, && (3.12) \end{aligned}$$

where F,A' and H' are matrices or parameters of the model with dimensions $(r \times r)$, $(n \times k)$ and $(n \times r)$, respectively. The first equation of model (3.11) is called *state equation* as far as it describes the state at which is 'nature' in the time point $t + 1$. The second model's equation (3.12) is called *observation equation*, because it describes the value of the observable variables at time t. The vector v_t of dimension $(r \times 1)$ and vector w_t of dimension $(n \times 1)$ represent white noise, so it holds:

$$\begin{aligned} E(v_t v_s') &= \begin{cases} Q & \text{if } t = s \\ 0 & \text{otherwise} \end{cases} \\ E(w_t w_s') &= \begin{cases} R & \text{if } t = s \\ 0 & \text{otherwise,} \end{cases} \end{aligned}$$

where Q and R are square matrices of dimensions $(r \times r)$, $(k \times k)$, respectively. Disturbances of v_t and w_s are under the assumption uncorrelated at all time points

$$E(v_t w_s') = 0 \text{ for all } s \text{ and } t. \tag{3.13}$$

The system of equations (3.11) through (3.13) is describing a finite number of observations $\{y_1, y_2, \ldots, y_T\}$ under the assumptions for the initial state vector ξ_1, that this vector is uncorrelated with all vectors v_t and w_t:

$$\begin{aligned} E(v_t \xi_1') &= 0 \text{ for all } t = 1, \ldots, T && (3.14) \\ E(w_t \xi_1') &= 0 \text{ for all } t = 1, \ldots, T && (3.15) \end{aligned}$$

The system of equations (3.11) through (3.15) is complete and can be used for further analysis, but before we do so, let's derive couple of characteristic, that need to hold beside. Recursively plugging the definition of state variable ξ_t, we can obtain ξ_t as a linear function depending on set $\{\xi_1, v_1, \ldots, v_t\}$ like

$$\xi_t = v_t + Fv_{t-1} + F^2 v_{t-2} + \cdots + F^{t-2} v_2 + F^{t-1}\xi_1 \quad \text{for all } t = 2, 3, \ldots, T. \tag{3.16}$$

So we can further imply that v_t is not correlated neither with lagged values of ξ and also w_t is uncorrelated with all unobserved realization of ξ. White noises v_t, w_t of both state and observation equations, respectively, are uncorrelated with all observed variables y_t except the given relation by observation equation between w_t and y_t.

3.3 The Kalman filter

The idea is to represent the analyzed model in the form of *State-Space representation*. For that we can employ a very useful tool of econometrics, named Kalman filter. That is a recursive filter that sequentially updates a linear projection of the model. First of all, we use the most simple form of the State-space representation with all the assumptions given above to be able to derive basic algorithm. Many of these assumptions were later on relaxed and model was modified into several ways to be more complex and be suitable for different kind of problems. Beside other benefits, this algorithm can generate exactly calculate forecasts or likelihood function for Gaussian $ARMA(p, q)$ processes. Further more, it can estimate a vector autoregression, that has coefficients changing over the time, what is the reason, why we do pay attention to Kalman filter.

3.3.1 Derivation of the Kalman filter

As we mentioned, Kalman filter is most of the times used to estimate the parameters of model, where observed variables y_t and visible, exogenous factors x_t are measured by analyst. However, we will assume the opposite to derive the model. All parameters will be known, and we will find formulas to estimate states and realization of the observations in given times. Algorithm is based on a least square error of the forecast of state vector ξ_{t+1} as a linear projection on the data set available till time t and constant,

$$\hat{\xi}_{t+1|t} \equiv \hat{E}(\xi_{t+1} \mid \nu_t), \tag{3.17}$$

where ν_t is information data set available

$$\nu_t \equiv (y'_t, y'_{t-1}, \ldots, y'_1, x'_t, x'_{t-1}, \ldots, x'_1)'. \tag{3.18}$$

That algorithm recursively generates estimates $\hat{\xi}_{1|0}$, $\hat{\xi}_{2|1}$, ..., $\hat{\xi}_{t+1|t}$ step by step. Together with estimate, method calculates the mean square error (MSE) of these forecast, that is denoted by $(r \times r)$ matrix:

$$P_{t+1|t} \equiv E[(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})']. \tag{3.19}$$

To be able to perform the recursion, we need to start forecast at beginning $\hat{\xi}_{1|0}$ where is no information available, so that is the reason why we need to have the initial state ξ_1 given or assumed. Than the first step is simplified into

$$\hat{\xi}_{1|0} = E(\xi_1) \text{ and } P_{1|0} = E[(\xi_1 - E(\xi_1)(\xi_1 - E(\xi_1))']. \tag{3.20}$$

Since now, all the steps can be repeated recursively in each time index.

Assume we know the estimate of the current state vector ξ_t we can calculate estimation of current representation of the observation y_t. Considering the assumption, that current exogenous variables do not enlarge the information about current state above the level of informations given in previous times, considering for the forecast of y_t following prescription,

$$\hat{y}_{t+1|t} \equiv \hat{E}(y_t \mid x_t, \nu_{t-1}) \tag{3.21}$$

and using the observation equation (3.12), we can derive forecast for observation y_t

$$\hat{y}_{t|t-1} = A'x_t + H'\hat{\xi}_{t|t-1} \tag{3.22}$$

and associated MSE

$$E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'] = E[H'(\xi_t - \hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'H] + E[w_t w'_t]. \tag{3.23}$$

Using given notation and characteristic of w_t, it can be simplified into

$$E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'] = H'P_{t|t-1}H + R. \tag{3.24}$$

Given the current value of observation y_t we can improve our forecast of the current state ξ_t as

$$\hat{\xi}_{t|t} = \hat{E}(y_t, x_t, \nu_{t-1}) = \hat{E}(\nu_t). \tag{3.25}$$

This can be simplified into expression

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}). \tag{3.26}$$

Updated forecast needs also updated MSE matrix, that can be simplified as well:

$$P_{t|t} = E[(\xi_t - \hat{\xi}_{t|t})(\xi_t - \hat{\xi}_{t|t})'] = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1}.$$

Next step is to forecast next state variables ξ_{t+1} using the state equation (3.11),

$$\hat{\xi}_{t+1|t} = \hat{E}(\xi_{t+1} \mid \nu_t) = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}). \tag{3.27}$$

The MSE of this forecast can be simplified into

$$P_{t+1|t} = FP_{t|t}F' + Q = F[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1}]F' + Q. \tag{3.28}$$

As we showed, we can express both state and observation by knowing the previous forecasts and all parameters, so that algorithm can be recursively used till we have available parameters.

3.3.2 Some economic applications of the Kalman filter

Kalman filter was apriori envented to show how to estimate the values out of the determinants that are subject to the changes. This approach was used in many fields, particulary it found applications in physics, technical science and robotics. During last decade its has been used all so mode the dynamics of within economic field. Here we will mention couple of examples. For reference we took article of Gurnain [35]. Even though, that basic model was restricted by many different assumptions, later research did derive augmented theory how to relax those assumption and correct the filter to response well.

Time Varying Parameters in a Linear Regression: Demand for International Reserves

Estimating macroeconomic relations within the country that has undergone structural changes during studied period became much easier using Kalman filter to estimate the time-varying coefficients. Bahmani-Oskooee and Brown (2004) used this approach to model the demand for reserves R_t as a specification of countrie's real imports IM_t, a variability measure of balance of payments VR_t and its average propensity to import m_t. The function formula was estimated as

$$\log(R_t) = \beta_0 + \beta_1 \log(IM_t) + \beta_2 VR_{+}\beta_3 m_t + \epsilon_t, \tag{3.29}$$

where βs are assumed to follow a random walk, The problem specification ignores the supply side and takes the level of demand quantity as a realized supply. This model is very similar to our specifications as far as it estimates the time-varying coefficients while given the data of observable factors.

Kalman Filter with Correlated Error Terms: Exchange Rate Risk Premia

Another mentioned example, wethat would like to mention did show applications for generalized Kalman filter where some of the assumptions may be relaxed. The the basic model assumes that the errors in the measurement and transition equation to be uncorrelated. This assumption fails in situation where increase in shock cause the movement of the observable variable as well as unobservable variable under the consideration. Such a problem was realized by Cheung (1993) with the exchange rates. The case of the market for exchange rate an additional, new information effects the rate by jump and at th same time it may lead the risk premium to change. The filter is initialized by unconditional mean and variance of risk premium.

Chapter 4

The Econometric Models

As we have showed in previous chapter, several different theories were examined in order to describe demand for money. Most of the research started in forming models for narrow money M1, that included cash currency, over night deposits and their close substitutes. As doubts about narrow money M1 demand stability continued, researchers used to study rather broad money M2, as we can see for example in many papers from Trehan [36]. We adopt the idea by trying to estimate demand function for real broad money M2.

4.1 Model for real money demand with constant parameters estimated by OLS

Starting from the Keynes' theory, we suppose the transactions and the interest rate are the main determinants of a demand for money balance, as they are defined by M2. The volume of money hold for transactions is measured by the volume of money produced in the country during the year, in other words, by the nominal gross domestic product (GDP) of the country. The speculative motive is captured by the interest rate on the deposits. We combine those motives into one linear function, where we include in addition to both factors also constant term. This simple form of the demand for money balances M2 is expected to have a positive coefficient with regard to output of the economy (GDP) and with regard to opportunity costs, we forgo by holding assets in the form of cash or other substitute included in M2, the coefficient should be negative.

Conventionally, we suppose that exists unity between the price level and the nominal need for money balance. This can be simply interpreted, as an assumption, that if prices rise twice while everything else stays constant, there is no other change in money demand except making it double, and vice-versa. This typical assumption of homogeneity of degree one is called the *homogeneity postulate*.

Note: A homogeneous function $x = f(y, z)$ is said to be homogeneous of degree μ with respect to y if and only if it has the property that:

$$\lambda^{\mu} x = f(\lambda y, z).$$

Therefore, the real money balances are calculated as nominal money balances divided by price level.

Summarizing all given assumption we construct a simple model to characterize demand for real money, where our liquidity function $L(Y, i)$ is given as

$$\frac{M}{P} = aY^{b}\exp(i)^{c}, \tag{4.1}$$

by counting natural logarithm of both sides we gain a linear model

$$\ln\left(\frac{M}{P}\right) = \ln a + b \ln Y + i \cdot c. \tag{4.2}$$

Obviously, the first part $\ln a$ represents the constant term.

Remember that an elasticity compares a percentage change of one variable x with the percentage change of the other variable y, i.e. $\frac{\mathrm{d}log(y)}{\mathrm{d}log(x)}$. While a semi-elasticity compares a level change in one variable with a percentage change of the second variable, i.e. $\frac{\mathrm{d}log(y)}{\mathrm{d}x}$.

Therefore, the second parameter b expresses the elasticity of the real money demand to the income. For example, if income rises by one percentage point, demand for real money will react by increasing with b percentage points. Furthermore, the real money demand semi-elasticity to the interest rate is c and it is expected to be negative, because of that, if interest rate rises by one base point, the demand for money balance will decrease by c percents. Applying this model to our data, we start with ordinary least square method to estimate those coefficients, the results are considerably weak as we will proof later on. Even though, that estimates turned out to be significant, in many cases we report disagreement between expectations over the signs of coefficients and their estimates, or the determinants end up to be insignificant. Disregarding the values of coefficients, that are in some cases satisfying, in some other not, we realize a certain level of inaccuracy of that definition.

4.2 Money demand models with the time-varying coefficients estimated by Kalman filter

Considering the theory of Keynesian approach, we see the root of imprecision in assumption, that these coefficients should be constant over the examined period. Due the financial development many changes were introduced to the financial market during previous two decades. Because of that, there might be some effect of a development reported in coefficients as well. Therefore we suggest a model with coefficients varying over the time, although the main cadre of model stays equal.

4.2.1 Model for real money demand

According to Keynesian approach, we examine linearized definition of money demand as we formulated above, with the only change in character of the coefficients. We allow them to behave as a random walks in order to be able to study possible changes in elasticity with output or semi-elasticity with interest rate. we are able to denote this concept mathematically due to the State Space representation in which we define log-linear model. Because of simplicity, let us denote logarithmic forms of variables for money demand M,price level P and domestic output Y by lowercase letters m, p, y, respectively. Interest rates i stay denoted in absolute levels. Afterwards, we formulate the observation equation as

$$m_t - p_t = rw_t + sv_t^Y \cdot y + sv_t^i \cdot i + w_t \tag{4.3}$$

and assume that state of the nature behaves as a random walk with regard to each considerable factor. Therefor, we gain a vector of states of the nature $(rw, sv^Y, sv^i)^T$ described by following state equations

$$\begin{aligned} rw_{t+1} &= rw_t + v_{t+1}^1 & (4.4)\\ sv_{t+1}^Y &= sv_t^Y + v_{t+1}^2 & (4.5)\\ sv_{t+1}^i &= sv_t^i + v_{t+1}^3, & (4.6) \end{aligned}$$

where all components of vector $(v^1, v^2, v^3)^T$ as well as w are assumed to be a Gaussian white noise. The upper index denotes the time point.

4.2.2 Model for nominal money demand testing the homogeneity assumption

As we argued in the first chapter, naturally is assumed that money demand should be homogeneous in price level. Therefore, we modeled demand for real balances, nominal demand for money divided by price level, as a reference value to be able to filter the states of the nature. At this point, we would like to check this assumption and verify, weather there exists unity, which is stable for all examined periods. We construct the observation equation of this model as

$$m_t = rw_t + sv_t^Y \cdot y + sv_t^i \cdot i + sv_t^P \cdot_t + w_t, \tag{4.7}$$

where variable m denoting logarithm of money demand is explained by logarithm of domestic output y, interest rate i and logarithm of price level P. Corresponding state of the nature is given by vector $(rw, sv^Y, sv^i, sv^P)^T$ and we keep assumption that state of the nature behaves as a random walk with regard to each considerable factor. The set of state equations is

$$\begin{aligned} rw_{t+1} &= rw_t + v_{t+1}^1 & (4.8)\\ sv_{t+1}^Y &= sv_t^Y + v_{t+1}^2 & (4.9)\\ sv_{t+1}^i &= sv_t^i + v_{t+1}^3 & (4.10)\\ sv_{t+1}^P &= sv_t^P + v_{t+1}^4. & (4.11) \end{aligned}$$

In case that there would be fluctuating character of the state related to price level, we will have to lower the explanation power of our previous results.

Chapter 5

The Results of Analysis

First, we will summarize the results from the model estimated by OLS methods and compare the results with literature available. The other models' estimation by Kalman filter allows us to use two different points of views to explain the gained results. By over viewing the estimates of the final states of tested models we will give a detailed description of the values and significance of related determinants for the model, while comparison of the graphical interpretation of the states' estimates over the time allows us to study rather the structural changes and stability of analyzed models.

5.1 Model for real money demand with constant parameters estimated by OLS

Testing a log-linear model of money demand with constant parameters brought us couple of interesting insights into the topic. First of all, we do confirm that this regression is significant for all countries, as it is proved also by information criteria.

Even though, only for two out of six estimated areas are all estimated parameters significant at 1% level, namely for Euro area and Slovakia. While in Euro area demand for money is relatively elastic for money with a coefficient equal to 1.14, in all studied new member countries is money demand relatively inelastic to the output of its economy. Slovakia has the lowest response in money demand to the change in gross domestic output. We believe, that a good result of Euro area is reflecting ECB's interest in being able to estimate demand for money well. For Hungary was filter significant in all determinants at the 10% level. Even though we emphasize the extreme size of positive effect of interest rate on the real money balances. Recalling the values of monetary aggregate M2 in Hungary (Figure 2.3), we can interpret it as a consequence of high depreciation of the Hungarian forint due to which we report an extreme increasing tendency in the volume of money balances.

Surprisingly, the semi-elasticity of the money demand with regard to the interest rate end up to be high and with positive sign in all significant cases. These results are contra the macroeconomic theory and that significantly lower the value and credibility and of given money demand function.

The absolute term in this equation is significant at least at 10% level and varies a lot through the countries. We do subscribe this to the factors that are not included among the explanatory variables, therefore their effect turns into that term. For the exact values reference, see the table A.1. The factors that were not specified might be some identical for the countries, but it may be also some country-specific factor.

Concluding this results, only the gross domestic product as a measure of country's output resulted to be significant for all countries. In case of interest rate, not even in one country it was significant and negatively related to the money demand. What is more, significant positive elasticity appeared couple of times that is contra any economical research done before, because it explains the increase of the money demand by the increase of opportunity costs. Further on, we find that definition restrictive for the countries that are implementing innovations, financial development activities and absorbing many reforms in order to enter the EU or later on EMU. Constant model brings unexplained gap in effort to describe the demand for money, therefore is not applicable for the reality and according to results it is neither confident way for estimating the future money demand. As we mentioned above, our theory finds the possible root of this gap in transformation mechanisms run of all examined countries, therefore, we searched for more flexible model, that would adapt for the development results during the experienced period and would allow us to study the process of structural changes made in countries.

5.2 Model for real money demand estimated by Kalman filter

The formulation of the model, where the coefficients can vary with the time, has several advantages above the common model with constant parameters estimated by

OLS.

Starting with significance at final states, recursive estimation over the examined period brings results comparable with the previous static model. First advantage, gained by time-varying specification of the model, are significant estimates of the states with the expected signs. As it is shown in the table B.1, this model has effects related to the gross domestic product significant and positive for each country except Hungary for which it is non significant determinant. The effect of output is the highest for Euro area, at the level close to 0.9, and Slovenia's effect is comparable, with level 0.74.

Second advantage, the significant effects related to the interest rate are significant in Euro area, Hungary and Poland and they are negative as was anticipated in the related theory. Although neither for any of the studied countries nor for Euro area are all of the specified determinants significant. Euro area has the highest coefficient of output's semi-elasticity to the interest rate, with the value close to -0.6 and the semi-elasticity of Poland and Hungary is nearly -0.4.

The random walk variable RW can be interpreted as the volume of shocks introduced to the desire of holding money balances. It includes factors that we do not specify explicitly among determinants, they are either common for all countries or they might be specific for each country as well. Therefore it is inaccurate to compare this effect among the different areas.

By looking at the graphical interpretation of the estimated states during estimated period, the results clearly show how Kalman filter adjusts his estimates of the nature and uses the information from previous period. As far as there was not available any previous information about the initial state of the nature, we started with the high level of variation for all countries. We allowed model for a wide diversity of possibilities for apriori informations and the structure of the Kalman filter did narrow this confidence interval by consecutive recursive iterations.

With regard to country's output (see the Figure B.1), we assess the trend as quite stable without any extreme fluctuations in all examined countries. It is important to realize, how much did introduction of the Euro stabilize this effect in 2001. The elasticity has been continuously raising by the time upto the final reported level. We experienced smoother estimations since 2001 for Euro area for estimated models so this documents a positive contribution of the policy of ECB. In addition, confidence intervals are positive for the most of the studied period in almost all countries. The only exception is Hungary, for which is this coefficient raising continuously and has reports positive values only since middle of the year 2003, but it has increasing trend on whole examined period. Slovakia seems to have estimated palliations in the trend for money demand elasticity to the output, but it still reports values comparable with Czech Republic. From this point of view, the most stable is evidently Poland. We can see, that in general, we confirm the results of OLS estimation, where only Hungary was insignificant, but we have a little difference in the size of the effects.

Referring to the set of graphs on the Figure B.2, we find states estimations related to interest rate insignificant in three countries out of six, namely for Slovakia, Czech Republic and Slovenia. That is result in agreement with the result of OLS estimations. For Euro area we report a significant period of three years between 2002q03 and 2005q03, where error bands of width equal double Root Mean Square Error (RMSE) lies in negative area. On the other hand, Hungary's interest rare state's estimate is clearly significant except the period between 2002q03 and 2005q03. For Poland we follow significant state estimate that is almost stable since 2002, significantly negative with the size of effect slightly above value -0.5.

As we reasoned earlier, there is no point in detailed comparison of the shocks between countries as far as we cannot adress them to the exact determinants. ome of them might be join for all countries, the other might be country-specific factors. Although, from the graphs in figure B.3 we can conclude that for each country is that term in model significant and has a relatively stable run. Only case of the Euro area it is insignificant it has a decreasing character. That confirms our assumption above, that this term includes other factors effecting money demand that are significant for the selected countries' explanation of their demand for money balace. The identification of those determinants would certainly contribute to precise estimation of money demand, but we did not get a clear results in this part.

Refering to the hidden determinants, it was already suggested in the research of Arango [9], there is also a clear evidence that demand for money is effected not only by changes in domestic variables such as permanent income, domestic interest rate and price expectations but also foreign interest rates can take a significant share in explanation of demand for a domestic currency. To explore augmented set of determinants, we included the interest rate in Euro area as an additional explaining factor into the model for money demand in selected five NMS. The results were not better, neither clear for explanation, because the effect of nominal interest rate together with Euro area interest rate might be related, therefore we do not present the results in detail and we confine to the approved determinants.

As an alternative model, we examined also dependence of real money demand on the real interest rate, that was defines as defference between nominal interest rate and inflation expectations ($r = i - \pi^{\star}$). The realation between inflation and real money gap was studied by Gerlach and Svennson [33] and had very good results. Is this work introducing inflation into our model did not improve the results therefore we do not present results in detail.

According to the paper of Dreger , Reimers and Roffia [37] the unidentified determinant for demand for money which causes gap between our estimations and real money demand could be the exchange rate.

5.3 Model for nominal money demand with time-varying coefficients estimated by Kalman filter

Model for nominal money demand with time-varying coefficients was estimated by Kalman filter in order to confirm weather the homogeneity postulate was raised appropriately above the relation of demand for money and the level of prices. Our main concern is therefore to study the character and any possible structural changes of the state related to the price level as far as according to the homogeneity assumption it should be a unit.

Reflecting the final estimates of the states related to the price level from the figure C.1, results lie within the interval $[0.68, 1.89]$ and are significant at the level of 1% for all countries except Czech Republic. We distinguish them into the 3 categories. First of all, Slovenia reports the highest elasticity to the nominal demand for money at the level close to 1.9 and the elasticity of Euro area is comparable (almost 1.75). This result was reported also by Dregen and Wolters (2008) at the symposium at Schloss Wartensee [1],who were able to identify the stable long run money demand relationship for M3 in the Euro area, but to obtain the result, they relaxed the short run homogeneity restriction between money and the prices. The second level of the output's elasticity to the prices that we report is around unit, as was assumed before, and it was estimated closely for Hungary and Poland. Third level we report in Slovakia, where the elasticity is below the expected level, and has the level close to 0.7 but it is highly significant as well. This lower effect in Slovakia could be possibly explained also as a result of new financial instruments. Even if prices rise, the demand for nominal money measured by M2 does not raise equally because of financial development. The

Czech results departure from the rule as far as its final state related to the price level is estimated as -0.15 and at the same time, it is the only insignificant price states among studies countries. We explain that as a result of problematic Czech data.

Considering the rest of the result given by filter estimation we confirm the significance of the other determinants as well, even though that their choice depends up on the country. In four out of six cases, the influence of output was significant at the 1% level and values vary from 0.45 to more than 1. One of the countries, where it is insignificant is Hungary, which reported lack of explaining power already in previous specification of the model, where real demand for money was estimated.

The nominal demand for money has same week relation to the interest rate as the real demand did. Only in Euro area it has the effect estimates for almost -0.6 and significant at 1% level. For Hungary and Poland it is significant at 10% level with the similar size of -0.4 which was reported also in real money demand model.

The random walk is still less significant for the Euro area an there it is estimated to be negative, while it has a high significance for Slovakia, Hungary and Slovenia and it reports much higher, positive values. For Czech and Poland appears to be insignificant.

Finally, considering the dynamics of the states during the examined period, we confirm analogous results for the entire period. Looking at the Figure C.3 we conclude that effects of price level were relatively stable for Hungary and Poland during entire period, while for the other countries became stable couple of years later. The homogeneity assumption cannot be rejected in majority of the cases, except the last year for Euro area and generally insignificant results for Czech Republic. For Euro area we subscribe the recent mall response to the prices as implication of the ongoing financial crisis. Hungary, Slovakia and Poland are comparable in their reaction to the price level, while Euro area and Slovenia have rather increasing tendency if the prices effect. Furthermore, we realize high level of stabilization of that influence in Slovakia since middle of the year 1999 while it experienced surprisingly high ans unstable levels in the period before. We believe this can be caused by the change in the government in 1998. During the transition of government changes were introduced into the monetary policy, which brought also flexible exchange rate among other reforms. This could led to the stabilization of the relation between price level and demand for money in Slovakia. Czech republic shows unexplainable results of this model even for entire period. The level of price state estimate is there negative for the whole period and non significant.

As we reasoned above, we see it as a result of problematic data.

The result of money demand elasticity to the gross domestic product described above can be generalized in most of the cases for all the periods (see Figure C.1). The only change was reported for Poland, where significantly positive values were filtered only since the last quartile of 2006. For Slovakia, Euro area, Poland and Slovenia is the effect close to 0.5. In addition, we report the rapid change for Slovakia in 1999 de novo.

The effect of interest rate resulted to be significant during studied period only for Euro area, Hungary and Poland and they reported relatively stable values around -0.5. For the rest three countries interest rate did not proof to be significant at all. This results confirms the findings of final states as well as the ones of model for real demand for money, where the results were almost the same.

Summarizing given results, we cannot reject in majority of case plausibility of homogeneity assumption. At the same time we conclude, that by relaxing the homogeneity assumption we estimated the demand for nominal money demand the same well or even rather better than we estimated demand for real money balances. All the related tables and graphs of states estimates and parameters together with information criteria of presented filters can be found in the appendix.

Conclusions

The aim of this work was to model demand for money. After the identification of its main determinants, we formulated and by OLS estimated the log-linear model with the constant parameters. Despite of model's significance for all countries, the results were partially in disagreement with studied theory and did show up a lack of explaining power to the demand for money. Only the gross domestic product resulted to be significant for all countries. The interest rate was estimated either as significant with positive sign or insignificant that contradicts macroeconomic theory. New models constructed in the state-space representation form allowed money demand to adjust for the changes in the economical conditions. The results estimated by Kalman filter showed the function form with dynamic coefficients to be significant in all cases and describe the effects of the determinants better than definition with constant coefficients.

Fistly, we conclude that both presented models purpose a reliable estimate of money demand in Euro area and they satisfactory well describe the demand for money balance in selected NMS of EU. Secondly, the effect of determinants is evidently more stable in Euro area after introduction of the common currency that is accordance with the expectations. The high relevance of the absolute term in NMS leads to conclusion, that there are still missing some important determinants, that might also vary across the countries. Thirdly, results comparison did prove the states diversity in the reaction for certain determinants. For Slovenia the reactions seems to be comparable with the ones for Euro area what is advantageous result for their entering into EMU. Slovakia and Poland are heading towards the results of Euro area, although there are still a small differences in size of their response to the identified determinants. For Slovakia the improvement for interest rate would apply while for Poland is rather expected higher influence of the output. Finally, for Hungary are the results poor even though they report continuous improvement and process of stabilization toward the value of Euro area. Nevertheless, demand for money in Hungary strongly appears to have additional significant determinants above the tested once. The results for Czech Republic are controvertible and partialy contra the theory what we subscribe to the problematic data.

To sum up these conclusions, we find this approach valuable and believe that although the results here are not perfect, but further search for new determinants might improve them in order to reliably estimate demand for money in developing countries. As the work of Dreger,Reimers and Roffia [37] suggests, the possible missing determinant for selected NMS might be exchange rate. Komárek and Melecký [38] confirm the same particulary for Czech Republic.

Appendix A

Estimates by OLS for real money demand

	Constant coefficients			$\bar{R}^2$	AIC
	c	c_Y	c_i		
Czech Republic	1.7885*** (0.4166)	0.5968*** (0.0309)	-0.0033 (0.1368)	0.9243	-3.2733
Euro area	-5.5327*** (0.3104)	1.1365*** (0.0212)	0.8592*** (0.3205)	0.9855	-4.6686
Hungary	-1.4064* (0.7877)	0.7470*** (0.0494)	2.3925*** (0.3257)	0.9250	-2.7968
Poland	-1.9580*** (0.5517)	0.8258*** (0.0435)	0.2463 (0.2050)	0.9699	-3.0676
Slovakia	5.8904*** (0.2152)	0.3308*** (0.0227)	0.6527*** (0.1720)	0.8195	-2.9160
Slovenia	2.9573*** (0.3181)	0.8297*** (0.0351)	-0.4650 (0.4270)	0.9659	-2.7757

c - absolute term, c_Y - output elasticity, c_i - interest rate semi-elasticity

AIC - Akaike information criteria, $\bar{R}^2$ - Adjusted R^2

***, **, * - significant at 1%, 5% and 10% level, respectively

standard errors are given in parentheses below

Table A.1: Real money demand model estimated by OLS

Appendix B

Estimates by Kalman filter for real money demand

	Final states			AIC	l
	RW	SV^Y	SV^i		
Czech Republic	3.7636**	0.4552***	0.0272	-3.9315	111.12
	(1.6656)	(0.1211)	(0.0514)		
Euro area	-1.8969	0.8957***	-0.6025***	-5.9029	168.28
	(1.1700)	(0.0799)	(0.2898)		
Hungary	9.0868***	0.0940	-0.3668*	-4.2004	118.51
	(1.0721)	(0.0677)	(0.2042)		
Poland	1.2084	0.5891***	-0.3843*	-3.6836	104.30
	(1.1082)	(0.0872)	(0.2307)		
Slovakia	5.9719***	0.3261***	-0.0037	-3.7689	114.18
	(0.7258)	(0.0745)	(0.0905)		
Slovenia	3.6269***	0.7444***	0.0222	-3.1984	79.76
	(1.0640)	(0.1187)	(0.2574)		

RW - random walk, SV^Y - state related to output, SV^i - state related to interest rate

AIC - Akaike information criteria, l - Log likelihood

***, **, * - significant at 1%, 5% and 10% level, respectively

standard errors are given in parentheses below

Table B.1: Final states of real money demand estimated by Kalman filter

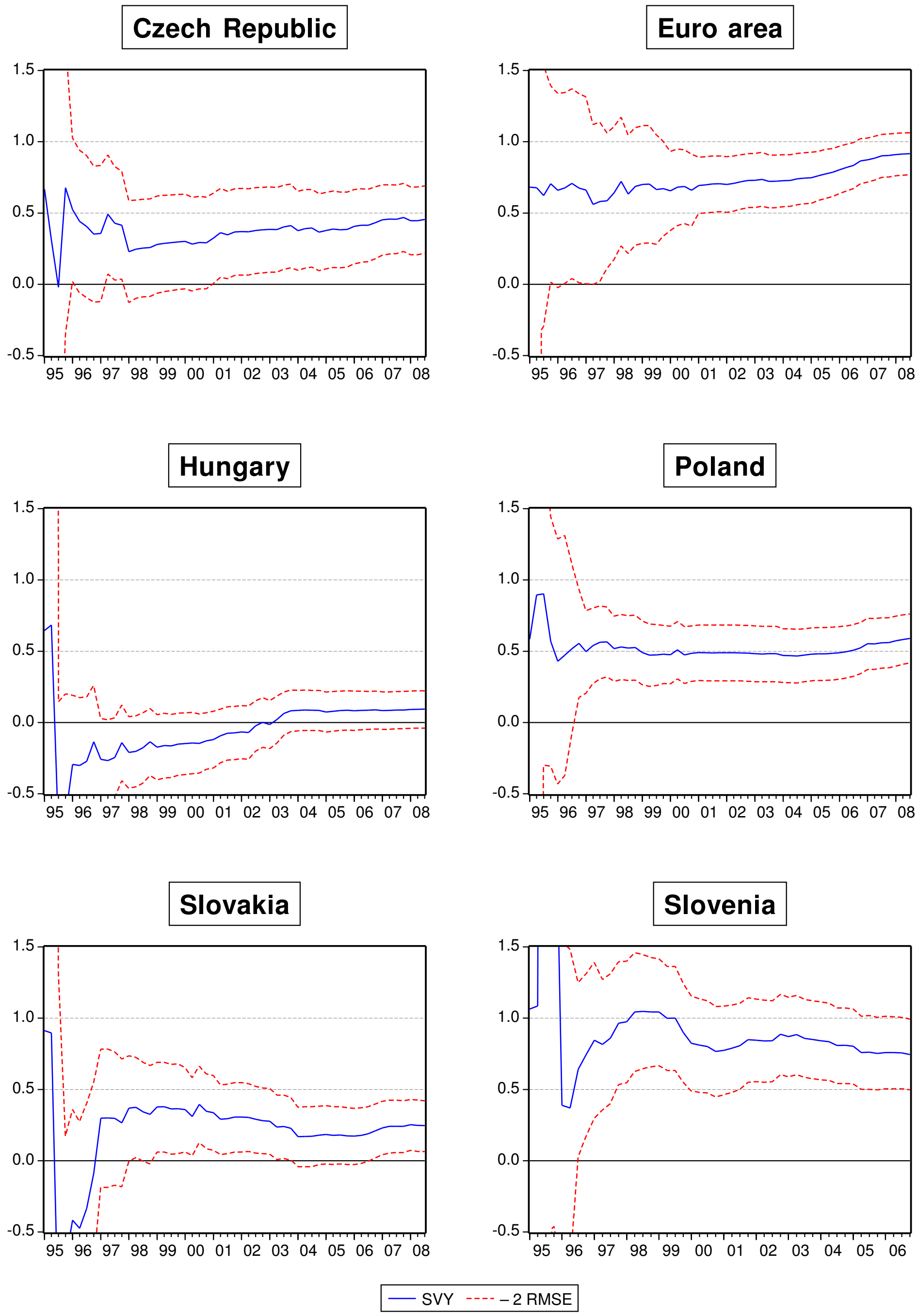

Figure B.1: Estimates of the state variable related to the output (SV^Y) in real money demand model for all considered countries

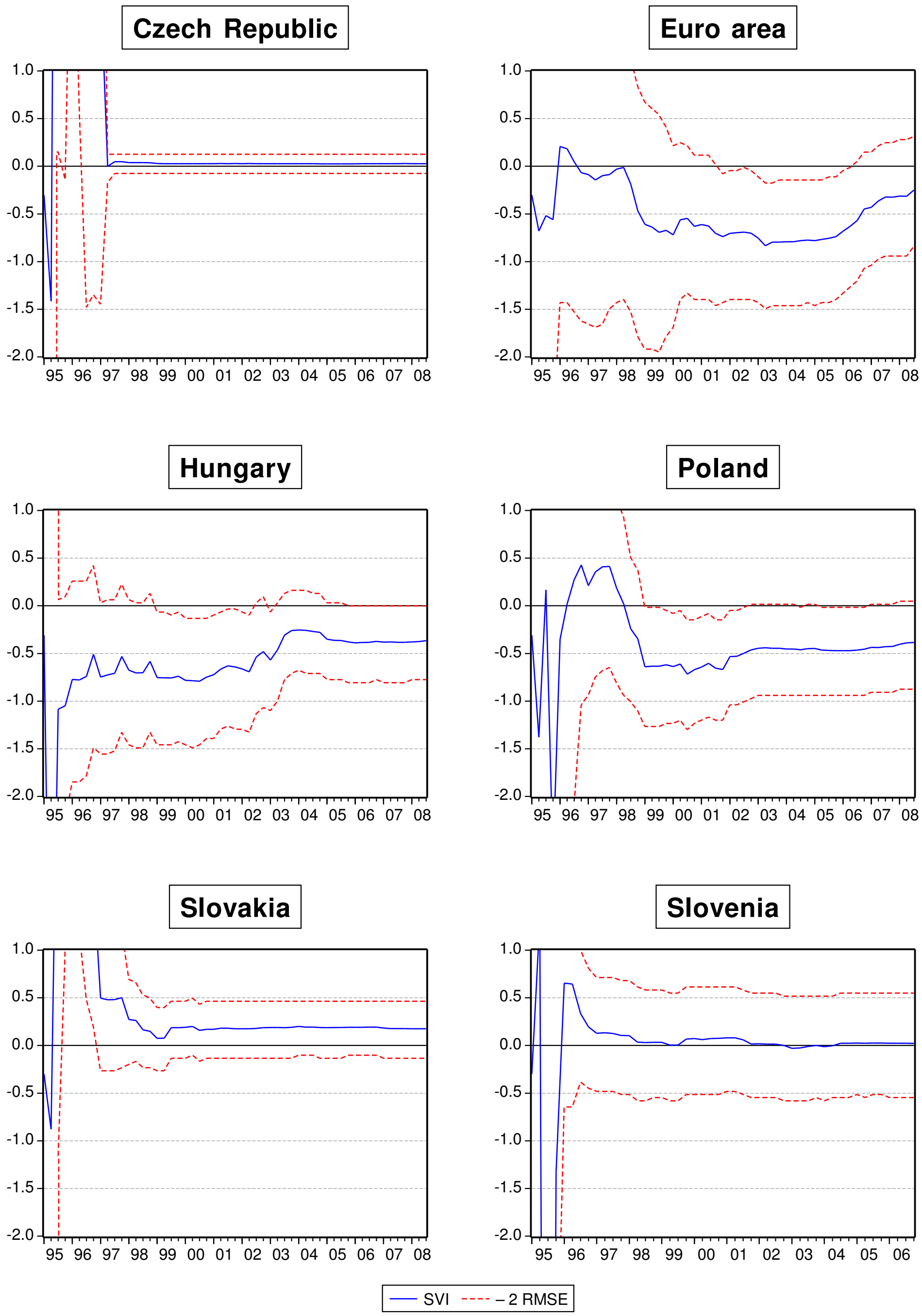

Figure B.2: Estimates of the state variable related to the interest rate (SV^i) in real money demand model for all considered countries

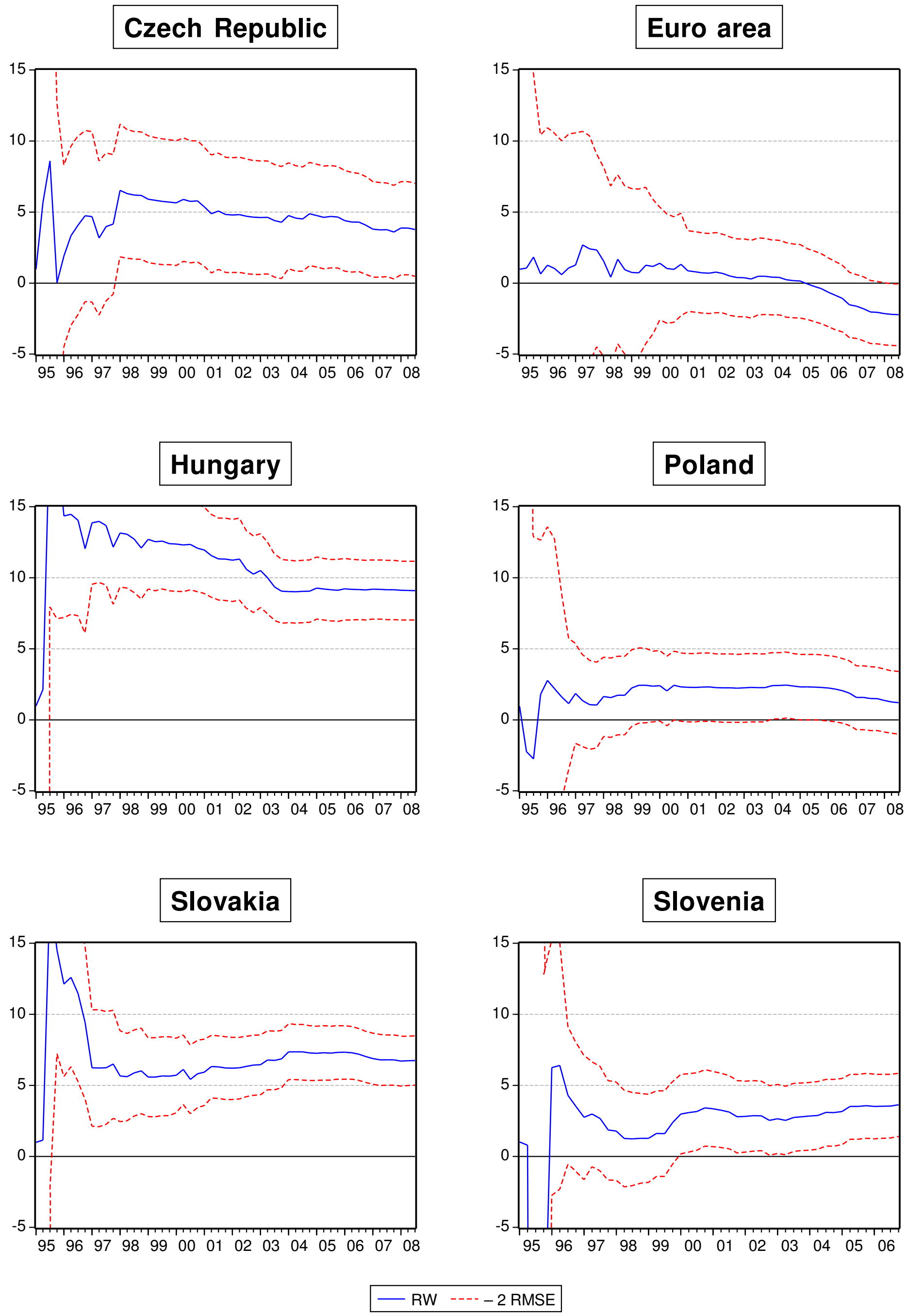

Figure B.3: Estimates of the random walk (RW) in real money demand model for all considered countries

Appendix C

Estimates by Kalman filter for nominal money demand

	Final states				AIC	l
	RW	SV^Y	SV^i	SV^P		
Czech Republic	0.4861 (1.6401)	1.0871*** (0.1887)	0.0038 (0.0448)	-0.1463 (0.2821)	-3.8586	110.11
Euro area	-1.5842* (0.9000)	0.6353*** (0.0904)	-0.5761*** (0.2222)	1.7461*** (0.1906)	-5.6606	162.50
Hungary	8.5975*** (1.3221)	0.1507 (0.1208)	-0.3798* (0.1949)	0.9165*** (0.1550)	-3.8526	109.95
Poland	1.7410 (1.2473)	0.4504*** (0.1702)	-0.3987* (0.2315)	1.2616*** (0.2549)	-3.3579	96.34
Slovakia	6.0996*** (0.7621)	0.4653*** (0.1127)	-0.0468 (0.0960)	0.6851*** (0.1874)	-3.4995	107.23
Slovenia	3.6517*** (1.0690)	0.2866 (0.2597)	-0.0778 (0.2634)	1.8823*** (0.4330)	-2.9227	74.15

RW - random walk, SV^Y - state related to the output, SV^i - state related to the interest rate,

SV^P - state related to the price level

AIC - Akaike information criteria, l - Log likelihood

***, **, * - significant at 1%, 5% and 10% level, respectively

standard errors are given in parentheses below

Table C.1: Final states of nominal money demand estimated by Kalman filter

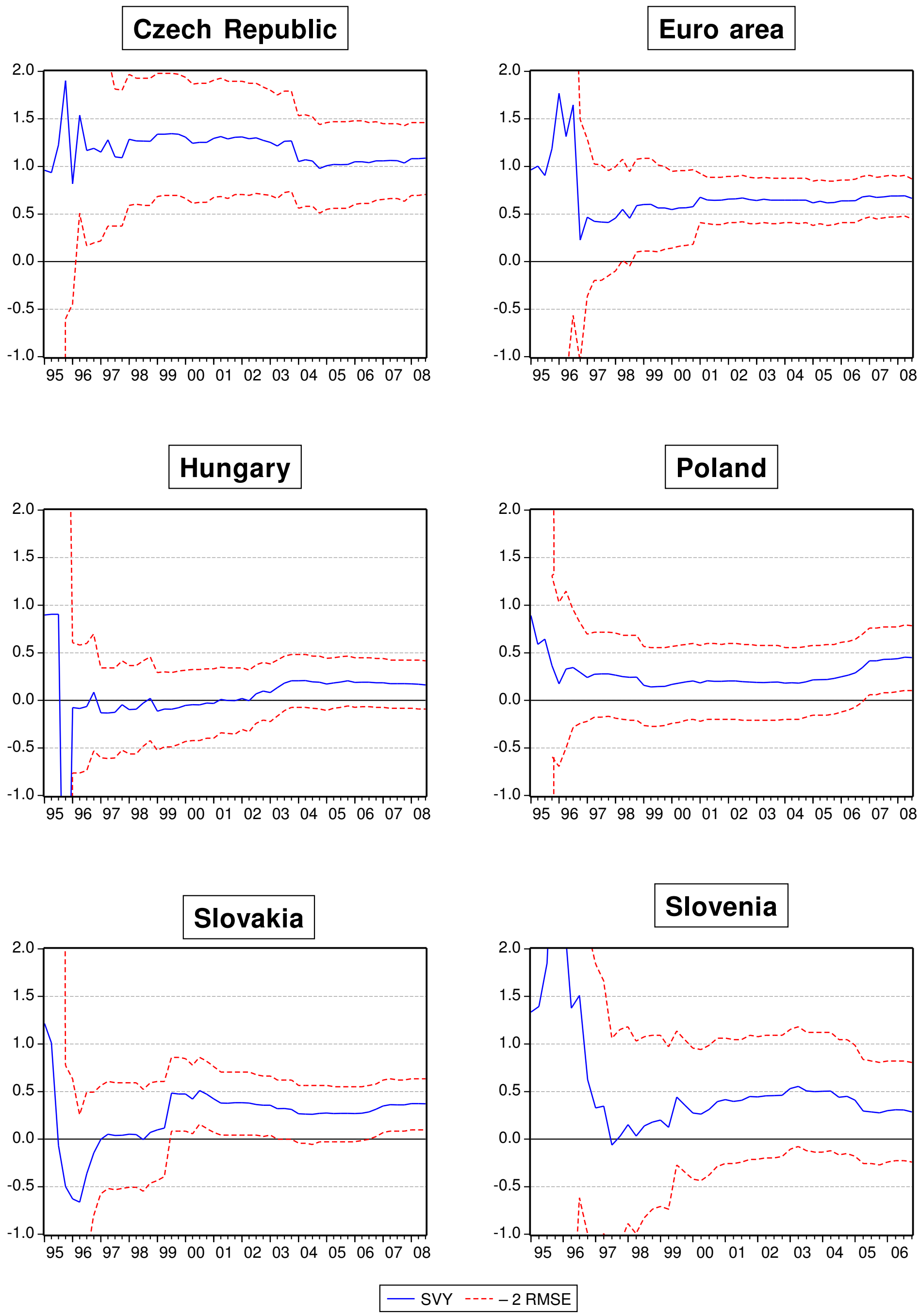

Figure C.1: Estimates of the state variable related to the output (SV^Y) in the nominal money demand model for all considered countries

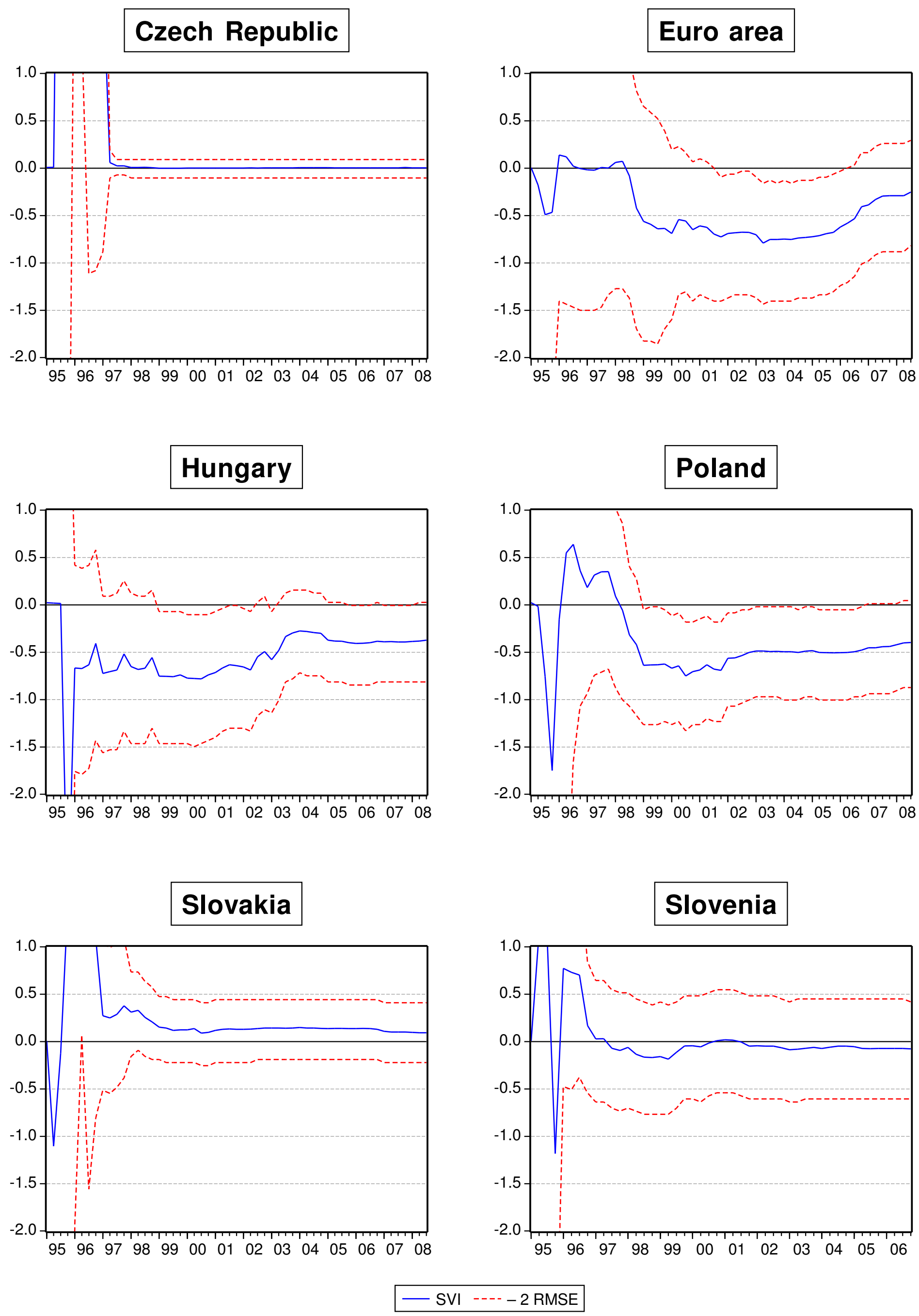

Figure C.2: Estimates of the state variable related to the interest rate (SV^i) in the nominal money demand model for all considered countries

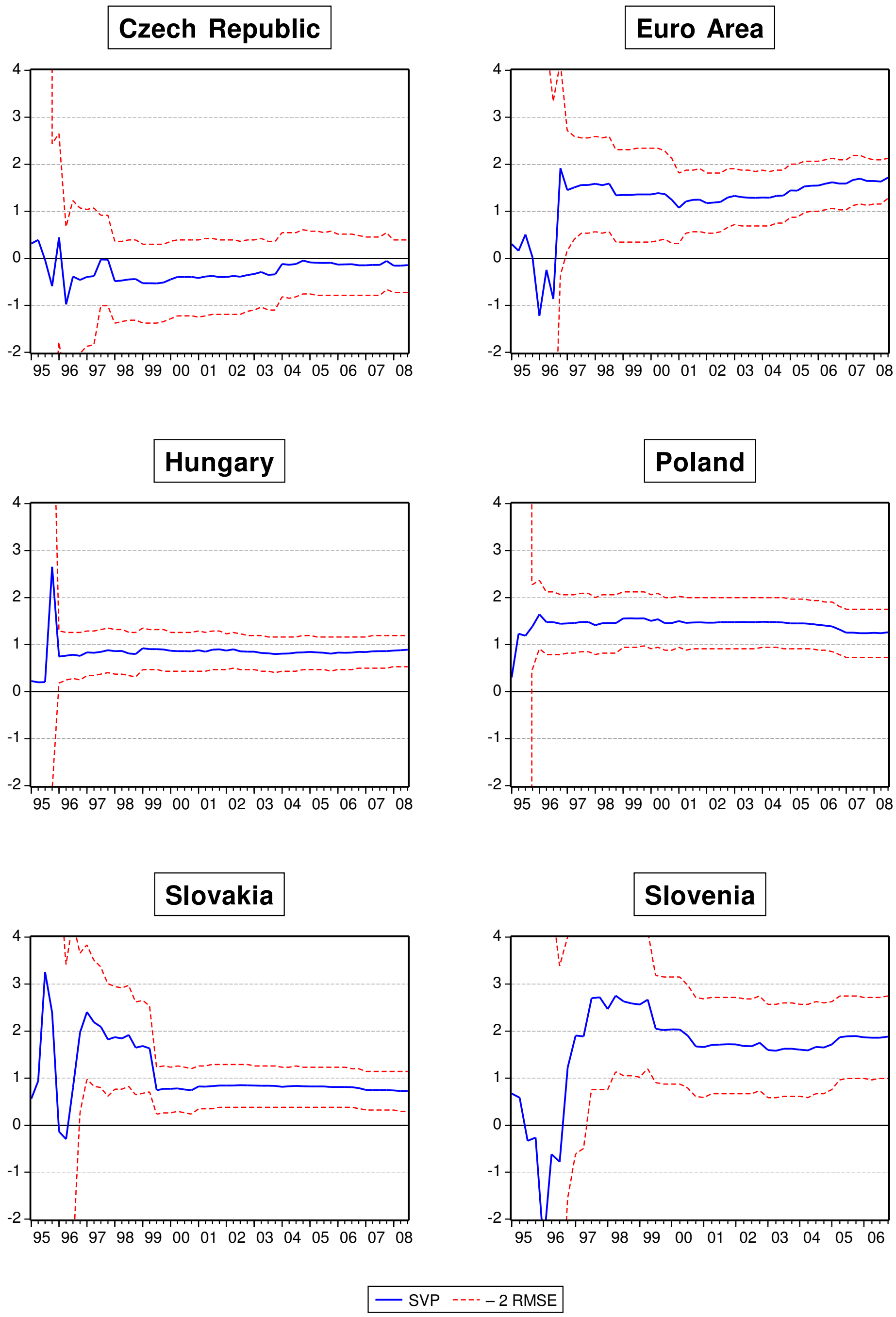

Figure C.3: Estimates of the state variable related to the price level (SV^P) in the nominal money demand model for all considered countries

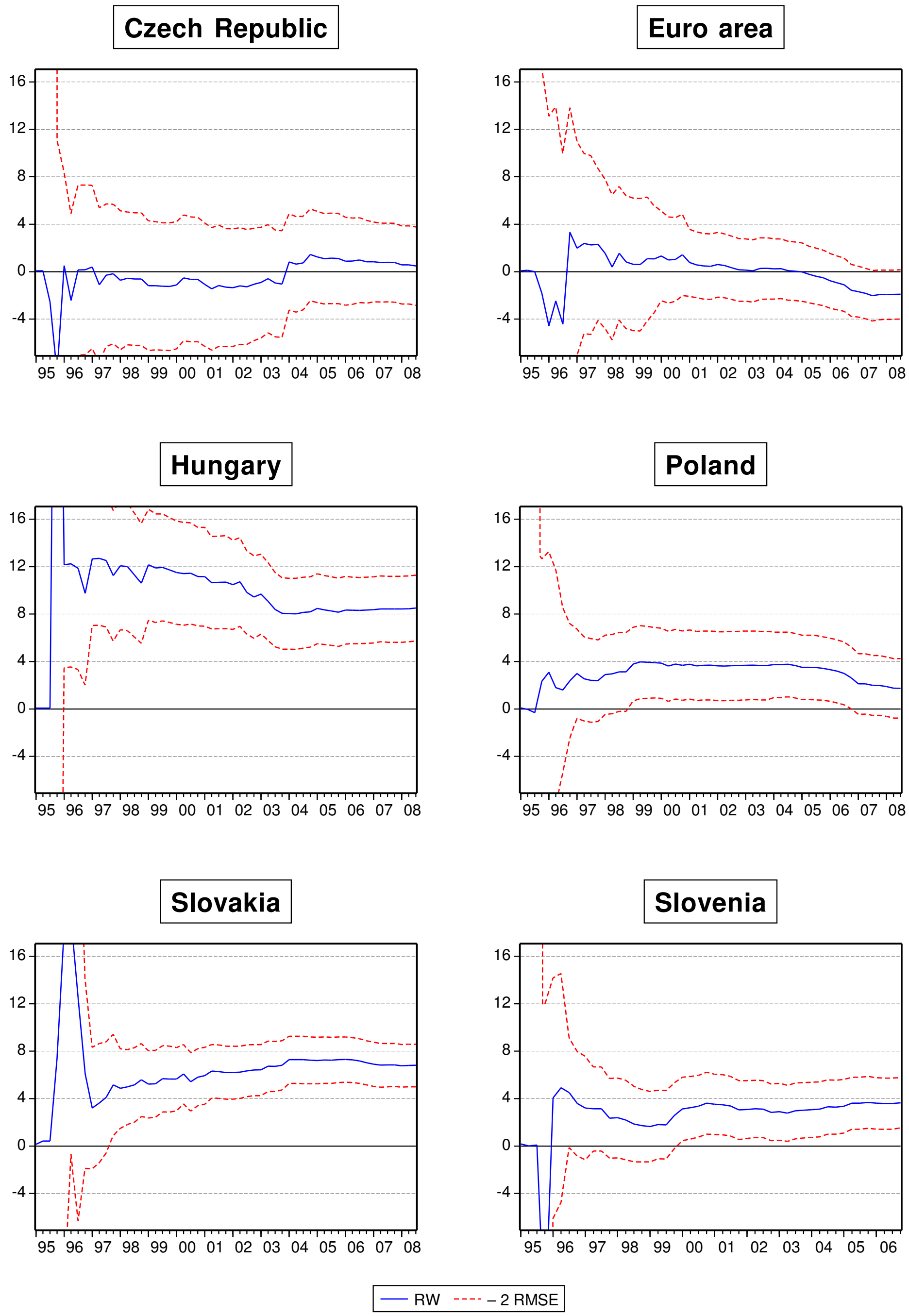

Figure C.4: Estimates of the random walk (RW) in the nominal money demand model for all considered countries

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