COMENIUS UNIVERSITY IN BRATISLAVA
FACULTY OF MATHEMATICS, PHYSICS
AND INFORMATICS

Stochastic model of collective behaviour of ants

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Stochastic model of collective behaviour of ants

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Economical and financial mathematics
1114 Applied mathematics
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Bratislava, 2014 **Bc. Miriam Malíčková**

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Stochastický model kolektívneho správania sa mravcov

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Cieľ: Práca rozširuje bezmriežkový stochastický dynamický model správania sa mravčích kolónií riadený feromónmi, ktorý bol navrhnutý v bakalárskej práci.
Diplomová práca má za cieľ:
1. kvalitatívne štúdium vlastností modelu, konkrétne schopnosť modelu prispôsobovať sa meniacim podmienkam, napríklad zmene pozície a kvality potravy,
2. kvantitatívne štúdium vlastností matematického modelu pomocou numerických experimentov, zahŕňajúc štúdium senzitivity modelu na parametre systému.

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I declare on my honour that this work is written on my own knowledge, references and consultation with my supervisor.

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Bc. Miriam Malíčková

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Abstract

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Colonies of ants are remarkable interacting living systems in which the distribution of roles of ants and interactions among individuals with an environment produce a reliable performance of a complex tasks. Particularly remarkable is the process of formation of narrow paths between the nest and food sources that is essential for the successful foraging. We have designed a simple mathematical stochastic off-grid model of ant foraging in the absence of direct communication. The motion of ants is governed by two components - a random change in direction of motion that improves ability to explore the environment and to find food, and a non-random global indirect interaction component based on pheromone signalling. Using numerical simulations we have studied the model behaviour in different parameters setting and its ability to adapt to changes in environment.

Key words: ants, pheromones, diffusion, randomness, synchronization.

Abstrakt

MALÍČKOVÁ, Miriam. 2014. *Stochastický model kolektívneho správania sa mravcov.* [Diplomová práca] Univerzita Komenského v Bratislave. Fakulta matematiky, fyziky a informatiky. Katedra aplikovanej matematiky a štatistiky. Školiteľ: Mgr. Katarína Boďová, PhD. Bratislava: UK, 2014, 67 s.

Mravčie kolónie sú pozoruhodné interagujúce živé systémy, v ktorých deľba práce mravcov a interakcie medzi jednotlivcami a prostredím prinášajú spoľahlivé riešenie komplexných úloh. Zaujímavým je hlavne proces formovania úzkych cestičiek medzi mraveniskom a zdrojmi potravy, ktorý je základom pre úspešné získavanie potravy. Vytvorili sme jednoduchý matametický stochastický bezmriežkový model mravcov získavajúcich potravu bez ich priamej komunikácie. Pohyb mravcov je riadený dvoma zložkami – náhodnou zmenou smeru pohybu, ktorá zlepšuje schopnosť prehľadávať prostredie a nachádzať potravu, a nenáhodnou globálnou nepriamou zložkou interakcie založenou na feromónových signáloch. Pomocou numerických simulácií sme študovali správanie modelu s rôznym nastavením parametrov a jeho schopnosť prispôsobovať sa zmenám prostredia.

Kľúčové slová: mravce, feromóny, difúzia, náhodnosť, synchronizácia.

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Introduction

The behaviour of ant colonies is a widely studied phenomenon in various scientific fields. Ant colony is an interesting system where simple individual ants along of direct and indirect communication and interaction with their environment are able to solve difficult specific problems on a group level. This behaviour is interesting for biologists to understand the processes in complex living organisms [23], [43]. The communication in ant colonies is primarily pheromone based and the characteristics and the influence of these chemical substances are of particular interest for chemists [17], [29]. Since ants are naturally able to optimize the trail they form, the approach of biomimicry in informatics and optimization theories leads to algorithms that are inspired by the behaviour observed in ants [10], [11], [34]. Furthermore mathematical modelling tries to reveal principles on which ant colony can perform in the manner that is observed in nature [7], [15], [47].

It seems that over years of evolution the nature was able to form a perfectly working living systems that are able to perform effectively in various difficult tasks. Nowadays we try to understand these systems and to learn from them when solving problems that arise in different fields. Ant colonies behaviour that is still not perfectly understood and also slightly varies for different ant species is a very good example of inspiring complex biological system.

In the previous work [27] we proposed a simple stochastic mathematical off-grid model of behaviour of ants. The focus was given to the ability of an ant colony to find the food and to form a trail between the nest and the discovered food source. Main principles that tried to keep a biological relevance of the model were continuous space and pheromone signal spreading by the diffusion. The influence and the importance of the randomness in the model was of a particular interest as well.

The behaviour of the ant colony system depends on the behaviour of single ants. Hence rules of motion of a single ant are an essential aspect of the model. As in the previous work we assume that the motion has two components that are present simultaneously, a deterministic component following detected pheromone signal and a stochastic component based on the behaviour of an ant in the environment with no pheromone signal.

Aims of this thesis are first, to improve the previous model to increase the biological relevance according to available studies in the field. Second, to study the system behaviour in different settings in more detail. In order to do so we need to perform multiple numerical simulations following the system evolution over time. We need to introduce some quantitative and qualitative indicators of the system behaviour and to compare simulations with different setting. Furthermore we want to study the system behaviour in presence of changes in the environment and the ability of the system to adapt to these changes.

1 Theoretical background of the model

This chapter gives a theoretical overview of the studied problem of the behaviour of ant colonies, especially the ability of the system to form and follow trails connecting the nest and the food source. First, we focus on processes observed in ant colonies that serve as a basis for the construction of a mathematical model. Second, we present a brief survey of the existing models that are either inspired by ant colonies behaviour or try to model ant colonies in order to get a deeper understanding of how the processes in colonies might work. The last part of this chapter is an introduction and s specification of the model that is studied in detail in following chapters.

1.1 Biological motivation

Motivation for mathematical modelling of biological processes arises from fascination with perfectly working complex systems observed in nature and essentially leads to two aspects of mathematical modelling.

On one hand, it might lead to deeper understanding of the processes in nature and the capabilities of systems that evolved by natural selection. On the other hand, we can learn from nature and use methods and principles that already work in other problems. This process of learning from nature is called biomimicry and it is already widely used approach in many fields [34].

Ant colony forms a complex system of cooperating heterogeneous organisms that has developed its functions through evolutionary processes to be able to survive, to fulfil colony needs and to adapt to changing environment. Despite extensive effort over last decades in studying ant colonies, many essential questions still remain unanswered:

- How do ants communicate?
- What are single ants able to do?
- How do ants organize themselves?
- How do they adapt to continuously changing environment?
- How it is possible that such a simple organisms are able to solve problems effectively?

When studying ants it is usual to use Pharaoh Ant (*Monomorium pharaonis*) as a benchmark. It is a very common ant species nearly all over the world. It is the most common house ant and nests mostly in building structures.

Its foraging system starts with a small number of scout ants searching independently for food, which consists of dead insects, scraps of human food and drops of water. When one of them finds food it returns to the nest and almost immediately a number of ants comes out of the nest and runs to the food. After some time one or more trails between the nest and the food source develop, where ants move in both directions carrying food to the nest [42].

Figure 1: Pharaoh Ant (*Monomorium pharaonis*), [49].

Naturally, different species of ants behave differently, but there are still enough similarities that might be generalized and used to formulate assumptions, models and hypotheses.

1.1.1 Self-organization

Self-organization is a concept assuming that complex behaviour of a colony or a group of animals is not centrally regulated but it emerges from the individual behaviour and interactions between these individuals. The concept of self-organization has been used to understand collective behaviour of animals. The main principle of self-organization is that simple repeated interactions between individuals can produce complex adaptive patterns at the level of the group.

Ant colonies are an example of systems of organisms behaving in a way where the whole colony is more important than a single ant. This is a biological feature of a particular interest - the ability of a large number of nest mates to function as a whole and to take collective decisions. Without having a sense of the whole workers solve daily problems such as an exploration of new areas, a foraging, a nest moving, a nest defence, a waste management or a brood care [9].

1.1.2 Trail formation

It is observed in nature, that ants are able to form narrow long trails from the nest to the food source as it can be seen in the figure. When studied in detail it seems that they are not just able to form trails but they are also good in the optimization of trails close to the minimum length.

Ants use formed trails in both directions in order to bring food to the nest. Articles focused on rules and principles of traffic control within the trail are also a part of the scientific interest in the topic of ant colonies behaviour [18], [26].

Figure 2: Photography of the trail formed by ants [48].

It is clear that ants must be able to adapt to the changing environment in order to find new food sources, to move from one place to another when it is needed and to overcome barriers which might be encountered while exploring the environment or even might appear in already explored environment. Ants were exposed to many experiments in laboratories, that aimed to find out what kind of problems are ants able to solve. For example, it is shown in experiments, that ants are able to find the shortest way in a labyrinth inspired by a famous Hanoi towers problem [35].

1.1.3 Perception

The self-organization and the trail formation are together already such interesting phenomena that they became widely studied topics by many scientists. One of the major questions on the way to understand mechanisms in ant colonies is how do ants communicate and what kind of information from environment are individuals able to process and benefit from.

Physiological attributes differ in species and so do abilities of ants. Some ants use vision other are hypothesized to have a sense for directional orientation and that they can evaluate information based on the geometric rules [19], [22].

Nevertheless it seems that the main mean of communication in ant colonies is represented by pheromones. Pheromones are chemical substances which ants are able to deposit and sense by their antennae. They serve to spread available information throughout the colony. Ants use multiple types of pheromones - different set of chemicals given species and also multiple pheromones are used within one kind to communicate diverse information [14], [40]. For example an evidence of attractive and repellent pheromones was already given [23]. Since pheromones are different chemical substances they also vary in their chemical qualities - some of them are recognizable for ants hours after deposition and on the other hand scent of the others disappear within few minutes. These qualities depend on a purpose of the specific pheromone [29], [41].

1.2 Overview of existing models

As it was already mentioned biomimicry is a commonly used approach in many fields. Biological interactions and mechanisms in ant colonies became an inspiration in optimization problems. Algorithms based on these principles are called shortly ACO (Ant Colony Optimization) algorithms. They are commonly used in problems like the shortest path search in graph theory [11], but also in many other dynamic and stochastic optimization problems [10].

However, these algorithms are not the object of the study in this work. The focus is given to mathematical models, which try to simulate behaviour of ants especially the process of a food search and a trail formation. Existing models vary in approaches to model behaviour, movement and interactions of ants and also in modelling the environment wherewith ants interact.

1.2.1 Grid models

One of the most common approaches in models of ant colonies behaviour is modelling in discrete space. Here, environment of ants is simplified to a grid, where ants decide in every step according to some principles to which cell of the grid they move from a current cell. Discrete random walk in combination with some reaction with pheromones is a commonly used method of decision making. Examples are shown in the Figure 3.

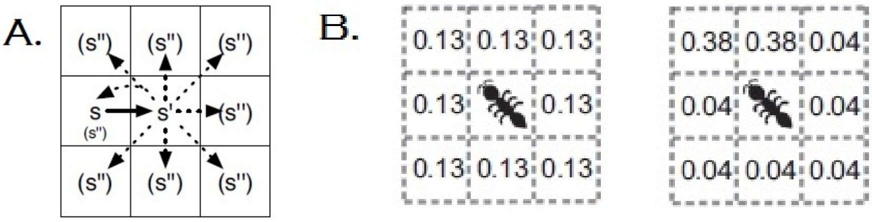

Figure 3: Random walk on grid principles. **A.** An ant moved from state s to state s'. Now deciding for new state s'' from the set of possible states, where the state s' is also included [32]. **B.** An ant decides with some probability distribution for the next step. In the first case the probability is equal because no pheromone is percepted. In the second picture the probability varies according to pheromone concentration (an ant is more likely to choose a path which follows higher pheromone concentration) [44].

The advantage of this approach is that it is relatively simple and changing probabilities allows to model the directional change in reaction to pheromones concentration. On the other hand in case of an uniform probability distribution for cells the natural movement of an ant is violated. It can be seen in experiments with real ants that it is more probable for an ant to move in a straight direction rather than to make a huge directional change. This is observed not only in ants following a pheromone trail but also in ants which are moving in an environment with no pheromone deposited [2].

Even if probabilities would be changed in order to prefer straight direction for an ant, there is one more problem with an unrealistic on-grid approach. The shape of possible trajectories is limited by the grid. An extreme example of trails formed in a lattice model is displayed in the Figure 4. Paths formed by artificial ants are mostly straight or diagonal, aligned with the lattice axes what is clearly an unrealistic result.

1.2.2 Pheromones

Pheromones are chemical substances and it is natural that in the real environment their concentration and placement change by physical principles - mainly the diffusion comes into play. Diffusion of pheromones provides two aspects which are essential for a pheromone concentration based navigation. First of all the diffusion allows a pheromone information to spread in the system so that more ants can catch and follow the signal. At the same time the diffusion decreases a local concentration of pheromone

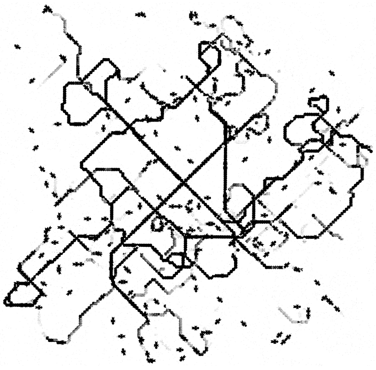

Figure 4: Lattice model trajctories in [47]

and smooths the concentration field that allows ants to ignore misleading signals and enhance the concentration at places where the pheromone following is more optimal.

Even though pheromones and their concentrations have the most important role in a food search and a trail following, a diffusion and a decay are often not well modelled. These phenomena are completely ignored in some models [16] or they are modelled in a very simplified way [2], [46]. On contrary we can also encounter works that attempt to model pheromones properly with a diffusion equation [7].

1.3 Proposed model

In the bachelor thesis [27] a stochastic model of a behaviour of ants was proposed. Since that, multiple improvements of the model were implemented. The main aim of this master thesis is a deeper (qualitative and quantitative study of the model). In this section we specify the basic model and included parameters.

The main idea of the model is based on the random movement of a single ant and the pheromone communication in ant colonies. The illustration of the basic principle in a food and nest search can be seen in the Figure 5.

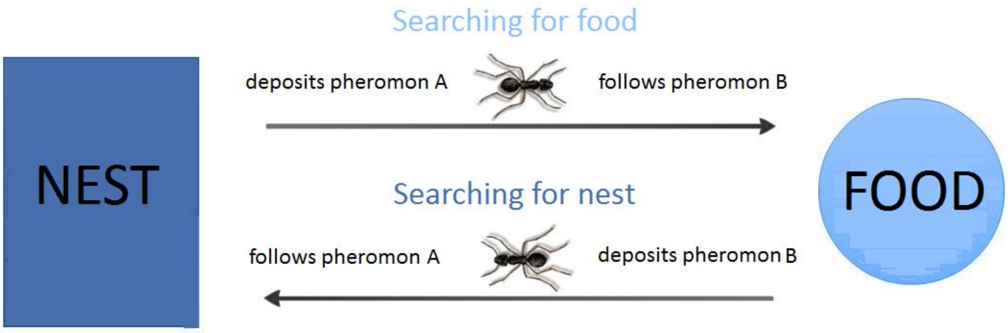

Figure 5: The basic principle of the model. Two main states of an ant (in search for food or in search for the nest).

1.3.1 Assumptions

There are few principles on which the model of a food search and a trail formation in ant colonies is built. These assumptions attempt to improve previous existing models. We also tried to keep the model realistic according to phenomena observed in nature, but not to make the model too complicated at the same time. The assumptions are:

1. **Continuous two dimensional space** - ants move in a two dimensional continuous space with no boundaries.

2. **Two types of pheromones** - there are two different types of pheromones used to mark food and nest that ants can distinguish.

3. **Two parts of an ant motion** - an ant motion consists of two components - a stochastic and a deterministic, while both are present simultaneously.

4. **Deposition of pheromones** - each ant deposits a pheromone according to its state (search for food/search for the nest). The deposition is time limited, i.e. an ant deposits a pheromone only for a limited amount of time since the last time was at the place where food/the nest is.
5. **Orientation of ants is pheromone based** - ants can sense both pheromones and are able to follow a direction with a stronger signal. We assume a threshold in pheromone concentration below which an ant is not able to sense the signal. Pheromones spread in a space by the diffusion and the decay is also present.
6. **Two phases of a trail formation** - a trail formation consist of two phases - the first phase is a foraging phase, where ants search for food in a space with no pheromone information and they form the pheromone field according to their exploration of the space. The second phase is the food search and the trail formation in the space where pheromone information made by foraging ants is already present.
7. **No interaction and constant velocity** - there is no interaction of ants, besides the pheromone communication, included in this model. Ants move with a constant speed.

1.3.2 Single ant motion

In nature, when ants do not follow any pheromone signal they move randomly. When observing ants which move in a space where no pheromone signal is present it seems that we can assume that they move on a principle of a correlated random walk [2]. Ants move in a straight direction with a random directional change in each step. This is the reason why the random walk they perform is called correlated - the direction which an ant takes depends on the direction which it came from. Observed randomness is crucial for the motion of a single ant, since it plays a considerable role in multiple phases of the processes in the whole ant colony:

- **foraging phase** - when ants are in search for food, a wide space is possible to be explored,

- **multiple food sources** - an ability of ants to find multiple food sources raises with the randomness in their motion,
- **trail formation** - an improvement of the trail in the sense of optimization in its length and width,
- **changing environment** - randomness improves a capability to adapt to changing conditions in environment.

In the mathematical model the motion of a single ant is an essential aspect to be determined. In the proposed model the motion consists of two parts which are present all the time in ant's motion when deciding which direction to take.

In general to simulate a trajectory of an ant given its current position (x_n, y_n) and current direction ω_n the only additional thing needed to know to determine its position (x_{n+1}, y_{n+1}) in the next step is an angle φ_n representing a directional change. To compute exact point we also need a step length s, but since we assume constant velocity, this parameter is also a constant. Difference equation for position of an ant can be now expressed as

$$x_{n+1} = x_n + \cos(\omega_n + \varphi_n) * s\,, \quad y_{n+1} = y_n + \sin(\omega_n + \varphi_n) * s\,,$$

$$\omega_n = \sum_{i=1}^{n-1} \varphi_i = \omega_{n-1} + \varphi_{n-1}\,,$$

where x_n and y_n are coordinates in $\mathbb{R}^2$ determining the position of an ant in step n. The current direction ω_n is a sum of directional changes performed in all previous steps.

The crucial aspect of the model is the way of modelling directional change φ_n. The main idea of a combination of a deterministic and a stochastic behaviour in one is implemented in a function of directional change, which can be written as follows

$$\varphi_n = \underbrace{f(c_{\frac{\pi}{6}}, c_{-\frac{\pi}{6}})}_{\text{deterministic part}} + \underbrace{\kappa\sigma^2\xi_n}_{\text{stochastic part}}\,, \tag{1}$$

$$\xi_n \sim N(0,1)\,, \quad \kappa = \min\left\{1, \frac{c_{\min}}{\max\{c_0, c_{\frac{\pi}{6}}, c_{-\frac{\pi}{6}}\}}\right\}\,,$$

where

- $f(\cdot)$ is a function of a deterministic directional change,
- $c_{\frac{\pi}{6}}$ is a concentration on the left antenna,
- $c_{-\frac{\pi}{6}}$ is a concentration on the right antenna,
- c_0 is a concentration in the current direction,
- $c_{\min}$ is the minimum level of a pheromone concentration when the random term starts to decrease,
- κ is a parameter set by a function determining the influence of a random directional change to the total directional change,
- σ^2 is a variance of a random directional change,
- ξ_n is a random variable with a standard normal distribution.

The whole concept of one step in an ant motion can be graphically represented as it is in the Figure 6.

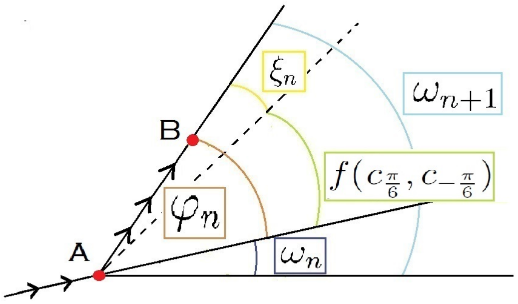

Figure 6: Directional change. Assume that an ant is at the point $A = (x_n, y_n)$ coming from the direction ω_n. In the next step it moves to the point $B = (x_{n+1}, y_{n+1})$. Its directional change is φ_n compounding of the deterministic directional change $f(c_{\frac{\pi}{6}}, c_{-\frac{\pi}{6}})$ that depends on the pheromone detected by antennae at the point A and the stochastic adjustment ξ_n from normal distribution with mean 0 and variance σ. The length of a line segment AB represents a step length s. At the point B an ant will make the directional change in the next step again according to the equation (1). It will do so in the same manner as it did at the point A, but now taking into account a concentration of a pheromone detectable at point B and the random directional change, present in each step, ξ_{n+1} draw from the same normal distribution.

Deterministic part of the movement

In equation (1) of the directional change φ_n in the step n the deterministic part is represented by a function $f(c_{\frac{\pi}{6}}, c_{-\frac{\pi}{6}})$. The form of this function is very important for the movement of an ant in the pheromone concentration field. According to the function an ant decides weather to continue in the current direction or to change the direction following the signal on antennae in the next step. The change of this function is one of the major differences to the model proposed in the bachelor thesis [27].

We decided to use a function based on the principle called Webber's law, because according to the article [33] there is a satisfying evidence that real ants move in the way which follows this rule.

In general Webber's law is the equation giving that the response U to the stimuli S_1, S_2 is proportional to the ratio of difference $(S_1 - S_2)$ and the total sum $(S_1 + S_2)$ of stimuli, which means

$$U = a\frac{S_1 - S_2}{S_1 + S_2}, \quad a \in \mathbb{R}. \tag{2}$$

In the case of our model the stimuli S_1, S_2 are concentrations on the left $(c_{\frac{\pi}{6}})$ and the right $(c_{-\frac{\pi}{6}})$ antennae. We assume that in the case with no stimuli and no stochastic process an ant just follows the straight direction and a noisy perturbation. If an ant catches some signal with its antennae the directional change may appear. We have not implemented continuous change in direction according to stimuli as it was studied in [33], but we adopted the idea of a ratio Γ for signals, where the difference of the detected concentration between antennae divided by the sum of these concentrations has to be higher than a given threshold α. This gives us a function $f(\cdot)$ in a form

$$f(c_{\frac{\pi}{6}}, c_{-\frac{\pi}{6}}) = \begin{cases} \frac{\pi}{6} & \Gamma > \alpha \wedge c_{\frac{\pi}{6}} > c_{-\frac{\pi}{6}}\,, \\ -\frac{\pi}{6} & \Gamma > \alpha \wedge c_{-\frac{\pi}{6}} > c_{\frac{\pi}{6}}\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \Gamma = \frac{|c_{\frac{\pi}{6}} - c_{-\frac{\pi}{6}}|}{c_{\frac{\pi}{6}} + c_{-\frac{\pi}{6}}}\,, \tag{3}$$

where $c_{\frac{\pi}{6}}$ and $c_{-\frac{\pi}{6}}$ equal to the concentration of pheromone on the left and the right antenna of an ant. It should be noted that an effect of a continuous change in a direction of motion according to the full Webber's law would not give significantly different results, since we count for a noisy perturbation as well.

Stochastic part of the movement

We assume that an ant moves randomly following the principle of a correlated random walk when there is no pheromone signal to follow. In the Figure 7 we can see histograms of the directional change in experiments with real ants, fitted to a normal distribution.

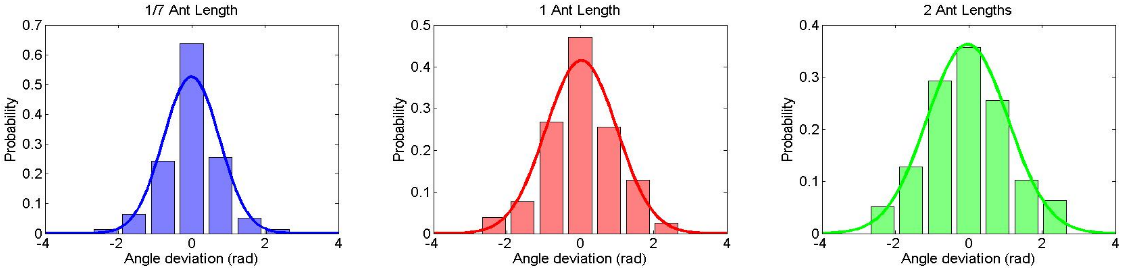

Figure 7: Histograms of directional changes for real ants fitted to the normal distribution. Each histogram is based on a different step length taken with respect to an ant body length. When two times body length is taken as one step, normal distribution has the best fit to the histogram. (data from [2])

Even though we already mentioned a step length s as a parameter in the model, in case of ants it is hard to define what exactly one step means. In the Figure 7 three different lengths are considered as a step length (one seventh of the ant body length, exactly the ant body length and two times the ant body length). As step size increases we observe an improvement in normal distribution fit to histograms of directional changes and it seems that when we take two times the length of a body of an ant as the step length, normal distribution fit is the best among the three. According to these findings we can assume that a random directional change of real ants has a normal distribution with mean 0 and variance $\sigma = 1.0991$ obtained from the last histogram and we define a step length as a two times length of an ant body. This is reflected by a random variable ξ_n in the equation (1).

If there is no pheromone field the motion of ants is described as a correlated random walk that gives many advantages to the effectiveness of the colony processes. However, when the signal of pheromones is strong and it is probable that the direction chosen according to information of high concentration is the correct direction, the influence of randomness decreases in order to keep effectiveness of the whole process.

Now, to explain the directional change equation completely, there is one more parameter left in the stochastic part of the equation, i.e. κ that decreases influence of randomness in case of a high level of the followed signal. Randomness plays an important role in all phases of the food search, the trail formation and the adaption to changing environment, but in the case, when the followed signal is strong enough (higher than a constant $c_{\min}$), so we can assume that an ant follows a correct pheromone trail, the randomness decreases since it is more certain that chosen direction is the right direction, but the randomness itself never disappears completely.

1.3.3 Pheromone field

Pheromones represent the way how ants communicate, interact within the colony and how they explore, mark and orient in environment. These skills are crucial for an ant colony, so the approach how we model pheromones is very important for the whole system.

Since pheromones are chemical substances, when they are deposited they spread in the system. We assume that they are transported in the two dimensional plane by diffusion. To model the diffusion mathematically the following partial differential equation of diffusion is used:

$$\frac{\partial c}{\partial t} = k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) , \tag{4}$$

with an initial condition in a form

$$c(x, y, t_0 | x_0, y_0, t_0) = c^0(x, y) ,$$

and boundary condition in $\mathbb{R}^2$

$$\lim_{x,y \to \pm\infty} c(x, y, t) = 0 ,$$

where $c(x, y, t)$ is a function giving the concentration of the pheromone at the point (x, y) in time t and k is a diffusion constant. Generally we can obtain an analytical solution to the given equation for each point (x, y) in every time t in a form

$$c(x, y, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c^0(x, y) \frac{1}{4\pi kt} \exp \left[- \frac{(x - x_0)^2 + (y - y_0)^2}{4\pi kt} \right] dxdy .$$

The partial differential equation is linear therefore with more initial conditions $c(x_i, y_i, t_i)$ for $i = 1, \dots, N$ to obtain the solution we just simply sum the solutions for each initial condition separately

$$c(x, y, T) = \sum_{i=1}^{N} c(x, y, T | x_i, y_i, t_i) .$$

Ants in our model deposit pheromone at any point of space synchronously in time, that can be represented by many initial conditions. For each deposition at the point $(x_i, y_i) = a \in \mathbb{R}^2$ in time $t_i$we set an initial value to

$$c^0(x, y | x_i, y_i, t_i) = m\delta(x - a) ,$$

where m is an amount of a deposited pheromone and $\delta(x-a) : \mathbb{R}^2 \to \mathbb{R}$ is a Dirac delta function defined as

$$\delta(x-a) = \begin{cases} \infty, & x = a \\ 0, & x \neq a \end{cases} ; \quad \int_{\mathbb{R}^2} \delta(x)dx = 1 ; \quad x, a \in \mathbb{R}^2 .$$

It has a property that

$$\int_{\mathbb{R}^2} f(x)\delta(x-a)dx = f(a) ,$$

which means that in our case with the initial condition $c^0(x, y|x_i, y_i, t_i) = m\delta(x-a)$ defined above we set the concentration at the point (x_i, y_i) in time t_i equal to the amount of pheromone deposited m.

In the proposed model there are two different pheromones used by ants. First pheromone (pheromone A) serves to mark the nest. Ants deposit this pheromone when leaving the nest to keep the information where the nest is. Second pheromone (pheromone B) serves to mark the place where the food source is. Ants use this pheromone when they reach the food in order to give the information to the rest of the colony. Both pheromones differ not only in why and when they are used but also in qualitative properties - diffusion constants (k_A, k_B) explaining the strength of diffusion are different and also the time for each pheromone to stay in the system (decay rate δ_A, δ_B).

Ants deposit specific pheromone according to their state to provide a valuable information for other ants about the system and the environment. However they do not want to give a misleading information that would cause some ants loosing their way due to following a wrong signal. When after some fixed time an ant still does not know where exactly the food source or the nest is it stops depositing pheromones and relies on information available in the system and new information produced by other colony members. This phenomenon is reflected in the model by the parameter of limited deposition time.

2 Numerical simulations

Proposed stochastic mathematical model of collective behaviour of ants has already been introduced in the first chapter. To study properties of this theoretical model numerical simulations are needed. This chapter provides approaches to simulations and their important attributes are explained. Numerical methods used for the purpose of simulations are studied in [25].

2.1 Ants and environment

A lot of existing mathematical models of ant colonies rely on modelling the environment of ants as a discrete grid of a given shape. This is a highly unrealistic assumption about the space where ants exist. In our proposed model we use a continuous $\mathbb{R}^2$ space, where ants can move to any point without restrictions.

There are two special regions in the space - the nest and the food source. At the beginning of any simulation all ants in the system are in the nest and the surrounding space is completely undiscovered, i.e. there are no pheromones in the system neither nest marking nor food marking. The food source is located at a given distance and the process of finding it is a part of a simulation.

Single ant motion is based on rules explained in the previous chapter. There are two constants concerning anatomy of an ant needed for the simulation - the spread of the antennae (to evaluate pheromone concentration at points of perception) and the length of the body (to set the step length). The anatomy of the modelled ant is described in the Figure 8.

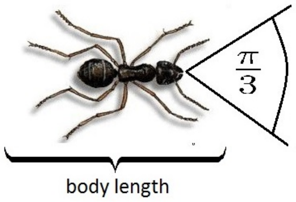

Figure 8: Modelled ant anatomy. The spread between antennae is $\frac{\pi}{3}$, which means that an ant can percept the local concentration at the points in the angle from the straight direction $\frac{\pi}{6}$ for the left antenna and $-\frac{\pi}{6}$ for the right antenna. The body length of an ant is set to 2.1 mm.

When these basic parameters are known we can simulate the trajectory for a single ant in the continuous space following the difference equation (1) for the directional change for each step. The trajectory produced by the simulation of the motion of an ant with anatomy as in the Figure 8 when moving in the space with no pheromones scent is comparable to the trajectory observed in real ants in the same conditions as it can be seen in the Figure 9.

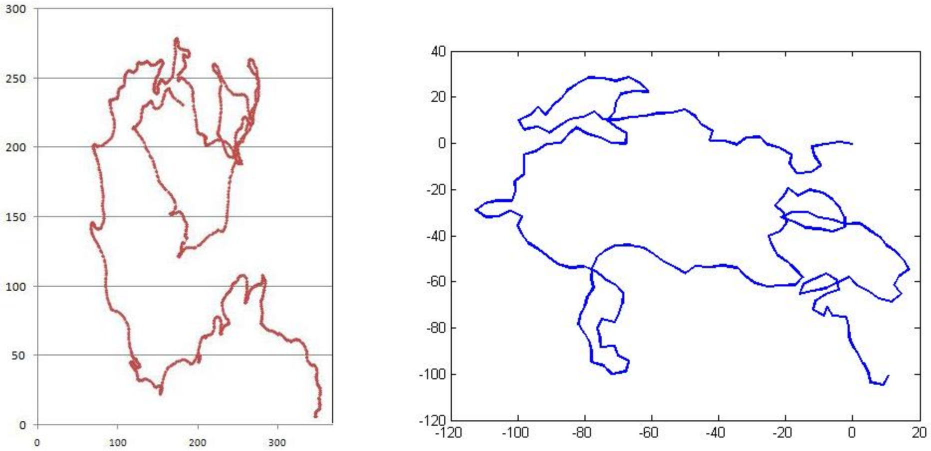

Figure 9: Single ant trajectory in the space with no pheromones. A trajectory of a real ant from [2] on the left. A trajectory of an ant from the simulation behaving by rules of motion of the mathematical model on the right.

2.2 Two phases of the simulation

In a food foraging there are two phases which might be naturally identified. In the case of ant colonies the first phase is a food search in an unexplored area and the second one is a food transport and a trail formation. The idea of two phases is implemented in numerical simulations for our model and it is important to remark that these phases are not artificially used only for needs of simulation but they are observed and defined in real ants [1],[8].

The first phase is to identify the food source. In our simulation the first phase is represented by ants leaving the nest to an unexplored space, with no pheromone information. These ants move randomly according to rules explained in the first chapter and they explore the surrounding space around the nest. They mark their trail from the nest using the pheromone A. When the food source is discovered by any ant, the ant starts to produce pheromone B to attract other ants and help them to find the food source. Consequently it starts to find its way back to the nest. The first phase ends when 10% of all ants in search for food find the food source and return to the nest.

Right after, the second phase begins - all ants in the system are situated in the nest and the process of the food search and the trail formation starts once again, now with the difference, that there is already a pheromone signal made by ants in the first phase.

Important note here is that all ants from the first phase are returned to the nest at the beginning of the second phase that is unrealistic, but for the purpose of the numeric simulation this inaccuracy does not matter. If we would keep foraging ants from the first phase and just add more ants to the system in the second phase, the only difference would be more ants that are lost. These are mostly ants from the first phase that moved too far from the pheromone signal of the nest and they are not of a particular interest any more since they do not modify the pheromone field any more. Also in nature few individual ants might get lost, but important is the effectiveness of the process in the level of a colony, not the effectiveness of the individuals. For this reason we used the same ants in both phases since this approach requires less memory and computational power for the simulation.

2.3 Diffusion

Pheromones play significant role in the communication of ants since they are used to spread available information from the environment to the whole colony. In our model pheromones are the main mean of communication because they ensure the interaction between ants by spreading information about the discovered environment.

Pheromones are chemical substances so they diffuse in the space which means that a signal can be detected by an ant not only at the exact point where the chemical was deposited but it also broadens in the system. The local concentration of the pheromone depends on

- time t ,

- distance from the point of deposition,
- speed of diffusion represented by diffusion constant k,
- the amount of pheromone deposited m.

The diffusion itself can be modelled by a partial differential equation (4) with a Dirac δ function as an initial condition as it was explained in the previous chapter. This equation has an analytical solution which can be obtained for any time t at any point $(x, y) \in \mathbb{R}^2$. However in numerical simulations when we want to compute current concentration at many points of the space, in order to have a complete information about the pheromone field, an analytical solution is computationally difficult because it always integrates the whole history to find a present value of a concentration at a given point. That is the reason why in simulations we rely on numeric approximation of PDE's (4) using finite difference scheme.

2.3.1 Numerical solution for diffusion equation

Suppose that we know the analytical solution for concentration at any point (x, y) at current time t and our aim is to compute the solution for time $t + \Delta t$. We can set a grid in the space and compute the current concentration at each point (x_k, y_k) of the grid. When finding the numeric solution for an equation (4) by finite differences method we need to discretize the partial differential equation that gives us a formula

$$\frac{C^{t+\Delta t}_{x_k,y_k} - C^t_{x_k,y_k}}{\Delta t} = k\left[\frac{C^t_{x_k-\Delta x,y_k} + C^t_{x_k+\Delta x,y_k} - 2C^t_{x_k,y_k}}{(\Delta x)^2} + \frac{C^t_{x_k,y_k-\Delta y} + C^t_{x_k,y_k+\Delta y} - 2C^t_{x_k,y_k}}{(\Delta y)^2}\right].$$

Our aim is to find $C^{t+\Delta t}_{x_k,y_k}$ when all the other values at time t from the formula above are known. We can set the grid such as distances in the direction of axes are equal $\Delta x = \Delta y = h$ and so we have the solution

$$C^{t+\Delta t}_{x_k,y_k} = C^t_{x_k,y_k} + \frac{k\Delta t}{h^2}\left(C^t_{x_k-h,y_k} + C^t_{x_k+h,y_k} + C^t_{x_k,y_k-h} + C^t_{x_k,y_k+h} - 4C^t_{x_k,y_k}\right).$$

For an effective usage of above formula it is important that a multiplicator $\frac{k\Delta t}{h^2}$ has to be satisfactory small – at least $< \frac{1}{4}$ – since this is a boundary when the approximated solution stays positive and the scheme does not develop an artificial instability. There are three variables in the multiplicator

- k is a given diffusion constant,
- h is the distance between points of the given grid,
- Δt is giving the time difference and telling us how often we need to update solution.

Δt is the parameter to be determined, when the others are given for the desired value of the multiplicator.

In this approach the analytical solution for the equation (4) is still needed. We compute the initial concentration on the grid points with respect to real concentration at these points in continuous space. However in some settings of experiments analytical solution for all the points in designed environment is not available so we need a complete numeric approach to evaluate pheromone concentration. The main aim in constructing a pure numerical solution for concentration field is to keep a continuous space, which means to construct the solution in the way that the pheromone concentration is identifiable not only at grid points but at all points $(x, y) \in \mathbb{R}^2$.

2.3.2 Linear interpolation for a continuous space

Even though we are using a numerical approach with a grid to calculate the pheromone field, we keep the model continuous in the space, so we still need a tool to calculate concentration at any point in two dimensional space even when the analytical solution is not available. We use linear interpolation for any point $A = (x, y)$ as it is demonstrated in the Figure 10. At time t concentration in the point A is calculated following the formula

$$C_A^t = \frac{d_{11}C^t_{x_1,y_1} + d_{12}C^t_{x_1,y_2} + d_{21}C^t_{x_2,y_1} + d_{22}C^t_{x_2,y_1}}{d_{11} + d_{12} + d_{21} + d_{22}}$$

The same idea, but in opposite direction is used for deposition of the pheromone. The amount of the pheromone is distributed to the four nearest grid points weighted by the distance of the deposition point from these points. The grid step size h is satisfactory small that the accuracy of this interpolation is enough for the purpose of the simulation.

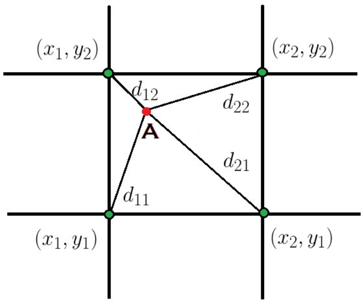

Figure 10: Linear interpolation of concentration at any point $A \in \mathbb{R}^2$ according to the numerical solution on the grid.

2.3.3 Numerical solution for diffusion equation with decay

If we want to include decay of the pheromone to the equation itself, we need to change equation (4) that leads to the partial differential equation of diffusion with a decay in a form

$$\frac{\partial c}{\partial t} = k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) - \lambda c \,, \tag{5}$$

where λ is a constant giving a decay rate. We can obtain a numeric solution using a finite difference method in the same manner as we did for the equation (4). This approach gives a discretization of the equation (5).

$$\frac{C^{t+\Delta t}_{x_k,y_k} - C^t_{x_k,y_k}}{\Delta t} = k \left[\frac{C^t_{x_k-\Delta x,y_k} + C^t_{x_k+\Delta x,y_k} - 2C^t_{x_k,y_k}}{(\Delta x)^2} + \frac{C^t_{x_k,y_k-\Delta y} + C^t_{x_k,y_k+\Delta y} - 2C^t_{x_k,y_k}}{(\Delta y)^2} \right] - \lambda C^t_{x_k,y_k} \,,$$

and the solution for the concentration at the gird point (x_k, y_k) at the time $t + \Delta t$ in a form

$$C^{t+\Delta t}_{x_k,y_k} = (1 - \Delta t \lambda) C^t_{x_k,y_k} + \frac{k \Delta t}{h^2} \left(C^t_{x_k-h,y_k} + C^t_{x_k+h,y_k} + C^t_{x_k,y_k-h} + C^t_{x_k,y_k+h} - 4C^t_{x_k,y_k} \right) \,.$$

This formula can be used to compute a concentration of the pheromone on all grid points numerically including the concept of the pheromone decay. The only question that is left is how to set parameter λ to resolve the same (or very similar) results as in the analytical approach, where the decay rate δ was introduced. δ_i is a time period for the pheromone i to stay in the system. Numerically, if the time when a pheromone was deposited is t and time now is T then this deposition is taken into account as a initial condition in solving the equation (4) only if $T - t < \delta$. Which means that we assume an amount of pheromone m deposited in one deposition at the point (x, y) at time $t \in (0, \infty)$ to stay in the system exactly for the time period equal to δ and then it disappears completely.

In the approach with a decay rate λ included in the equation (5) the pheromone deposition initial condition does not disappear completely in some given time but it evaporates gradually. The amount of pheromone in the system from one deposition can be expressed in both cases as a function of time $c(t)$

$$c(t) = \begin{cases} c_0 & t < T\,, \\ 0 & t \geq T\,, \end{cases} \quad \text{and} \quad \bar{c}(t) = c_0 e^{-\lambda t}\,,$$

where c_0 is the amount of pheromone deposited at time $t = 0$. In order to keep these two functions equal in some sense we decided to scale those functions as a density functions, i.e. integral over the time must be equal to 1

$$\int_{-\infty}^{\infty} c(t)dt = \int_{0}^{T} c_0 dt = c_0 T = 1\,,$$

$$\int_{-\infty}^{\infty} \bar{c}(t)d = \int_{0}^{\infty} c_0 e^{-\lambda t} dt = \frac{c_0}{\lambda} = 1\,.$$

This scaling gives us two distributions that can be represented by density functions

$$\tilde{c}(t) = \begin{cases} \frac{1}{T} & t < T\,, \\ 0 & t \geq T\,, \end{cases} \quad \tilde{\bar{c}}(t) = \lambda e^{-\lambda t}\,.$$

As it is clear from functions $\tilde{c}(t)$ is an alternative distribution and $\tilde{\bar{c}}(t)$ is an exponential distribution. Our primary question was how to set parameter λ in order to keep it equal in some sense to the approach with decay δ. We use knowledge of the expected value for both distributions and we set λ in the way that expected value for distributions equals, i.e.

$$\tilde{E}(t) = \frac{T}{2} = \tilde{\bar{E}}(t) = \frac{1}{\lambda}\,,$$

$$\frac{T}{2} = \frac{1}{\lambda} \Rightarrow \lambda = \frac{2}{T}\,,$$

where T is a time period for pheromone to stay in the system in the first analytical approach. This means that for numerical calculations with decay we set the parameter λ_i for pheromone i equal to $\frac{2}{\delta_i}$.

2.4 Setting of parameters

There are some parameters in the model needed to be determined. Most of them were already explained in the previous chapters. The summary of all the parameters with their value in the basic simulation and the references is in the Table 1.

PARAMETER	VALUE	REFERENCE	EXPLANATION
ANT			
body length	2.1	2 - 15 mm [2]	body length of an ant (step size is defined as two times the body length)
velocity	8.4	5 - 13 mm/s [2]	velocity of an ant is constant
antennae angle	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{6}$ [7]	angle in which antennae differ from straight direction
antennae length	0.7	$\frac{1}{3}$ body length	length of the antenna needed to set the exact point of pheromone perception
MOTION			
α	10^{-5}	[33]	threshold from Webber's law
σ	1.0991	data from [2]	standard deviation of random directional change
directional change	$\pm\frac{\pi}{5}$	$\pm\frac{\pi}{5}$ [33]	in equation (3)
PHEROMONES			
k_A, k_B	1 , 5	1 mm^2/s [17]	diffusion constants
δ_A, δ_B	200, 100	s	decay of pheromones
m	0.01	g	amount of pheromone deposited in one deposition
c_{tresh}	10^{-11}	10^{-11} g [17]	minimum detectable
$c_{\min}$	10^{-5}	g	κ for decrease of randomness in equation (1)
deposit limit	80	s	for how long pheromone is deposited
SIMULATION NUMERICS			
ants number	150	-	number of ants in the system
h	0.5	-	step size on the numeric grid
multi	0.1	-	multiplicator in the numeric computation of pheromone field
λ_A , λ_B	0.01, 0.02	$\frac{2}{\delta}$	decay parameter included in PDE
$[x_{nest}, y_{nest}]$	[0,0]	-	position of the nest
$[x_{food}, y_{food}]$	[0, 150]	-	position of the food

Table 1: Parameters of the model with references and their settings for the basic simulation.

3 Results

The mathematical model of ant colony food source discovering and trail formation proposed in the bachelor thesis [27] led to the similar behaviour of the constructed system as it is observed in the nature in real ant colonies.

Ants were able to find the food source in an undiscovered environment with no information about the food source position. A trail between the nest and food was formed by the communication based only on two types of pheromones. Furthermore established trail was narow and close to the optimal in length. However the model was improved and studied in more detail from that time. We improved the biological relevance and numerical approach to simulations of the model by:

- adding a minimum threshold for the concentration of pheromones that may be detected by a single ant,
- changing the rule of directional change following Webber's law,
- implementing two phases of a food foraging,
- adding quantitative indicators for the behaviour of the system.

The minimum threshold for the concentration of pheromones was set because it increases the biological relevance in comparison with the previous approach. The diffusion and unlimited pheromone detection caused that the pheromone signal spread in the system with no boundaries so the information about the food source or the nest was available everywhere in the space when at least one deposition of pheromone appeared. This assumption made it easier for ants to find their way with nearly no dependence on the distance from the source of a signal.

Even though such changes added to the model to satisfy biological relevance are more restrictive, the model still works well. A successful food foraging and a trail formation is observed in numerical simulations.

Furthermore quantitative indicators allowed us to study the system behaviour in more detail. They also serve as a new tool to compare simulations in different settings and to study the influence of its parameters.

3.1 Graphical representation

The state of the system at some time point can be graphically well represented by a pair of graphs in $\mathbb{R}^2$, where one of them shows the pheromone concentration field and the other shows trajectories of ants in the system such as in the Figure 11. Upper panel shows the field of pheromone concentration where the sum of concentrations of both pheromones A and B is represented, so the complete pheromone signal information is given. Lower panel shows last 15 positions for each ant and demonstrates the recent motion of ants. While plotting trajectories we also plot the position of the nest and the food source. The nest is represented by a black circle and food source by an orange one.

3.2 Basic simulation

A basic simulation of the model shows a behaviour of the system in the simplest setting with parameters set to chosen benchmark values mentioned in the previous chapter. These values were mainly set to follow biologically relevant references and were balanced to obtain satisfactory results by a trial and error method with performing simulations.

The basic simulation is a simulation of an ant colony consisting of 150 ants exploring the space with one limitless food source located 15 cm from the nest. The simulation has two phases [3],[4], [39]. The first phase is the beginning of the simulation where there is no pheromone signal and all ants are placed in the nest. This phase ends when 10 % of successful food foragers return to the nest. In the given setting this process lasted for number of steps that correspond to approximately 220 seconds of the system behaviour.

Right after the first phase the system is restarted and the second phase begins. At the beginning of the second phase all ants are placed to the nest and they start to explore the environment in which a pheromone signal is already present from the pheromone field made by ant foragers in the first phase. The trail formation can be observed in this phase using the sequence of the pheromone concentration fields and trajectories at different times. After some time the system reaches a stable state when the pheromone trail leading ants from nest to the food source and back is formed. The basic simulation follows the evolution of the system for 2400 steps that with the scale of all the parameters corresponds to 20 minutes of behaviour of the system.

3.2.1 Graphical results for the basic simulation

Figure 11 shows the results for the initialization part of the basic simulation. All panels in the figure appertain to the first phase of simulation which was initialized with no pheromone in the system. Panels show the state of the system after 100, 200 and 220 seconds.

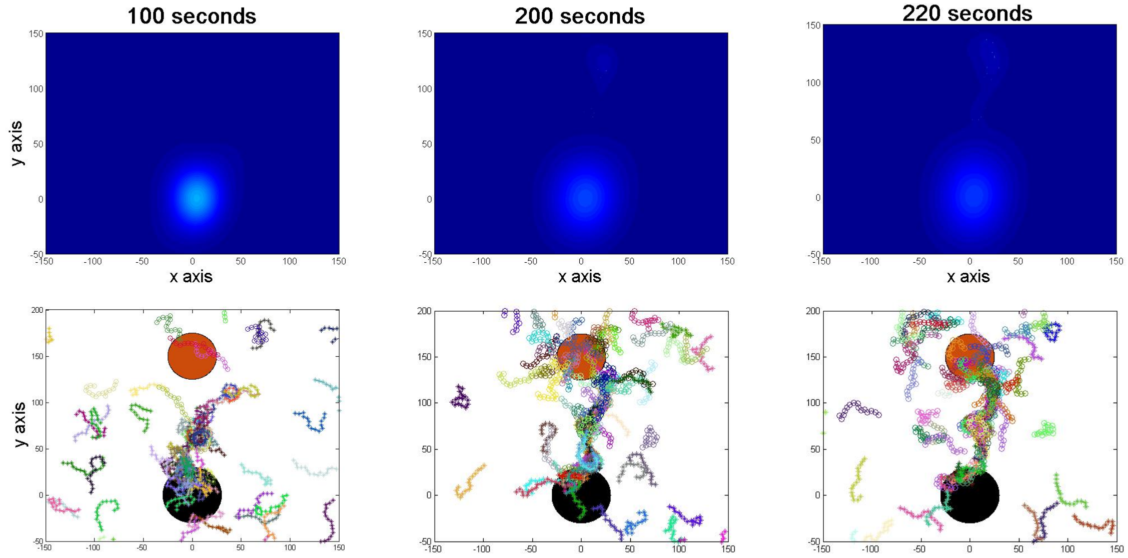

Figure 11: The first phase - the pheromone concentration field and ant trajectories at the beginning of the basic simulation. The left and the middle panel represent the state of the system after 100 and 200 seconds from the beginning of the simulation. The very right panel shows the state at the end of the first phase that is in this case at the time 220 seconds from the beginning.

At the beginning of the simulation we can observe a Gaussian like distribution of pheromone at the concentration field graph around the place where the nest is. This signal was made by initial pheromone A deposition from foraging ants leaving from the nest. The pheromone signal around the food source can not be observed yet. The signal around the food source comes mainly from the deposition of the pheromone B by those ants that already reached the food source. The number of ants that reached the food source in the first 100 seconds of the simulation is very low so the signal from them is not observed in the left panel. Trajectories show more or less random behaviour of ant foragers, even though we can observe a trend in some trajectories - movement towards the food source, since some small unobserved signal has already been established.

The middle panel represents the state of the system 200 seconds from the simulation initialization. Simulation is still in the first phase and we can observe a pheromone trace both around the nest and around the food source. There are few ants already forming a trail between the nest and the food source as it is observed in the trajectorie's panel. The last very right panel in the Figure 11, represents the state of the system at the end of the first phase, when 10% of all ants found the food source and also the way back to the nest. The concentration field formed at the end of the first phase serves as the initial pheromone distribution in the system for the the second phase. When the first phase comes to the end, the system is restarted and the second phase begins.

When entering the second phase the system stays in this phase. There is no other reset and system evolves just by itself. The evolution of the basic simulation in the second phase is displayed in the Figure 12.

At time 300 seconds from the beginning of the simulation a very strong pheromone signal can be observed. The reason for this strong signal comes from the reset of the simulation after the first phase. At the beginning of the second phase a pheromone signal for the food source was already formed and ants coming from the nest were able to follow the signal to the food source. Furthermore, there was also a nest leading pheromone signal around the nest from the first phase which was strengthened at the beginning of the second phase by all the ants in the system coming from the nest and depositing the pheromone.

The trail formation and its improvement in the second phase can be observed in the sequence of panels representing the pheromone concentration field. As the simulation proceeds a strong pheromone signal forms a narrow trail between the nest and the food source.

Looking at ant's trajectories we observe that after some time the majority of ants in the system is following the pheromone signal. Ants follow and improve established trail in both directions. However, there is a small fraction of ants that are further away from the trail and those are still exploring the environment for further food sources.

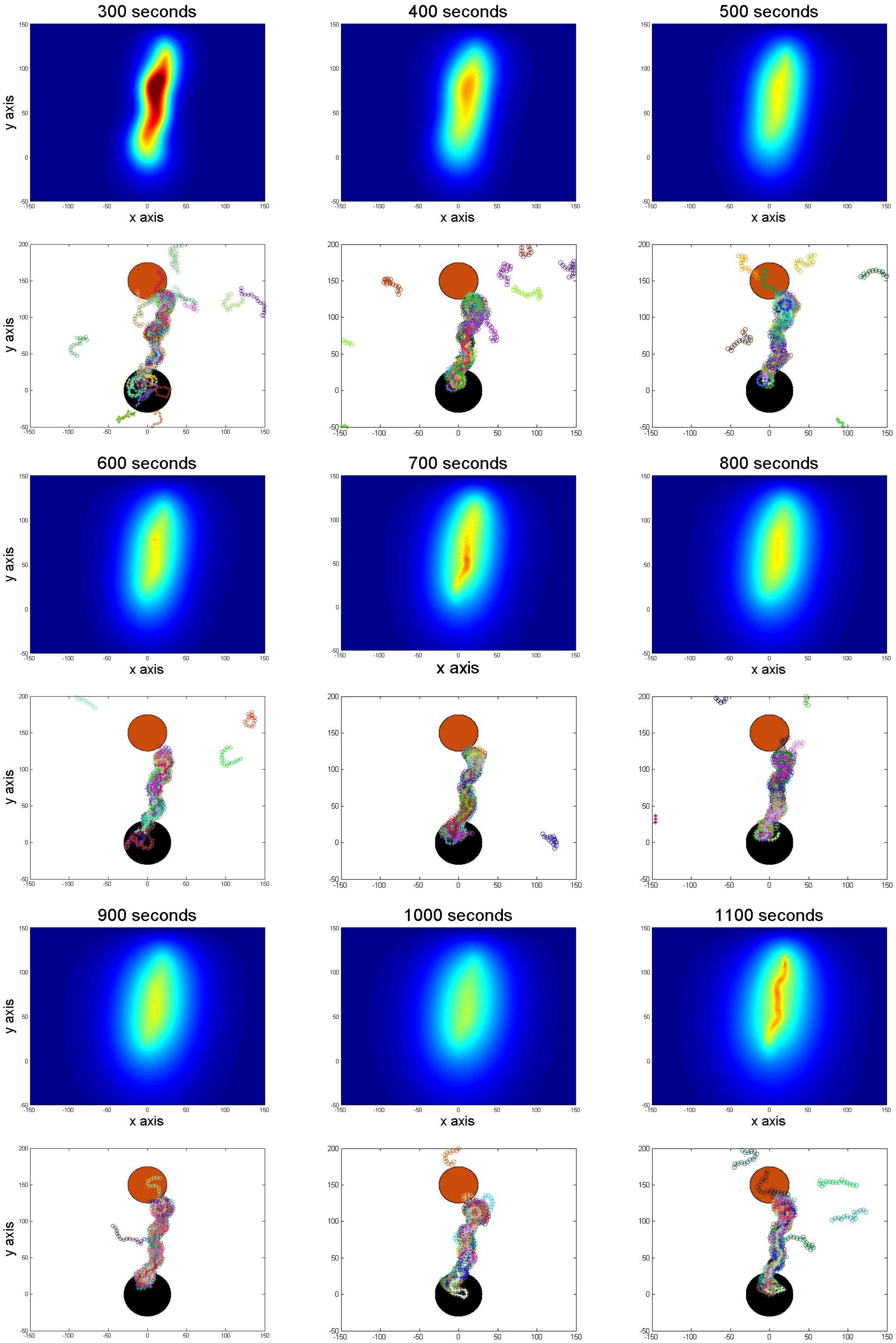

Figure 12: Second phase - evolution of the system in the basic simulation. From the left to the right states of the system after 300, 400, 500, 600, 700, 800, 1000 and 1100 seconds from the beginning of the simulation.
The colormap range is from 0 to 0.0095 g/mm^2 (dark blue to red)

3.2.2 Quantitative results for the basic simulation

A quantitative indicator that describes the state of the system is the ratio of ants that are in search for food or in search for the nest. Since in our model each ant has to be in one of those states the mentioned ratios add to 1. In the Figure 13 the dynamics of both these ratios is plotted for the basic simulation.

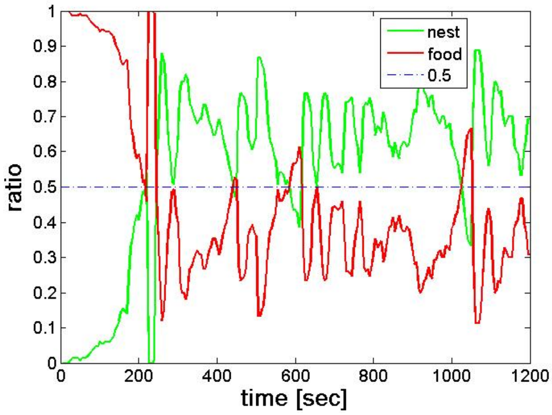

Figure 13: Ratio of ants in search for food and in search for the nest. The graph shows the course of ratios over time in the basic simulation. Around the time of 220 seconds of the simulation the reset of the system after the end of the first phase causes a dramatic change. As simulation proceeds with time we can observe the oscillation of the ratio close to the value 0.5, where the bias towards the ants in search for the nest is observed. This is partly explained by the ratio of lost ants which are in search for food and in search for the nest (Total number of ants: 150, Lost: 9, Food: 3, Nest: 6). The system does not stabilize at some equilibrium over time but it shows a quasi periodic behaviour.

At the beginning of our simulation all ants are foraging that means that all of them are in search for food. When the first ant reaches a food source (approximately after 50 seconds from the start) it changes its state to nest searcher, so the ratio of ants in search for food decreases. The pheromone B signal giving the information about the discovered food source is present in the system so more ants are able to discover the food source more quickly and the ratio of ants in search for food declines more rapidly.

The first phase reaches its end around the time of 220 seconds from the beginning of the simulation. The reset of the system is made, when all the ants are placed back to the nest and are given a status of food searching. This reset explains the dramatic return of the ants ratio in the Figure 13 around the time 220.

After the reset, second phase begins and since there is already a pheromone B present from the first phase, many ants can find their way to the food source and the ratio changes quickly. We can observe that far more than a half of all ants in the system got to the state of searching for the nest after reaching a food source. As time passes the system stays in the state where the ratio oscillates around half, but mostly with more ants in search for the nest than in search for the food.

There are two aspects that we have to keep in mind when drawing conclusions from this graph. The first one is that the system has a stochastic behaviour and the graph is a representation only of one sample of the simulation setting. When performing more simulations with the same parameters we can get a slightly different graph because the randomness has a relevant impact on the system.

The second aspect is that the graph shows the ratio of all ants in the system, including those which are not following the trail but are lost in the space. It is also observed in nature that some ants get lost during foraging so at some level it is not considered as an error of the model [8]. However, lost ants in our model do not change their state over time and may cause a bias in the ratio. For instance in the presented sample of the basic simulation 9 ants out of the total 150 are considered to be lost, since they stayed in the same state for a very long time (more than 3/4 of the simulation length). In those lost ants 6 were lost in the state of search for the nest and 3 in search for the food source. This partly explains a bias in ratio towards ants that are in search for the nest.

The second indicator of the system state is the time period for which an ant stays in some state - either in search for food or in search for the nest. In the Figure 14 these time scales are represented by a scatter plot. Every dot plotted in the graph represents an ant entering and quitting time for the given state. In the left scatter plot we can observe that many ants entered the state of being in search for food at the beginning of the simulation and around the time 220 seconds from the beginning. These observations are consistent with the run of the simulation - at the beginning all ants are in the state of food search and at the end of the first phase after the system reset all ants are again set to this state.

As time of the simulation proceeds we can observe that ants tend to reach the food source in approximately the same time - dots that are horizontally clustered in the scatter plot for the food time scales. This phenomenon gives us an explanation of dots vertically cumulated in the nest time scale scatter plot, since when ants quit one state they switch to another. The bipolarity of the system corresponds to the oscillating behaviour observed in the Figure 13.

The phenomenon observed in the food time scale scatter plot can be interpreted as a system synchronization, when most of the ants reach the food in the same time. The phenomenon of ant colony synchronization was also observed in nature [3] and studied in some theoretical models [4], [39]. Even though the pulses observed in nature were mostly due to different roles of ants within the colony whereas here they are induced by an asymmetry between the diffusion properties of pheromones A and B as we will see later. The synchronization in our model is considered asymmetric as we do not observe the same behaviour in the the nest finding as in the case of food discover. In the basic simulation only the food finding is synchronized.

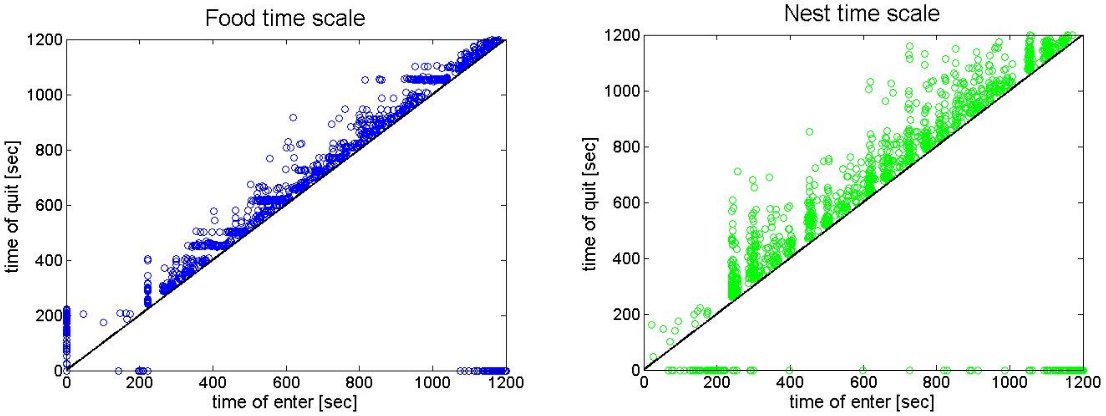

Figure 14: Time scales for ants being in the state of food search and in the state of the nest search. Dots on the x–axis are for ants that entered a status and have not left it yet either because they got lost in the space without a pheromone signal or they did not have enough time to reach the other state, because the phase or whole simulation ended.

The most probable explanation of the synchronization and its asymmetry arises from the difference in diffusion of pheromones. This hypothesis is also supported in the further example of simulation with different diffusion settings. It seems that diffusion constants for particular pheromones play important role in ant‘s synchronization.

There is also the other aspect that might have an impact on synchronization. Even though we assumed no interaction among ants an interaction is present in the system by the local pheromone attraction and single ant confusion. The confusion arises from the very strong but misleading signal of the pheromone when an ant which is in the state of search for food passes by an ant in search for the nest. Both of these ants deposit a pheromone that is attractive to the other ant hence they might give a misleading information to each other. These ants are sometimes stuck in the cycle when following each other. Nevertheless these cycles are eventually broken since in our model ants deposit a pheromone just for the fixed time period. When the period of depositing for the ant ends it stops to deposit pheromone and does not confuse the other ant any more. Both ants may just simply follow the pheromone signal from the surrounding space with no misleading interaction. Similarly if ants use diminishing amounts of pheromone the cycling behaviour would eventually stop.

The concept is in correspondence with diffusion constants impact. Hypothetically when the diffusion is slower the local attraction is less likely and so the asymmetry emerge. The lower diffusion constant k implies the slower diffusion, so the local interaction around the nest is less probable than the misleading pheromone attraction around the food source.

3.3 Impact of changes in the basic simulation

All assumptions and parameters in our model are set to reach effective behaviour of the system with attempt not to violate biological background and relevance. To satisfy this demand multiple simulations with various settings were performed to study the system behaviour.

In upcoming sections results for some simulations with a particular changes in comparison to the basic simulation are shown. The impact of changes can be studied from the graphical representation of the system evolution. In all cases that are mentioned the system still worked well at some level - ants formed the trail between the nest and the food source. Even though we consider the setting for the basic simulation as a better benchmark choice as it is explained particularly for each change performed.

3.3.1 Diffusion constants k_A, k_B

Diffusion of the pheromone is one of the most important assumptions in the model. There are two attractive pheromones with different physical properties present in the model. The diffusion constant k has an impact on the particular pheromone information spread. The smaller diffusion constant k the slower the diffusion of the pheromone is and the signal is more cumulated.

When considering two pheromones – pheromone A indicating the nest position and pheromone B giving information about the food source – we can logically assume that $k_A < k_B$. In short run the nest is fixed in space and there are many ants that can produce pheromone A around the nest - the information about the nest placement is strong and not changing dramatically over time. On the other hand food source is a dynamic part of the environment. It varies in its position and size therefore ants need to spread the information about the discovered food source quickly and since the food source is not limitless and it changes over time the information does not need to persist long time but to spread and disappear quickly. Given reasoning also has a biological background where we have an evidence ([14], [36]) about the diffusion constants of pheromones supporting the choice $k_A < k_B$.

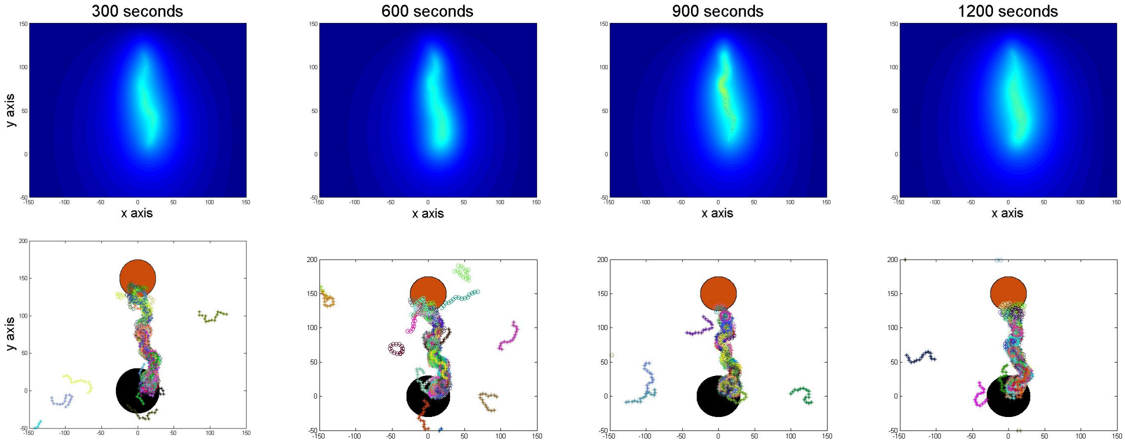

Figure 15: Simulation with changed values of diffusion constants $k_A > k_B$ ($k_A = 5$, $k_B = 1$) in the time 300, 600, 900, 1200 seconds from the beginning of the simulation.

In the Figure 15 and the Figure 16 results for the simulation where we changed the diffusion constants in a manner that $k_A > k_B$ are shown. Ants were still able to find the food source and the way back to the nest and to form the trail as we can see in the concentration field and trajectories graphs.

Even though when we look at the plot in the Figure 16 representing the ratio of ants in the two states - the food search and the nest search – we can observe that the process was not as fast and effective as in the basic simulation. We can observe the opposite bias in the ratio as we observed in the basic simulation. There are more ants in search for food than in search for the nest. The bias is again explained by lost ants, where in this simulation we had 16 lost ants out of the total number of 150, what is more than 10%. All of these ants where in search for food. The probable reason is that since the diffusion constant for pheromone B (k_B) was set to a lower value compared to the basic simulation the information about the food source was more concentrated and not enough widely available for ants, that many of them have not found their way even though the pheromone was present in the system.

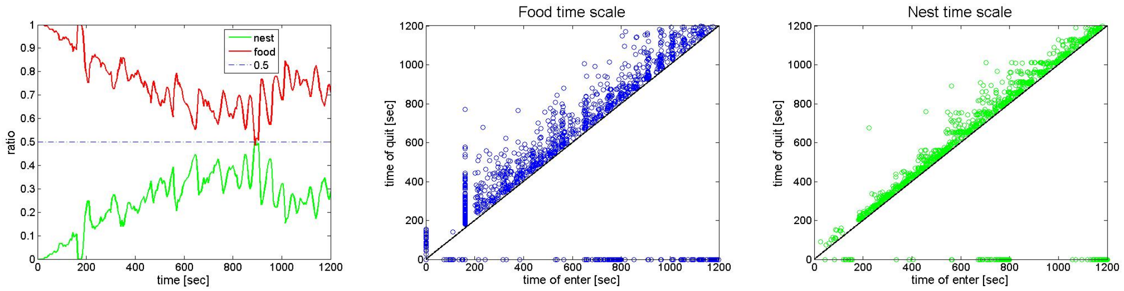

Figure 16: Ratio and time scales for the two states of ants in the simulation with pheromone diffusion constants $k_A > k_B$ ($k_A = 5$, $k_B = 1$).
(Total number of ants: 150, Lost: 16, Food: 16, Nest: 0).

When we study the time scales for ants in the particular states we can notice an opposite behaviour in comparison to the results obtained in the basic simulation. The asymmetric synchronization of the system is again observed, but now with ants reaching the nest in the same time. This gives an evidence of impact of diffusion constants on the system synchronization. Hypothetically the local information which does not spread so fast gives a lower local interaction between ants and they do not interact that much.

As time passes we can say that simulation gets nearly to the same pattern as it does in the basic simulation. The reason is that the food source is limitless and not changing so after some time there is no difference between the food source and the nest. We reach the same state as in the basic simulation setting but with reverse role of food and the nest. Comparison of synchronized behaviour in the basic simulation and the simulation with changed diffusion constants is given in the Figures 17 and 18.

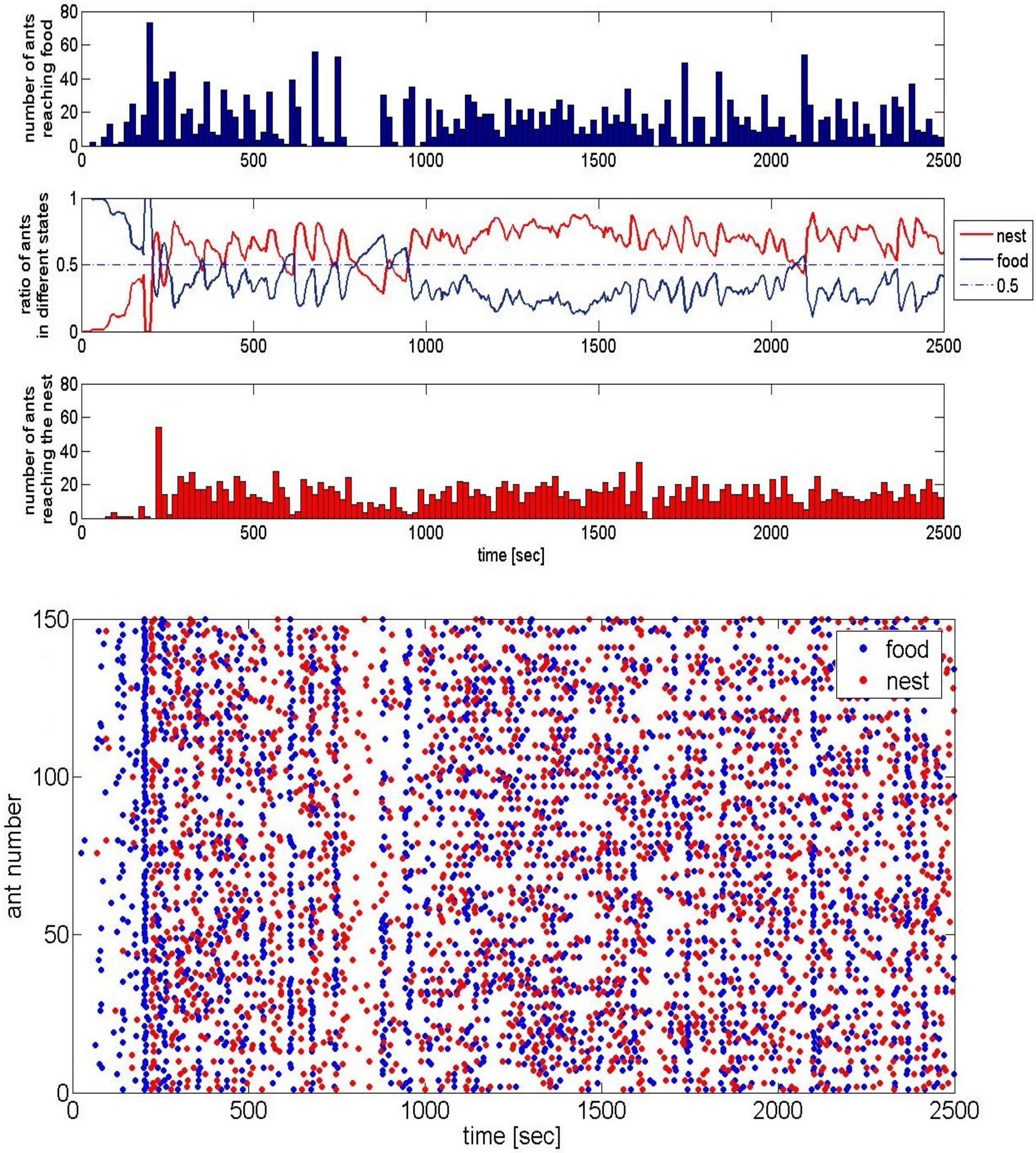

Figure 17: Synchronization in the basic simulation. Many ants reach the food source at the same time. Differences in number of ants reaching the nest do not vary that much over time, we observe an asymetric synchronization. Histograms shows the number of ants reaching food/the nest at a given time. The synchronization corresponds with oscillations in ant‘s state ratio. The lower panel is a raster plot for individual ants reaching the food or the nest over time. The synchronization can be observed in vertically cumulated food marking dots.

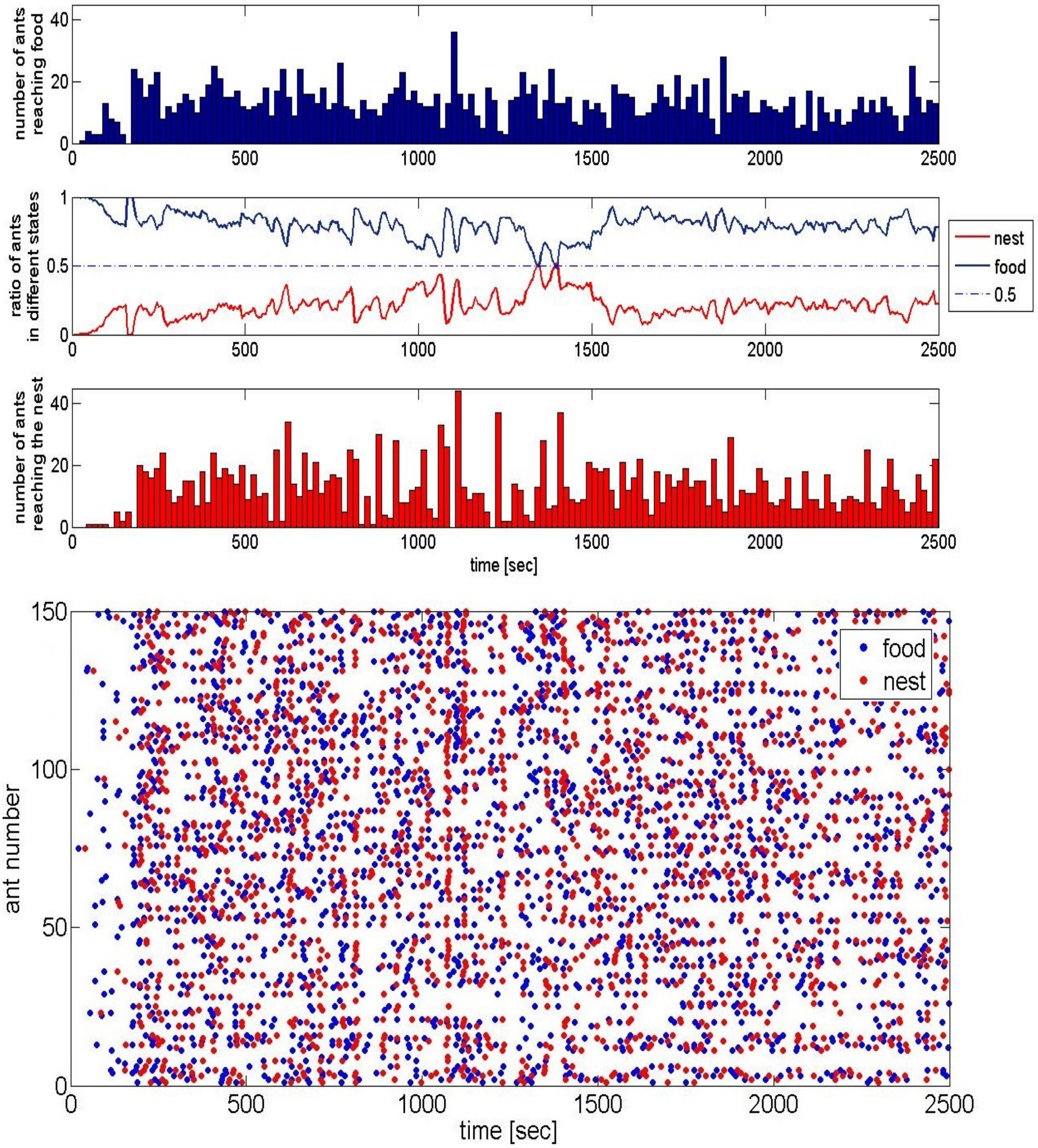

Figure 18: Synchronization in simulation with changed diffusion constants $k_A > k_B$. Ants are more likely to reach the nest at the same time than reaching the food source at the same time. We observe an asymetric synchronization again. The lower panel is a raster plot for individual ants reaching the food or the nest over time. The synchronization can be observed in vertically cumulated nest marking dots.

3.3.2 Randomness decrease κ

Randomness has an important role in the model. It is present in the single ant motion by a noisy perturbation in each step which is set according to the behaviour which should correspond to the motion of a real ant. The stochastic behaviour in ant colonies allows to effectively discover unknown environment, to find new food sources and to adapt to changing environment.

However when the food source is discovered and ants are able to share valuable information about the environment, there should be a mechanism which allows ants to follow the strong signal more precisely and not to lose their way in order to form and maintain the trail between the nest and the food source.

In our model this mechanism is represented by the randomness decrease factor κ, which in the case of high concentration of pheromone detected by an ant causes that the impact of stochastic part of motion in the next step for this ant is reduced. In the Figure 19 we can see results for the simulation where the effect of randomness is never decreased, so the parameter $\kappa = 1$ is constant.

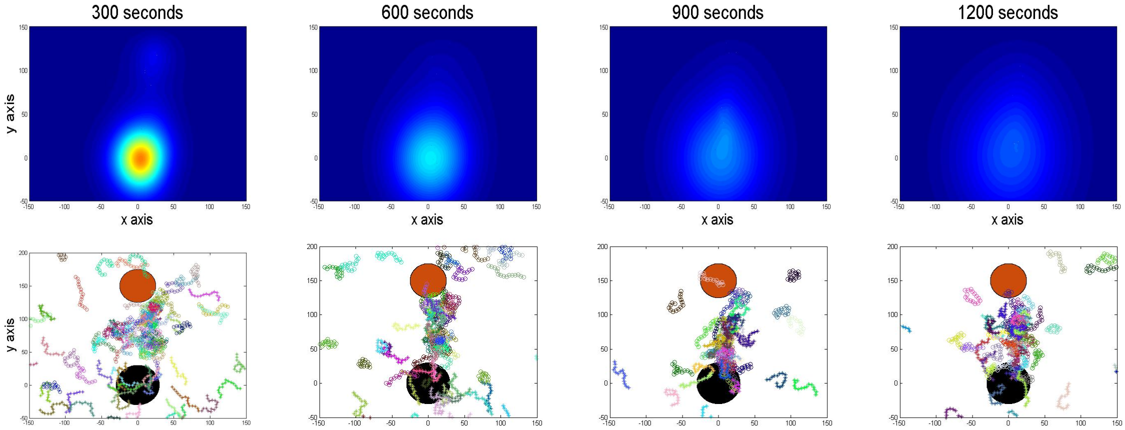

Figure 19: Simulation without the randomness decrease, $\kappa = 1$, in the time 300, 600, 900, 1200 seconds from the beginning of the simulation. Colormap range values are set half in comparison with the basic simulation, i.e. from 0 to 0.0045 g/mm^2.

In the second row of Figure 19 we can observe that even though a trail between the nest and the food source is formed that it is not narrow, straight and it is not improving over time. Ants move between the nest and food and are able to find both, but in a less effective way. The pheromone trail appears weakly with a small and dispersed concentration. There is a pheromone trace around the nest and the food source but it is not concentrated and does not form a trail. The overall pheromone concentration after 1200 seconds is also much lower than in the basic simulation, since the graphs were obtained with a colormap maximum value set to half of a maximum value compared to the basic simulation.

The reason why ants are not able to form a narrow trail in the simulation with constant random influence is that they are not able to strictly follow the pheromone signal. When ants all the time randomly change desired direction it is impossible to concentrate the signal and to enhance the trail formation.

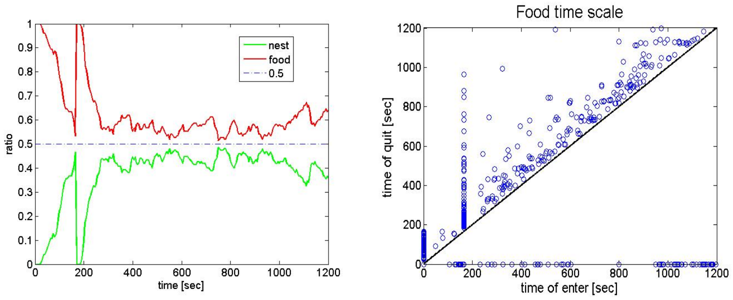

Figure 20: Ratio and time scales for two states of ants in the simulation with no randomness decrease, where parameter $\kappa = 1$. (Total number of ants: 150, Lost: 51, Food: 39, Nest: 12)

Figure 20 shows the plot of the ratio of ants in both states over time in the simulation without the randomness decrease. We can see that as simulation proceeds the ratio oscillates close to the value 0.5. We can again observe a bias for more ants in search for food. This bias can be again explained by higher proportion of ants in search for food in lost ants. In this simulation more than one third of all ants was lost that is another indicator of the non-effectiveness of the system. Furthermore in time scales of ants being in particular states we do not observe synchronization any more, so the discovery of the food source and the nest is more random.

The simulation without the mechanism of randomness decrease in the case of strong pheromone signal is less effective and more ants are lost in the space. Remaining ants are not able to form a narrow trail between food and the nest.

3.3.3 Simulation without the first phase

In our model we used the approach where we consider two phases of the environment discovering and the trail formation in ant colonies. There is a reset of the system at the end of the first phase that may look as a very artificial part of the simulation. There are two main arguments for the division of simulation into two phases.

First, there is a biological evidence that ant colonies foraging process consists of more phases. There are ants responsible for food source discovery that later recruits other ants from the nest to reach the food source in the environment where the food source is already marked by a pheromone signal.

Second, explaining the simulation approach of returning ants back to the nest after the first phase. When the first phase ends there are ants which are lost and they would not later contribute to the trail formation. These particular ants are not of the interest any more. We could ignore them and add more ants from the nest at the end of the first phase in order to exactly copy observed ant colony behaviour. This would increase the memory and computational requirements for the simulation. For that reason we use the ants that we already have in the system and do not add new ones. With returning ants to the nest the effect remains the same as with addition of new ants and we do not need to increase computational endurance. As a result of the reset ants synchronised leave the nest at the beginning of the second phase but this might be considered as a phenomenon observed in nature as well [3],[4], [39].

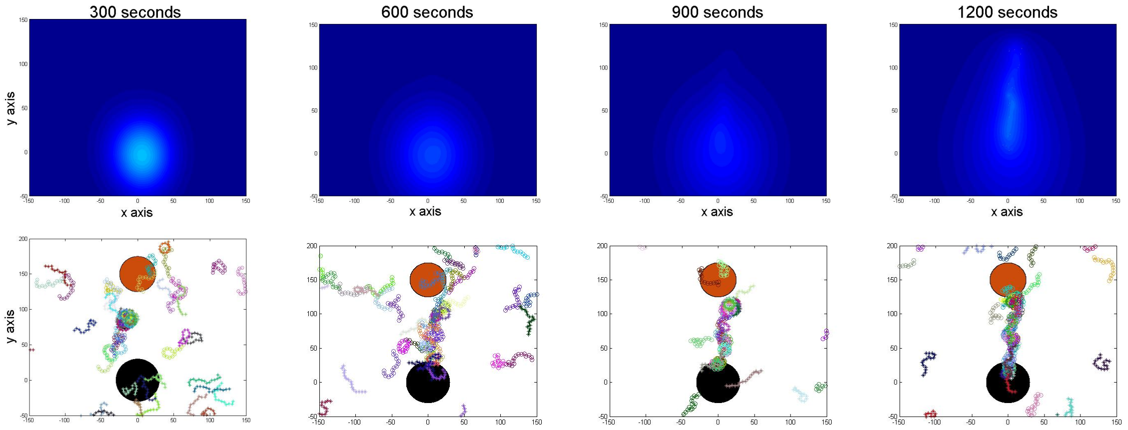

Figure 21: Simulation without the first phase in the time 300, 600, 900, 1200 seconds from the beginning of the simulation. Colormap range values are set half in comparison with the basic simulation, i.e. from 0 to 0.0045 g/mm^2.

In the Figure 21 we can see results for the simulation without the first phase. The conditions for this simulation were exactly the same as in the basic simulation, but we did not use the reset when a given number of ants find the food source and the way back to the nest and left the system to evolve on its own.

It is obvious from the pheromone concentration field graph and from the plotted trajectories as well that ants were able to form a narrow trail between the nest and the food source. Nevertheless there is one main difference in comparison to the basic simulation. The overall pheromone signal is much weaker. The reason is that there are less ants forming the trail and enhancing the pheromone signal, because many ants got lost at the initial part of simulation – they are already far away from the nest and the food source they are unlikely to find their way back.

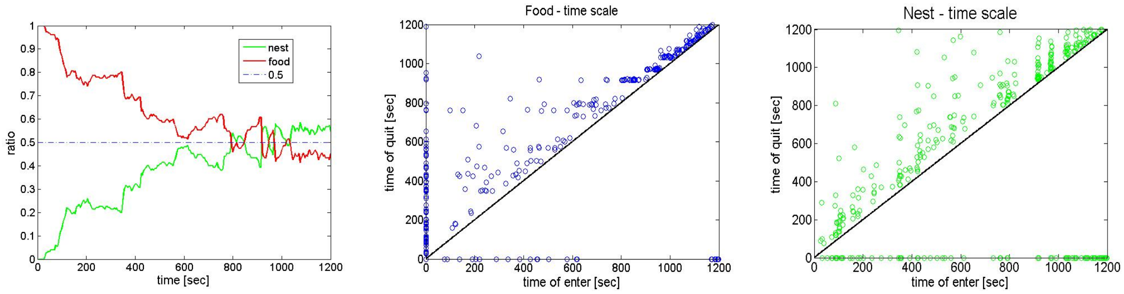

Figure 22: Ratio and time scales for two states of ants in the simulation without the first phase. (Total number of ants: 150, Lost: 63, Food: 46, Nest: 17)

The system got to nearly the same state as it did in the basic simulation where the concept of two phases is used. Also the synchronization of ants in the same manner as in the basic simulation can be observed. The main difference is the effectiveness measured in number of ants that are not lost and so contribute to the trail formation and in the time needed to reach the synchronized state. We can see in the Figure 20 that the ratio of ants in different states started to move around value 0.5 only around the time 600 seconds from the beginning of simulation. In the scatter plots where the synchronization of ants can be seen in the same manner as in the basic simulation we observe this phenomenon even later – approximately after 900 seconds from the beginning of simulation.

In conclusion, the first phase is not inevitably needed for the model to work well and to perform in the manner where trail formation and ants synchronization is observed. However the two phases approach helps to improve the effectiveness of the system without the violation of biological principles on which is the model built. Furthermore it is also an easy improvement of the algorithm which shortens the time of simulation needed to reach a synchronized state without the addition of new ants to the system.

3.4 Two food sources

Until now we have studied scenarios with original parameters and with changed parameters in comparison to the basic simulation. The reasoning for the choice of parameters in the basic simulation has already been given. In all upcoming results for different simulations the same parameters as in the basic simulation was used if not stated otherwise.

This section displays results of our simulations where two limitless food sources are available in the environment. We performed simulations where the food sources were not equally distant from the nest but since both were limitless and the closer food source was always discovered sooner the trail was always formed only between the closer food source and the nest. The second food source might get discovered by chance by few ants, but the pheromone information given by them was never strong enough to attract ants that had already been following a formed pheromone trail. For that reason results for those simulations were similar to ones obtained from the basic simulation.

The question of system behaviour when two equidistant food sources are present in the environment arises. In the Figure 23 the results for these simulations are presented. In all cases results reflect the state of the system after 1200 seconds. Even though the probability that ants reach any of the food sources is the same we can observe that after some time ants tend to prefer one of them and to ignore the other. This is the case even when the food sources are relatively close to each other. In all simulations ants initially discovered both food sources but after some time they stabilized pheromone trail between the nest and just one of the food sources where the choice was random (could be any out of the two choices). This is demonstrated in the first two simulations in the Figure 23, where the same environment setting for the simulations was given, but the different food sources were preferred.

In all simulations even after both sources have been discovered one of them was preferred. The behaviour when one food source is preferred even among the others equivalent to the one was already observed in some ant colonies [30]. In this sense the simulations with two equidistant limitless food sources are very similar to the basic simulation and not much influenced by the presence of the second food source. The system behaviour in this setting can be considered, especially when the trail to one of the food sources is already formed, as another sample of the basic simulation behaviour.

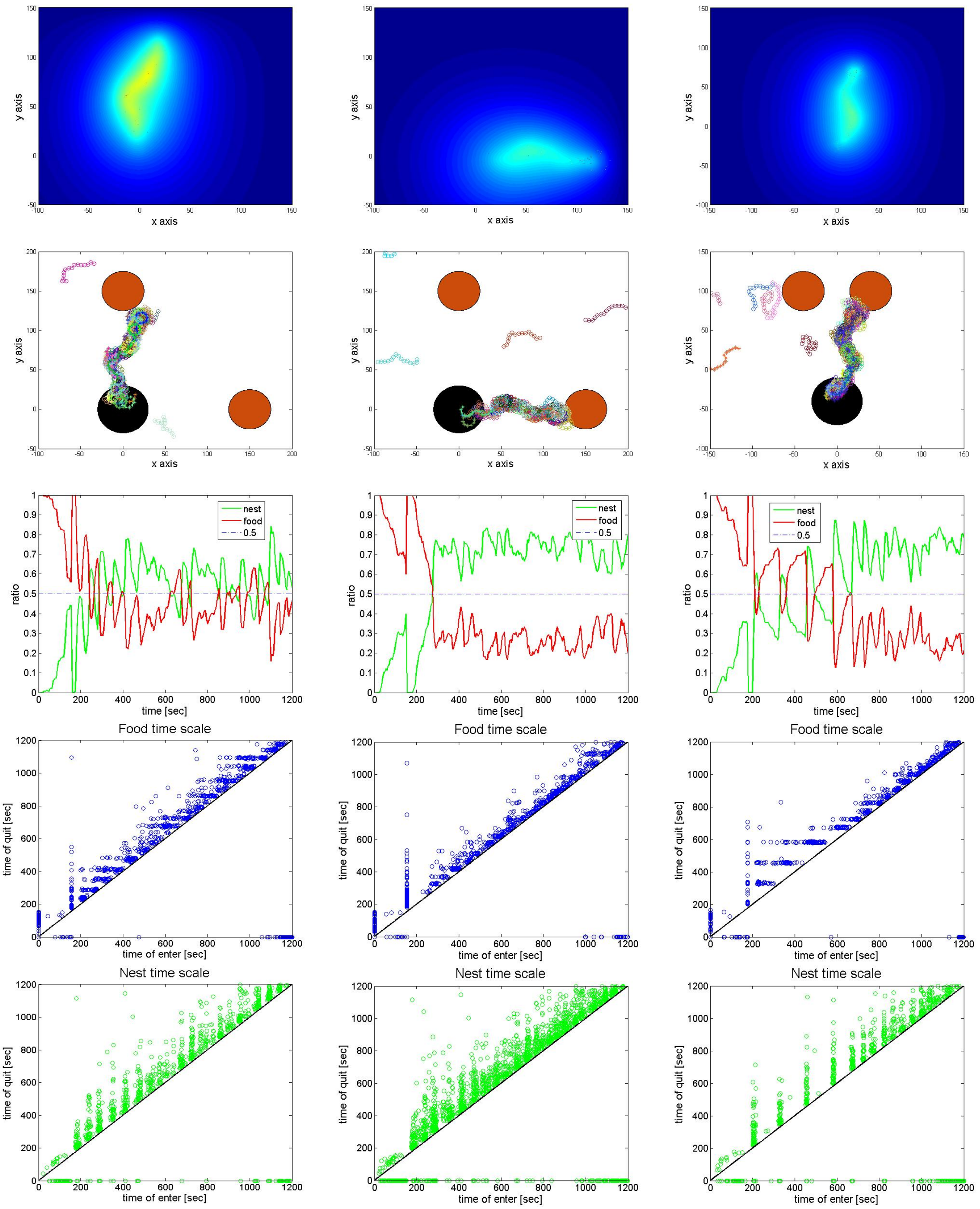

Figure 23: Pheromone concentration field and trajectories of ants in the environment with two equidistant food sources at the time 1200 seconds from the beginning of the simulation. Dynamics of the ratio of ants in two states of search and scatter plots for the time scales of ants being in a particular state for all three simulations.

Even though simulations with two food sources suggest that ants tends to prefer one food source among the two this phenomenon might be considered as a result of an optimization. For the small number of ants it is easier to maintain just one trail between the food source and the nest. Even if the second food source is discovered there is no reason to form the second trail when the first food source is limitless. The effect of non-limitless food sources is discussed later.

Nevertheless it shows up that ants are able to maintain two trails when those are already formed and are far apart. We set the simulation where we used the pheromone trail formed by ants in the basic simulation with one food source and placed another food source symmetrically through the center of the nest and copied the formed pheromone trail as well. We increased the number of ants to 200 and set all ants back to the nest. Ants started to forage in new environment. We observed that one of the food sources was preferred as more ants started to follow the trail to that food source. This might be considered as a random choice of one of food sources. The difference in the choice of one out of the two food sources in comparison to all simulations performed with two food sources was that also the second trail lasted till the end of the simulation. Ants followed both trails and as time proceeded trails stabilized with nearly equal amount of pheromone signal and so the comparable number of ants following each trail. The evolution of the system can be seen in the Figure 24.

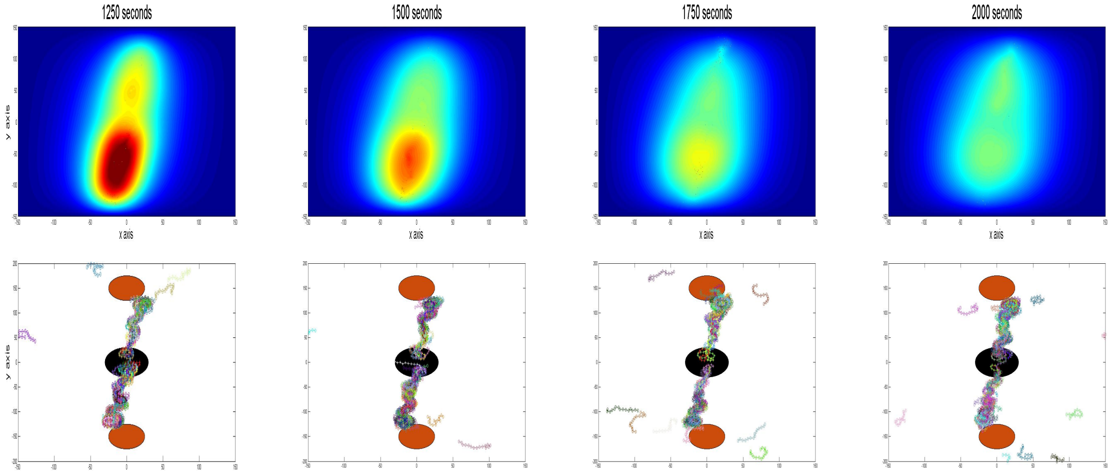

Figure 24: Simulation with two equidistant food sources when the pheromone trail from the nest to food sources is initialized as a symmetric distribution of the pheromone through the nest after 1000 seconds of the basic simulation with one food source. States of the system at time 1250, 1500, 1750 and 2000 seconds from the beginning of the simulation.

3.5 Changing environment

We have seen that ant colony in our simple mathematical pheromone based model with a single ant motion influenced by a random directional change is able to perform successful food foraging and to form a narrow trail between the nest and the food source. Since the food source is limitless and does not change its placement the system reaches a stable state in some sense. Once the trail is formed majority of ants follow the trail to reach the food source and find the way back to the nest.

In nature the ability to find the food source and to form the trail is very important for the ant colony to survive. But in the environment where the conditions change dynamically this is not the most difficult problem that an ant colony faces. Mechanisms that control ant's behaviour should give ants ability to adapt to the changing environment conditions. Ants should be able to discover new food sources and to decide for the better food sources as an optimization on the colony level.

The scientific studies about the abilities of ant colonies to adapt to changing conditions have already been made [8], [13], [46]. It seems that one of the most important mechanism that gives ants ability to adapt to changing environment is the randomness present in their behaviour.

In our model the randomness is present in each single ant motion. The level of the noise is controlled by the strength of pheromone signal detected. The dynamics of deposited pheromones provided by the diffusion and decay allows us to consider the strength of a signal as a measure of the information about the environment at the given space point.

Results in upcoming sections shows that ant colony behaviour based on the rules described by our model enable ants to adapt to some changes in environment. All further described simulations were performed with the same parameters as the basic simulation with the same rules based on the assumptions made about the model. Changes were made only in environment - the food source placement changes and the impact of the massiveness of the food source.

3.5.1 Food source placement

In the basic simulation the food source is limitless and when the trail is formed the system behaviour does not change. We performed a simulation in which we changed the food source placement while the pheromone trail between the nest and the original food source was already made. We used the basic simulation but after 1000 seconds from the beginning of simulation we changed the placement of a food source and observed the system behaviour in the next 1000 seconds.

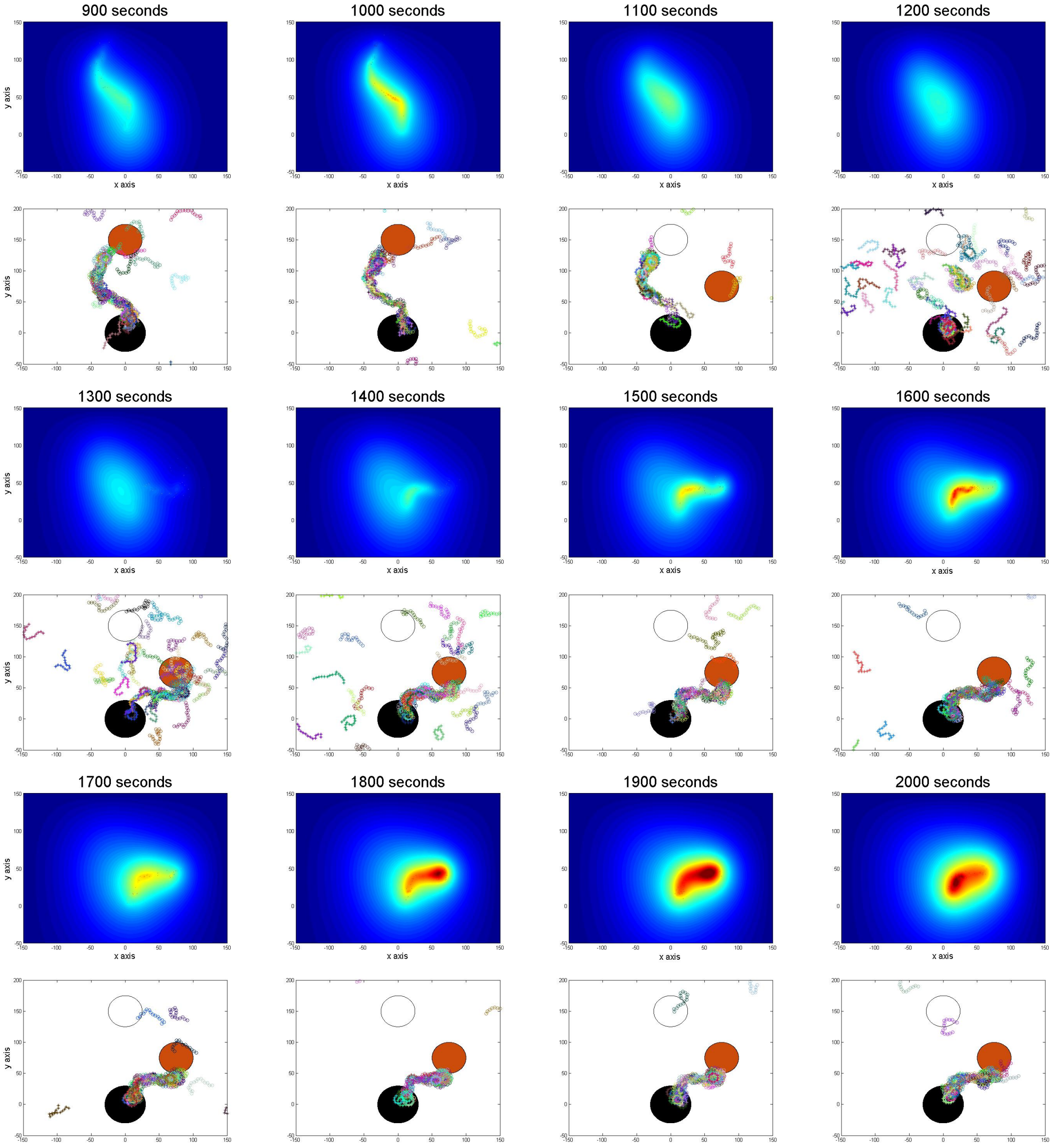

Figure 25: Pheromone concentration field and parts of ants trajectories in the simulation where we change the food source placement in the basic simulation in time 1000. Times: 1100, 1200, 1300, 1400, 1500 and 2000 seconds.

After the first 100 seconds from the change of food source placement ants were still following the old pheromone trail. Since the trail did not lead to a food any more new pheromone B was not added. Ants in search for food were less attracted by the trail because the pheromone B concentration decreased and they started to explore the environment randomly. This behaviour can be observed 200 seconds after the change of a food source placement. The new food source was discovered and after 300 seconds from the change we can already observe a new trail formation between the nest and the new food source. After some time the trail is established and the majority of ants is following it in both directions - from the nest to the food source and back. The system evolution in the case of changed placement of a food source is demonstrated in the Figure 26 by the sequence of panels describing the pheromone field and ant‘s trajectories.

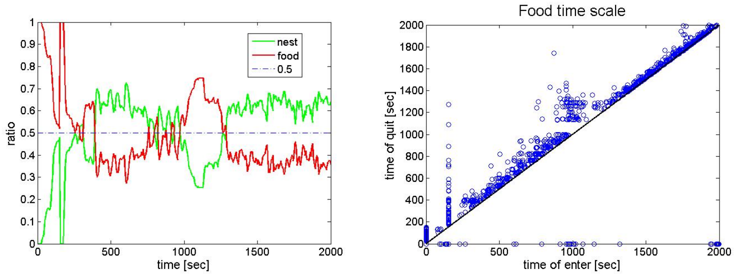

Figure 26: Dynamics of the simulation with changed food source placement at time 1000 seconds from the beginning.

The dynamics of a simulation with the change of a food source after 1000 seconds is described in the Figure 26. The overall behaviour is similar to the behaviour of the basic simulation except the behaviour between the time from 1000 seconds to 1300 seconds. When the food source was displaced there is an increase in ratio of ants in search for food since there is no information about new food source yet and the old pheromone trail leads ants to non-existing food source. After approximately 300 seconds the new food source is discovered and new trail is formed. The ratio of ants gets the same trend as before the food source displacement.

The synchronization of ants was observed at the trail formed to the original food source. When the food changed its placement we can observe a desynchronized behaviour – ants do not reach the food at the same time, because the trail for the new food source is not established yet. But again after 300 seconds from the food displacement ants get back to the synchronized state. Panels of food time scale and nest time scale shows this behaviour.

3.5.2 Mass of a food source

In all previous simulations we had a limitless food source of the same size. In simulations presented in this section we have studied the behaviour of the system when there are food sources of different size.

It shows up that in the case when we place the food source at the same distance from the nest we do not observe any significant difference in results when we change its size. Simulation with the food source smaller and larger than in the basic simulation were performed. The trail formation was observed in the same manner as in the basic simulation. The state of the system with different food size setting after 1200 seconds from the beginning of the simulation is shown in the Figure 27. Results for the ant‘s ratio in both states and for the time scale are comparable to ones obtained for the basic simulation as it is demonstrated in the Figure 28. In both figures the left panel is for the simulation with small food source, the middle one is the basic simulation and the very right panel shows results for the simulation with larger food source.

The probable reasons why the size of a food source had no influence are that first, there is only one food source available in the environment and second, even though the food source is smaller or larger it is still limitless and so ants can form the trail to the exact point with no food fading overtime.

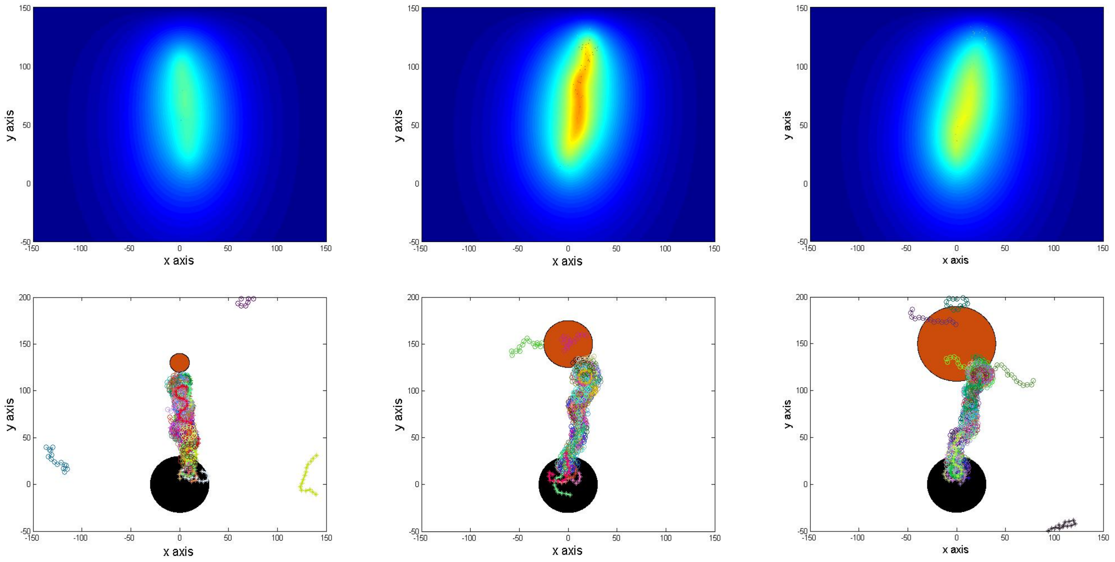

Figure 27: Pheromone concentration field and trajectories for simulations with different sizes of a food source at the time 1200 seconds from the beginning of simulation.

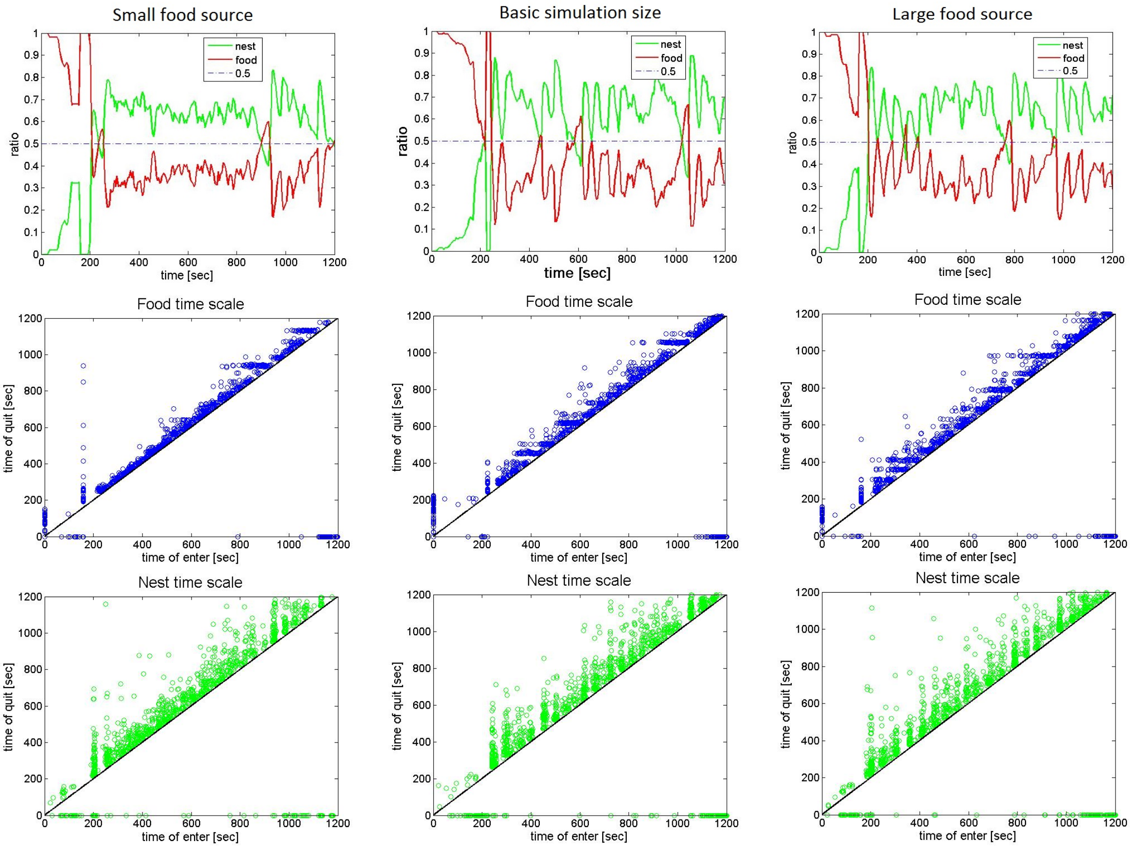

Figure 28: Ratio of ants in both states (in search for food and in search for the nest) and time scales for being in these states for simulations with different sizes of a food source.

The more interesting results we obtained for the simulation with two dynamic food sources. We placed two food sources of the same size in the same distance from the nest. The difference from the simulation with two equidistant food sources described in previous section is that those food sources are not limitless – every time an ant reaches a food source it reduces its size by reducing a diameter of the food source. This approach has a biological relevance since we imitate the dynamics of environment with food source which fades back. Results for this simulation are represented in the Figure 29.

In the initial phase ants were able to discover both food sources since those were placed at the same distance from the nest. Even though the pheromone signal was present for the both food sources from the first phase ants started to prefer one of them and formed a trail from the nest to the preferred food source.

The preference of the food source lasted for approximately 500 seconds of the simulation and then the food source became very small along of its diminishing property when reached by ants in search for food. Ants that followed the trail started to search for another food source. Relatively quickly ants rediscovered the second food source and formed new trail from the nest to more plentiful food source. At time 700 seconds from the beginning of the simulation a new trail was already formed. Ants following the trail to the food source and back to the nest used up the second food source within 500 seconds. When just nearly undetectably small food sources were left ants stopped to enhance the pheromone trails and started to move randomly as in the first foraging phase in order to discover new food sources in the environment.

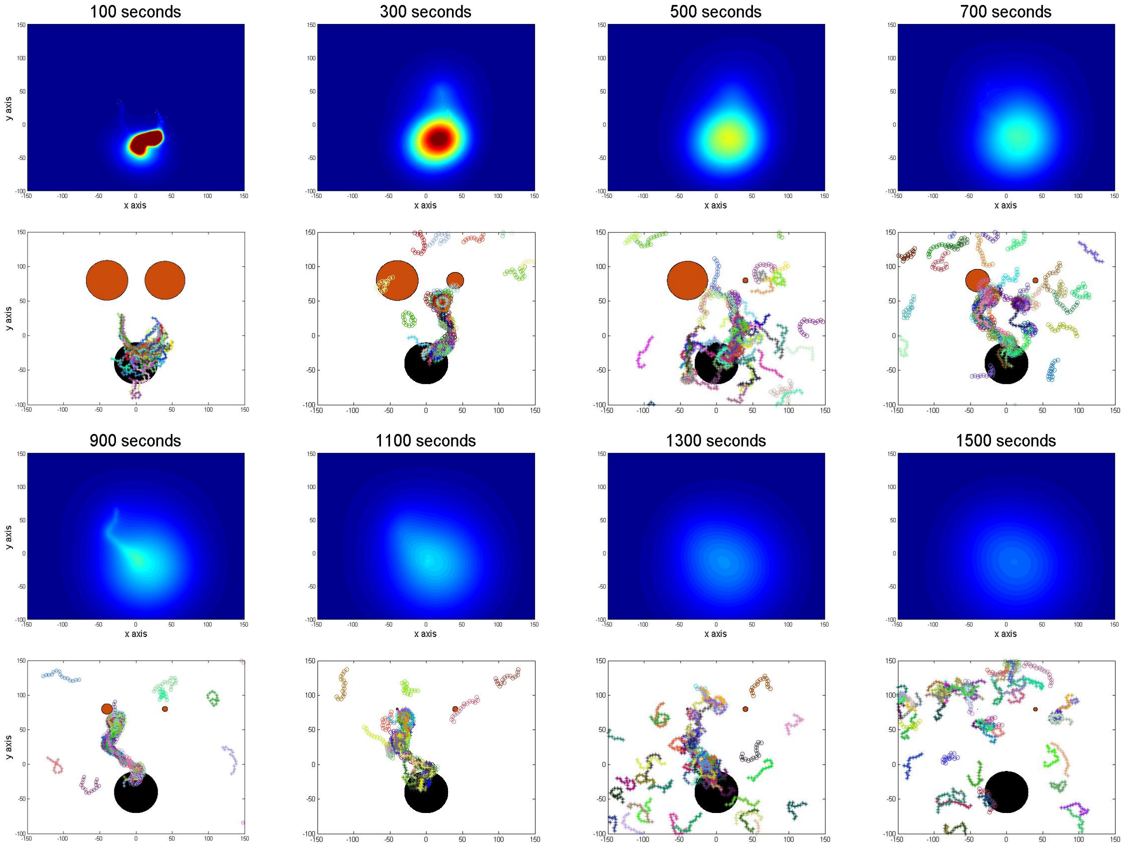

Figure 29: Evolution of the simulation with changing mass of the food source when two food sources are available.

Figure 30 gives another evidence of the system behaviour in the setting with two diminishing food sources. Ratio af ants in state of search for food declines in the first part of simulation since ants can find both sources and form an effective trail to one of them. The preferred food source is faded back and the ratio of ants reaches its local peak around the time 500 seconds from the beginning of the simulation. Right after the second food source is discovered and ratio declines. Approximately at the time 1100 ratio starts to increase again, because both food sources are nearly diminished and there is no more food left in the system. Ratios in both states stabilize, but the ratio of ants in search for food is not equal to one since we still have ants in the system which lost their way in search for the nest and are not foraging.

Synchronized behaviour is observed in the simulation as well. Ants tends to reach the food source (either first one or second one) in at the same time. There are less reaching of the food source or the nest observed in comparison with the basic simulation. The reason is that food sources are not limitless and so the number of ants that might successfully reach it is limited. After the time 1000 seconds from the beginning of the simulation there are very few food or nest reaching since the pheromone trail does not exist any more and majority of ants in spread in the environment either foraging for non-existing food source or lost on the way to the nest.

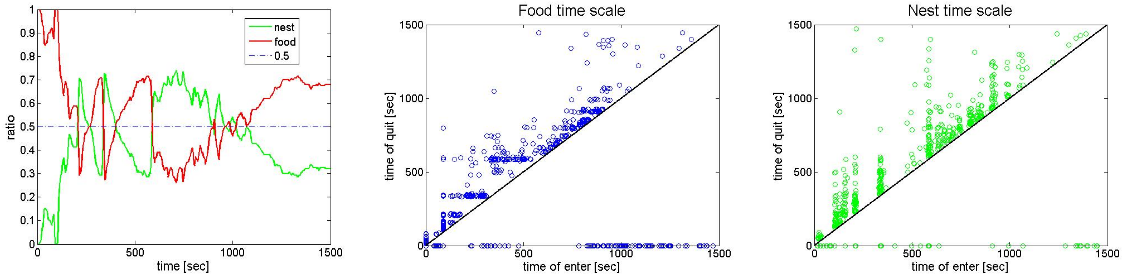

Figure 30: Ratio of ants in both states (in search for food and in search for the nest) and time scales for being in these states for the simulation with changing mass of the food source when two food sources are available.

Conclusion

The main aim of this work was to extend the stochastic model of the collective behaviour of ants proposed in the bachelor thesis [27]. We improved the biological relevance of the model and performed multiple numerical simulations in order to study the model behaviour in more detail and to study its ability to adapt to changes in environment.

In the first place the simple stochastic mathematical two pheromones signalization based model improved to follow a higher biological relevance shows the behaviour of successful food foraging and trail formation between the nest and the discovered food source.

The process in simulation is divided in two phases - initial phase with no pheromone signal in environment, where ants explore the environment and deposit pheromones to remain the information about the discovered environment. The second phase is initialized with the pheromone signal from the first phase – ants form and enhance the narrow trail between the food source and the nest.

Long simulations with different parameters setting were performed. We introduced some quantitative indicators over time such as a ratio of ants in different states – either in search for food or in search for the nest – and time scales representing the time an ant spends in a particular state. Interpreting this data we got to deeper understanding of the system behaviour and the impact of changes in parameters.

The interesting finding based on observations in simulations is that ants which by the assumption of the model do not directly communicate or interact, perform a synchronized behaviour. Experiments with various parameters setting indicate that different physical properties of pheromones relate to the synchronized behaviour, mainly the speed of diffusion numerically represented by the diffusion constant. The evidence of synchronized behaviour in ant colonies has a scientific references [3], [4], [39].

The crucial aspect in the model is the stochastic part of a single ant‘s motion. The motion of an ant influenced by the random directional change describes well the behaviour observed in real ants. The randomness in ant colony system gives it the ability to explore the environment and to find multiple food sources. It enhances the narrow trail formation and last but not least it gives an advantage of dynamical adaption to changes in an environment.

Simulations gives an evidence that the system is able to adapt to changes in the placement of the food source and its dynamic change of strength. Regarding the diffusion and the decay of pheromones formed trails that are no longer leading to the food source are abolished and ants are not determined to follow the misleading signal. By contraries ants start to randomly explore the environment and thus are able to discover new food sources and to form new trails.

The model performs well in all simulations presented in this thesis. However further study of its properties could be done. One of the logical suggestion is to study the effectiveness of the model in different distance scales with a need to form longer trails. In our model an ant deposits a pheromone just for a limited time. In order to be able to transfer an information for longer distances we would suggest a decaying depositing amounts of the pheromone instead of complete stop of depositing.

In our simple model two attractive pheromones are used. Pharaoh ant which is considered to be a good benchmark for the ant colony behaviour uses three types of pheromone - two attractive and one repellent [37]. The possibility to use one more pheromone in the model might give an advantage to the system to solve problems more effectively or to solve even more complicated tasks.

Other improvements of the model at the level of ant agents could be a new assumption in the model describing an interaction among ants. At the level of the system‘s adaption to the environment new findings could be done in a simulation with more food sources or with an obstacle in the environment.

Since all simulations are stochastic processes obtained results for one simulation are just one sample of the stochastic process. In future work multiple simulations should be performed in the same parameter setting to obtain more data in order to do more relevant and statistically robust conclusions.

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