COMENIUS UNIVERSITY IN BRATISLAVA

FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

THE IMPACT OF QUANTITATIVE EASING ON INFLATION

DIPLOMA THESIS

2015 Bc. Martin KUPEC

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Study programme:	Economic and Financial Mathematics
Field of study:	1114 Applied Mathematics
Department:	FMFI.KAMŠ - Department of Applied Mathematics and Statistics
Supervisor:	RNDr. Juraj Zeman, CSc.

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THESIS ASSIGNMENT

Name and Surname:	Bc. Martin Kupec
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Aim: In the context of the current economic and financial crisis is often mentioned impact of quantitative easing as the main instrument of monetary policy. The aim of the thesis is to clarify what quantitative easing is and what may be its impact on inflation and on the real economy. Then, building on the work of S. Reynard (2007, 2012) dedicated to similar issues, the thesis should develop a model based on the quantity theory of money with the single aim - predicting inflation. We evaluate the possibility of other factors in given model, backtest all outputs and the result will be applied in the context of quantitative easing period during last years on real data from Slovakia, other European countries and eventually the USA.

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Abstrakt

Ekonomický vývoj viacerých krajín je po finančnej kríze neobvyklý. Rast stagnuje, inflácia je nízko a v niektorých krajinách sme mohli pozorovať dokonca aj defláciu. Centrálne banky reagujú uvoľnenými monetárnymi politikami s účelom stimulovania ekonomickej aktivity. Úrokové miery sú ale už teraz veľmi nízko a museli byť prijaté aj širšie opatrenia akými je kvantitatívne uvoľňovanie (QE). Nakoľko takáto monetárna politika doposiaľ nebola používaná v tak širokej miere, jej dôsledky je ťažko odhadnúť. Hlavným cieľom tejto prace je posúdiť možné dopady QE na budúcu infláciu vo viacerých ekonomikách. Naša analýza vychádza z práce S. Reynarda, ktorý sa venoval posúdeniu vplyvu QE v krajinách ako USA, Švajčiarsko, Japonsko a Argentína.

Kľúčové slova: Monetárna politika, Kvantitatívne uvoľňovanie, Kvantitatívna Teória Peňazí

Abstract

The economic development in most countries is rather unusual after the financial crisis. It is characterised by stagnating grow, very low inflation or outright deflation in some cases. Central banks react with accommodative monetary policies to stimulate economic activity. But because interest rate has been already very low, they had to use to unconventional instruments namely quantitative easing (QE). Because this policy has not been used before in such a large extent, the consequences are uncertain. The main objective of this paper is to assess possible impact of QE on future inflation development in various countries. Our analysis is based on the work of Reynard, who analysed possible impact of QE in US, Switzerland, Japan and Argentina.

Key words: Monetary Policy, Quantitative Easing, the Quantity Theory of Money

Contents

Introduction

Monetary policies of central banks have changed a lot since 2008. Today, we can see practically 0 interest rates; in some special cases they even turned to be negative. Several macroeconomic theories suffered and became untrustworthy. Shocks in financial markets are hard to explain by mathematical models. Central banks have only limited experience with crisis times and situations similar to recent years. How to ensure sustainable growth? How to stabilize financial markets? How to perform monetary policy? These are the main questions.

First step in order to keep economy rolling since 2008 turmoil was lowering of interest rates. But they hit zero lower bound very quickly. Due to uncertainty, low demand for financial assets and low liquidity of markets, central banks had to do more. They established new lending facilities and entered financial markets from buyer's side. Introduced strong expansive monetary policy can be also called **Quantitative Easing (QE).** But situation is tricky. On the one hand, funds of central bank appear to be unlimited. It can help the economy, protect it from short term shocks and preserve long run equilibrium. On the other hand, printing money does not produce any value and does not change any real output. Strong Quantitative Easing increases total amount of money in circulation. It can lead to massive inflation and can have fatal consequences on economy. We saw effect of high inflation in the past. Recall the situation of the US or Germany in 20s or more recent example of Argentina.

Given historical examples have not prevented central banks from Quantitative Easing today. Luckily, we did not see growing inflation yet. But it is hard to set safe boundary what banks can do and what is too much, "too dangerous". Even theories of great economists diverge at some point. Our main objective will be to assess the relationship between nominal Money in economy and Price level. Fundamental basis for us to review this link is classical Quantity Theory of Money. Our model will stem from work of Samuel Reynard. His recent study, namely his last work (Reynard, 2012), is focused on examining Equation of Exchange $M * V = Y * P$ in current environments. This will be our starting point. The topic of referred paper is in line with the main aim of our study.

Chapter one describes the theory of Vector Auto Regression necessary to set up our model. Second part deals with data selection, development of the model and discusses all its properties. In chapter three we present all results and findings, discuss explanatory power of constructed model and extract possible consequences in the future. We answer questions if QE is affecting real economy, if it is observable and whether is there a potential risk of Inflation or any other exposure. We would like to emphasize the parts about the United States and the Economic and Monetary Union. We discuss recently introduced Quantitative Easing of European Central Bank in the EMU and assess possible effects on European economy in comparison to the USA.

1. Introduction to VAR analysis

Our goal is to analyse several times series (GDP growth, money aggregates, interest rates, etc.). Apparatus of linear regression or univariate auto regression models are not enough. We are going to introduce mechanism of multivariate time series analysis. For model developed from the second chapter, we would specifically need to introduce used mathematical basis of Vector AutoRegression (VAR) models. Following chapter and all theoretical background of VAR models is based on the book (Enders, 2004).

Firstly, we should start from easiest case of 1 dimensional autoregression equation.

$$\boldsymbol{x_t = a_0 + a_1 x_{t-1} + \epsilon_t} \qquad \textbf{1}$$

$$\epsilon_i \sim N(0,\sigma^2), \forall i \; and \; cov(\epsilon_i, \epsilon_j) = 0 \; for \; i \neq j$$

There is a time series x_t dependant on its own previous value (time lag), on some time independent constant and some unpredictable error or shock (also called white noise). We can make this simple system more difficult with making the lag bigger, adding x_{t-2} and older values to this equation. We get AR equation of the order p s.t.

$$\boldsymbol{x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \ldots + a_p x_{t-p} + \epsilon_t} \qquad \textbf{2}$$

$$\epsilon_i \sim N(0,\sigma^2), \forall i \; and \; cov(\epsilon_i, \epsilon_j) = 0 \; for \; i \neq j$$

Consequently, if we can observe some time series in real word, e.g. series of interest rates daily - $i_1, i_2, i_3 \ldots i_n$, we can try to fit model (coefficients a_i) in „best possible way“. Goal is to describe the movements of interest rates such that we minimize$\sum_{i=1}^{m} \epsilon_i^2$. This method of fitting coefficients for AR model is called ordinary least squares and is one of basic statistical methods.

Since we want to analyse more series of data, mentioned case is not enough. Classical model in equation2 can describe only one independent sequence of data, one variable. We would like to analyse 4 different time series with several interdependencies. Solution is to vectorize given equation to get VAR model, see example of lag 1, 2 dimensional model below.

$$\begin{pmatrix} \boldsymbol{y_t} \\ \boldsymbol{z_t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a_{10}} \\ \boldsymbol{a_{20}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{pmatrix} \begin{pmatrix} \boldsymbol{y_{t-1}} \\ \boldsymbol{z_{t-1}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon_{yt}} \\ \boldsymbol{\varepsilon_{zt}} \end{pmatrix} \qquad \textbf{3}$$

$$\epsilon_i = \begin{pmatrix} \varepsilon_{yi} \\ \varepsilon_{zi} \end{pmatrix} \sim N\left(0, \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}\right), \forall i \; and \; cov(\epsilon_i, \epsilon_j) = 0 \; for \; i \neq j$$

In given case, each variable (e.g. y_t) is dependant on its own values from past (y_{t-1}), but also on values of second given variable (z_{t-1}). In more generalized case, we can assume bigger time lag to the order of p, more variables and most interestingly interdependencies of variables in given time t. It means that value of y_t can be influenced by value of z_t and vice versa. As a result, we can rewrite 3 in given way

$$\boldsymbol{Bx_t = A_0 + A_1x_{t-1} + A_2x_{t-2} + \ldots + A_px_{t-p} + \epsilon_t} \qquad \textbf{4}$$

$$B = \begin{bmatrix} 1 & b_{12} & \ldots & b_{1n} \\ b_{21} & 1 & & \\ \vdots & & \ddots & \vdots \\ b_{n1} & & \ldots & 1 \end{bmatrix};\ \epsilon_i \sim N\left(0, diag(\sigma_1^2, \ldots \sigma_n^2)\right)\ \forall i$$

Given statistical model is describing system of n time series in best possible way. However, there is problem of implicitness, we would need to represent vector x_t explicitly – VAR system in standard form to be able to fit our model from given dataset. To achieve this goal, simple trick can be used. We multiply system 4 by matrix B^{-1} to get the explicit formula. Idea of this trick is to transform interdependencies from matrix B to residuals and their covariance, residuals will no longer be independent. Variance matrix of residuals will be $B^{-1}diag(\sigma_1^2, \ldots \sigma_n^2)(B^{-1})^T$, normality and zero mean stays. Standard system will look like

$$\boldsymbol{x_t = M_0 + M_1x_{t-1} + M_2x_{t-2} + \ldots + M_px_{t-p} + \epsilon_t} \qquad \textbf{5}$$

$$\boldsymbol{cov(\epsilon_t)} = \begin{bmatrix} \boldsymbol{\vartheta_1^2} & \boldsymbol{\vartheta_{12}} & \ldots & \boldsymbol{\vartheta_{1n}} \\ \boldsymbol{\vartheta_{12}} & \boldsymbol{\vartheta_2^2} & & \\ \vdots & & \ddots & \vdots \\ \boldsymbol{\vartheta_{1n}} & & \ldots & \boldsymbol{\vartheta_n^2} \end{bmatrix}$$

Solution of fitting model 4 is very simple, but problem is that we cannot fit all coefficients as they are in4. If we count them, we can get the number $(\mathrm{p}+1)\mathrm{n}^2 + \mathrm{n}$. On the other side, if we count up coefficient in 5 we can easily get to the number $(\mathrm{p}+\frac{1}{2})\mathrm{n}^2 + \frac{3}{2}\mathrm{n}$. In case of n=2 and p=1. We have 10 against 9, so we have one coefficient less in standard system. More presumptions have to be considered. One of possible way how to restrict our system is the following. Set of time series x_1^t, x_2^t, ... x_n^t have to be organized in a way that x_1^t depends only on values x^{t-1} and older ones. Series x_2^tcan be affected instantly onlybyx_1^t and by lagged values x^{t-1} and so on. Hence matrix B can be considered to be uppertriangular (or lower, effect would be same).We get exact number of parameters as in standard system. With these assumptions we are also decomposing the residuals. In 2 dimensional case, we have residuals

$$Var(\varepsilon_1^t) = \sigma_1^2 + b_{12}^2\sigma_2^2$$

$$Var(\varepsilon_2^t) = \sigma_2^2$$

$$Cov(\varepsilon_1^t, \varepsilon_2^t) = -b_{12}\sigma_2^2$$

As one can see, instantaneous effect between variables in Structural VAR model was transferred into covariance of residuals in VAR model of standard form. For our purposes, 2 dimensional model in standard form is sufficient. We were able to line up our variables in such a form, to have B matrix triangular and instant dependencies well-organized.

1.1. Estimation and identification

In practice, observation of some real data is followed by estimation of proper model, in most general form described by equation 5. As stated in (Enders, 2004), Box-Jenkins approach is to provide a methodology that leads to parsimonious models. Final objective of making proper and most accurate short-term predictions can be done only by purging model for insignificant parameter estimates, shortening the order of model, setting insignificanta_{ij} as 0. On the other hand, (Sims, 1980) argues for alternative estimation strategy. (Enders, 2004)has noted, "Sim's methodology entails little more than a determination of the appropriate variables to be included in the VAR and determination of the appropriate lag length. The variables to be included in the VAR are selected according to the relevant economic model." On top of that, we will use one of lag length tests to determine appropriate lag length.

Unquestionably, in discussed way our model with $n + p * n^2$ coefficients will be overparametrized (over fitted). Many of estimated coefficients are not significant but our goal is to find important interrelationships, general patterns in our data series. We will see later that making even short term forecasts on "real world" time series cannot be so accurate and finding perfect foresting model is almost impossible. In other words, VAR model is rather qualitative than precise quantitative approach. Making model with more zero restrictions than necessary can waste important information. Moreover, the regressors are likely to be collinear. The t-tests for individual estimates should not be reliable guides for pairing down the model.

In order to estimate the model in equation 5, note that the right-hand side contains only predetermined variables and the error terms are assumed to be uncorrelated with constant variance for each. Hence, each equation (each line) in the system can be estimated separately using OLS, what is the consistent and asymptotically efficient way. Moreover, even in the case of correlated residuals, "seemingly unrelated regressions (SUR)[1]method does not add to the efficiency of the estimation procedure since all regressions have identical right-hand side variables," claims Enders. Outlined idea gives us explanation how VAR model can be estimated using computational statistical software. Especially in the case of instant effects, we can firstly estimate standard form of VAR, assume necessary restrictions for the B matrix from equation 4and finalize form of the Structural VAR afterwards.

1.2. Stability and stationarity

We can observe a direct analogue between the stability conditions for univariate AR model (requirements for roots of the process) and the conditions for multivariate VAR(p) model from equation 5. We will illustrate the idea on VAR(1) model. Using the brute force method to solve the system, we iterate it backwards to obtain

$$x_t = A_0 + A_1(A_0 + A_1 x_{t-2} + e_{t-1}) + e_t = (I + A_1)A_0 + A_1^2 x_{t-2} + A_1 e_{t-1} + e_t \,.$$

After n iterations, solution looks like

$$x_t = (I + A_1 + \ \ldots + A_1^n)A_0 + \sum_{i=0}^{n} A_1^i e_{t-i} + A_1^{n+1} x_{t-n-1} \,.$$

If we continue to iterate backward, it is clear that convergence requires that the expression ${A_1}^n$ vanish as n approaches infinity. It basically means that our process x_t is stationary in long term. If this condition is met, we can rewrite formula for solution.

$$x_t = \overline{\mu} + \sum_{i=0}^{\infty} A_1^i e_{t-i} \qquad \mathbf{6}$$

Where $\overline{\mu}$ is representing unconditional mean of x_t, some long term average. Here we suppose that each of variables x_{1t} , x_{2t}, ... is stationary on its own. Taken separately, each of our variables fulfills conditions defined in univariate time series analysis.

It can be shown that in our case of VAR(1) model, we can transform 2 dimensional case of order 1 with lag operators L as follows

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \mathbf{L} \begin{pmatrix} y_t \\ z_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix},$$

getting 1 dimensional case of order 2. Consequently, the roots of the polynomial $(1 - a_{11}L)(1 - a_{22}L) - (a_{12}a_{21}L^2)$ must lie outside the unit circle. In similar way, we can derive conditions for higher orders of our model. Equations are much more complicated

[1] Method for estimating system of equations if we expect error terms being correlated, for further information see (Zellner, 1962)

but basic idea with long term mean and stationarity of separate time series holds in the very same way.

On the other hand, there is a clash whether the variables in a VAR should be stationary in real estimates. Recommendation from (Enders, 2004) based on Sims's, Stock's and Watson's work is not to differentiate even if variables did not pass unit roots tests. The argument is once again that we want to determine the general patterns in our data sequences, not to have most precise estimates. With differencing we risk "throwing away" information concerning the comovements in our set. Furthermore, it is argued that the data need not be detrended. In a VAR, a trending variable will be well approximated by a unit root plus drift. To conclude, general idea is to form VAR model in such a way, where it describes the true data generating process in best possible way (trends, non-stationarity, etc). It is particularly true if the aim is to estimate Structural VAR model. However, this is the topic of cointegration and much more complex vector error correction models. These are out of our previously defined scope. For other theoretical concepts of VAR analysis introduced later, we will assume our variables to be stationary.

1.3. The impulse response function

In univariate auto regression model we are familiar with Wold Representation Theorem with a basic idea that we can rewrite given stationary process as an infinite sum of errors (white noise) $\varepsilon_t$and some deterministic predictable partμ_t

$$x_t = \sum_{j=0}^{\infty} d_j \varepsilon_{t-j} + \mu_t$$

To have stationary process, it is quite reasonable to have these infinite sums convergent. There must be condition for $d_j$that$\sum_{j=0}^{\infty} d_j{}^2 < \infty$. Furthermore, values of d_j have much more to tell about our process. If you look closer, these coefficients carry some information how a shock in given time t can affect future values of the process. For example, if they are decreasing slowly, shock of time t can be affecting our system after many periods. On the other hand, if sequence of d_j is zero for index j and all following, then we can say some shock in time t has effect only in limited couple of period up to time j-1.

Comparatively, we can make representation with error terms for VAR model as well (VMA – Vector Moving Average representation). For VAR (1) case we have already seen the form of VMA in equation 6. In general, it is possible to compute VMA representation and write it down in following form

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \bar{y} \\ \bar{z} \end{pmatrix} + \sum_{i=0}^{\infty} \begin{pmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{pmatrix} \begin{pmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{pmatrix} \; in\; R^2 case$$

$$x_t = \mu + \sum_{i=0}^{\infty} \Phi(i)_{n\times n} \varepsilon_{t-i} \quad \Phi(i) = \{\phi_{rl}(i)\}_{r=1..n\,;\, l=1...n} \qquad 7$$

This representation is in multivariate case especially useful tool. We can examine the impact of ε_{yt} shock not only on y_t series but also impact onz_t. VMA representation is used to explore interaction between all n series in our system $x_t \in R^n$. In a given notation, elements $\phi_{rl}(0)$ are impact multipliers. For example, the coefficient $\phi_{12}(0)$ in two dimensional case is instant effect of one unit change/shock in variablez_t on variable y_t. Similarly, coefficient $\phi_{12}(1)$represents one-period response in same set of variables. As one can see, effect of a unit shock at time t can have cumulative effect after n periods. This accumulated effect can be obtained with appropriate summation of all coefficients up to this time. For example effect of unit shock in ε_{zt} on the value of y_{t+n} would be just $\phi_{12}(n)$ but if we want to know effect of this shock on whole sequence $\{y_t\}$, it is the sum

$$\sum_{i=0}^{n} \phi_{12}(i)$$

Letting n approach infinity yields the long run multiplier. Since the $\{y_t\}$ and $\{z_t\}$sequences are assumed to be stationary, it must be the case that for all indexes r and l,

$$\sum_{i=0}^{\infty} \left(\phi_{rl}(i)\right)^2 < \infty \; (is\; finite)$$

We can look at described $\Phi(i)$elements as functions of i. Particular coefficients of this matrix are called **impulse response functions (IRF).**

In practice we can see very often the plot of these functions, each element of $\Phi(i)$against i, as analysis tool for interdependencies. However, this methodology is not available to the researcher since we do not have estimated VAR system. Moreover, as we discussed in previous subsection, knowledge of the various A_i coefficients and variance/covariance matrix Σ is not sufficient to identify the primitive (Structural) VAR structure. We would need to introduce additional assumptions and restrict B matrixin equation 4 to know what the actual interdependencies are. For example having the B matrix upper triangular in 2 dimensional case makes second variable causing and first dependant. So we know that

$\phi_{21}(0)$ must be zero in this case. Not to mention that we do not have this information from correlation/covariance matrix.

For illustration, consider simulated simple system

$$y_t = 0.4y_{t-1} + 0.3z_{t-1} + \varepsilon_{yt}$$
$$z_t = 0.2y_{t-1} + 0.5z_{t-1} + \varepsilon_{zt} \quad \textbf{8}$$

You can observe effects of y and z shocks in Figure 1. As expected, in both equations shocks have direct positive effect decreasing with time lag.

In practice, we do not have exact parameters as in equation 8. When we work with real world datasets, we can just estimate coefficients with some uncertainty. As a consequence, impulse response functions have some uncertainty as well. In Figure 1 it is possible to draw some confidence intervals around plotted functions. The issue is to construct confidence intervals around that allow for the parameter uncertainty inherent in the estimation process. The problem is much more complicated in higher-order systems since the estimated parameters can be correlated. Furthermore, we cannot assume normality in general, especially if we have variables with drift. Well-established **method for**

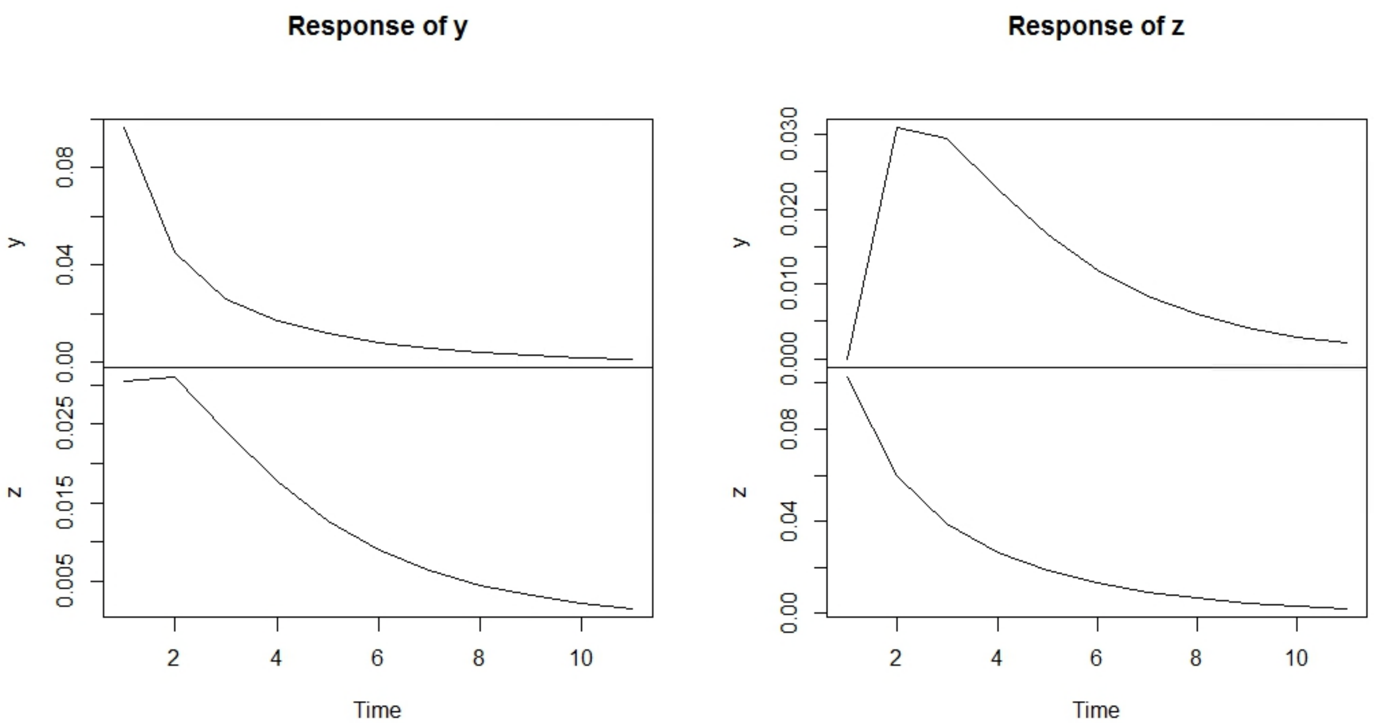

Figure 1 – Impulse response functions of system 8, artificially generated data

confidence interval estimates is **Monte Carlo bootstrap**. Main idea is to estimate VAR model $\widehat{M}$ from original data series $\{x_t\}$ and then look closer at error terms. When we have a set of $\varepsilon_t$for $t = 1, \dots, T$, we can draw T random number from them (with replacement, in bootstrap logic) and create new data series $\{\widetilde{x_t}\}$ (simulated sequence) using model $\widehat{M}$. Now act as if you do not know the parameter values in $\widehat{M}$, approach new $\{\widetilde{x_t}\}$ independently and estimate new model $\widetilde{M}$ with its new response function $\widetilde{\Phi}(i)$. After repeating whole process from drawing T random errors sufficient times, we can estimate confidence intervals from set of $\widetilde{\Phi}(i)$ we generated.

Every researcher's next step, after examining impulse response function, should be to focus on variance of our sequence in given time. IRF describes the absolute effect of unit shock on observer sequences, whether it is positive or negative, if it decreases slowly and shock stays in system or not. However, hand in hand with this study should be examining the relative effect of some shock in given time. Our intension should be to know what part of variance in e.g. $\{y_t\}$ is caused by its own shocks and what is the part of variance caused by residuals in $\{z_t\}$ sequence. In other words, it is exploring of forecast error. If we rewrite equation 7, we can express n-period forecast error $x_{t+n} - \mathrm{E_t}[\mathrm{x_{t+n}}]$ as

$$x_{t+n} - \mathrm{E_t}[\mathrm{x_{t+n}}] = \sum_{i=0}^{n-1} \Phi(i)\varepsilon_{t+n-i}$$

Where both sides are random variables. If we focus purely on $\{y_t\}$ sequence and look at the variance, equation transform into:

$$\sigma_y(n)^2 = {\sigma_y}^2(\phi_{11}(0)^2 + \phi_{11}(1)^2 + \dots + \phi_{11}(n-1)^2) + \\ + {\sigma_z}^2(\phi_{12}(0)^2 + \phi_{12}(1)^2 + \dots + \phi_{12}(n-1)^2) \qquad \mathbf{9}$$

And σ^2 on the RHS are variances of residuals ε_t. Because all values of $\phi_{11}(i)^2$ are non-negative, the variance of the forecast error increases as the forecast horizon n increases. In order to get relative proportion, we can divide equation 9 by total variance $\sigma_y(n)^2$. Note that it is possible to decompose LHS into the proportion due to $\{\varepsilon_{yt}\}$sequence (LHS, first expression) and due to $\{\varepsilon_{zt}\}$ (LHS, second expression). Finally, it is again possible to plot the result against time lag (forecast horizon) i. This process is called **forecast error variance decomposition (FEVD)**. You can see an example of variance decomposition for system 8 in Figure 2.

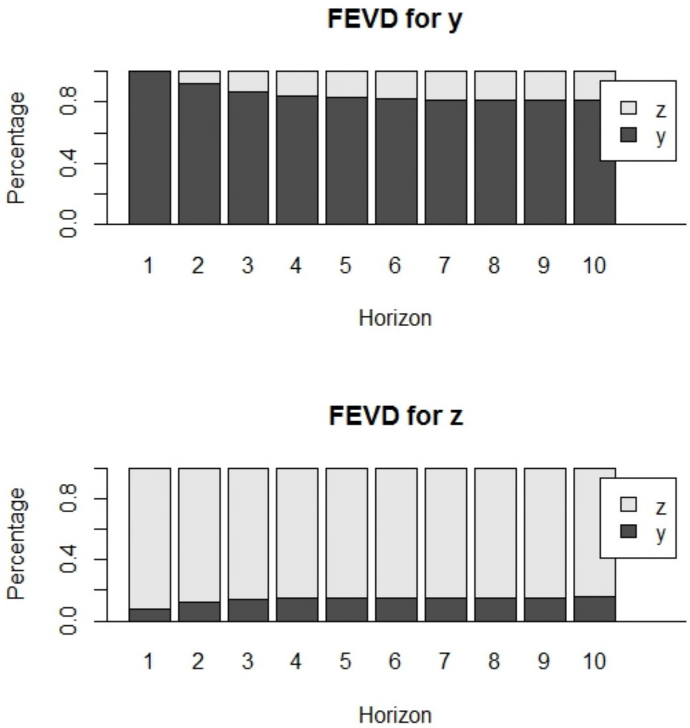

Figure 2 – Forecast error variance decomposition of system 8, artificially generated data

Consequently, IRF and FEVD, together called **innovation accounting,** can be useful tools to analyse the relationships among observed real data, especially economic variables. If the correlations among the various innovations are small, the identification problem is not likely to be especially important. The alternative orderings should yield similar IRF and FEVD.

1.4. Testing hypotheses, Granger Causality

Multidimensionality and intention to have sufficient historical effect can make VAR model very large. Degrees of freedom can grow really fast with adding new variables or higher time lag. For example, we want to simulate set of 6 (n) time series monthly. If we want to include at least 1 year effect, we would need to have time lag of 12 (p). There are $p * n^2 + n$ coefficients to estimate, in this case it is $12 * 6^2 + 6 = 438$ parameter estimates in standard form model. It is significantly higher than univariate case (e.g. linear regression with 2 or 3 variables has up to 5 coefficients, ARMA(12), univariate time series model has up to 30). With higher number of parameter estimate, total uncertainty grows as well. As we discussed before, VAR analysis based on Sims methodology is rather qualitative than quantitative. We will operate with and take into account bigger level of uncertainty in hand with more complex model. However, we would like to have some methodology to check variables and their impact in order to make conclusions and talk about some outcome at the end of the analysis.

We want to proceed with test for some parameters restrictions. Quite usual logic is to make likelihood ratio test for this cross equation restriction. In regular approach, we estimate original model together with restricted VAR model over the same sample period to obtain the variance/covariance matrix of residuals for both. Regular likelihood statistic looks like:

$$T * (ln|\Sigma_r| - ln|\Sigma_u|)$$

However, (Enders, 2004) based on Sims Methodology recommends to use

$$(T - c) * (ln|\Sigma_r| - ln|\Sigma_u|)$$

Where:

T – Number of usable observations

c – Number of parameter estimated in each equation of the unrestricted system

$|\Sigma_u|/|\Sigma_r|$ – Determinants of unrestricted/restricted covariance matrices respectively

Given test statistic has asymptotic χ^2distribution with degrees of freedom eqal to the number of restrictions in the system.

With possibility of restricting model, we can test several different things. Appropriate lag lengths (e.g. restriction from VAR(12) to VAR(6)), appropriate seasonality (e.g. have only lags 12,24,36 for monthly data) or many others. But one of most important tests is causality analysis. In terms of likelihood ratio test, it means to check for significance of parameters representing interdependency. Take an example of 2 dimensional VAR(p). We can investigate whether coefficients $a_{21}(1) = a_{21}(2) = \cdots = a_{21}(p) = 0.$ Moreover, introduced hypothesis is typical for so-called **Granger Causality test**. Once again, in this meaning we can say that the sequence $\{x^1{}_t\}$ Granger cause sequence $\{x^2{}_t\}$ if and only if all of the coefficients of $A_{21}(L)$ are equal to zero (H0 hypothesis). So if first sequence does not improve the forecasting performance of the second, than first one does not Granger cause the second one. However, Granger causality refers only to the effects of **past** values of one sequence on the **current** value of other. To illustrate the distinction in terms of a VMA model, consider the following equation for $\{x^2{}_t\}$ (upper index):

$$x^2{}_t = \overline{x^2} + \phi_{21}(0)\varepsilon^1{}_t + \sum_{i=0}^{\infty} \phi_{22}(i)\varepsilon^2{}_{t-i}$$

Forecast error for the second sequence t+1 value, given that we know information up to time t, is $\phi_{21}(0)\varepsilon^1{}_{t+1} + \phi_{22}(0)\varepsilon^2{}_{t+1}$. But if we consider introduced restriction for $A_{21}(L) = 0$, than passing the test (H0 is not rejected) only means that historical values does not affect current values of second sequence, conditional expectations are $E[x^2{}_{t+1}|x^2{}_t] = E[x^2{}_{t+1}|x^2{}_t; x^1{}_t]$. When you look closer, this holds for our VMA example process even though there is effect from first sequence. Therefore, one has to be careful before taking any big assumption after running this test.

Clearly, we can use proposed logic of 2 dimensional case in set of n series. Restriction will be $A_{ij}(L) = 0$ in general, if we are investigating the effect from j sequence on i sequence.

2. The model development

2.1. Quantity theory of money

Our goal will be to review impact of quantitative easing (QE) on inflation. We should introduce classical monetarist theory, Quantity Theory of Money. This is the most fundamental theory explaining the impact of the money supply change (we can consider QE as positive shock on supply side) on price levels (in other words - Inflation). The very basic equation, the Equation of Exchange, to describe relationship between money supply and the price level is:

$$M \times V = P \times Y \qquad \textbf{10}$$

Where:

- M represents monetary aggregate, i.e. money supply steered by central bank
- V describes velocity of money, average frequency across all transactions
- P is price level in the economy
- Y is total real output, gross domestic product

Equation is rather abstract, but in the economy, where all product are bought for money it makes perfect sense. Let's take an example. Consider closed economy with one central and several commercial banks. There is given amount of domestic product in year 1 and we have our own currency for trading produced goods. If we produce twice as much products in year 2 and price stays the same, either number of transaction paying for these products will double or total amount of currency/money doubles in order to pay for this increase of productivity.

Consequently, main use of this theory should be steering of money supply with the aim of controlling price level, assuming that velocity of money is constant in short term.

However, this theory was challenged much in 1980s and 1990s. Biggest argument against quantity theory is that **money velocity is not stabl**e. Not event in short term. In addition, also **prices are sticky**, e.g. shocks in GDP or money are not reflected directly in price level. As a consequence, it all results in unstable relationship between money supply and price level. For example in the paper (Benjamin Friedman, 1992) was written: "Including data from the 1980's sharply weakens the post-war time-series evidence indicating significant **relationships between money** (however defined) and **nominal income** or **between money and either real income or prices** separately. Focusing on data from 1970 onward destroys this evidence altogether." Similar observation is stated in the work (De Long, 2000) as follows: "Controlling the particular money supply that was relevant for total spending turned out to be very difficult indeed." Also finding as "... for the velocity of money turned unstable in the 1980s..." show us, that driving monetary policy focusing on monetary aggregates is no longer popular. Connection between any of monetary aggregates and price level assuming constant velocity is unstable. With the main objective of central bank to keep inflation – price level growth below 2%, we are losing the reason to control aggregates so closely.

Beliefs in weak money – price transmission mechanism allowed central banks to introduce several economy-stimulating programs. In practise, we can see lending facilities and support of commercial banks by cheap credits, resulting in higher money growth. In addition, directly regulated short term interest rates are steered towards zero levels. We should observe plenty of liquidity in commercial banks and significant growth of money balances. All caused by high money supply.

2.2. Major assumptions of the model

As we have seen in previous section, there is quite strong evidence that quantity equation does not hold. However, if we consider some other assumptions, mostly on the side of velocity of money, we can get much more reasonable results. Thus, our model will be based on Quantity Theory of Money. Similar approach can be seen in work of Samuel Reynard, who dedicated several studies[2] to adjusting Quantity Theory for practical use deriving very interesting results. He argues in (Reynard, 2012) that equation 10 stays in place if we consider some other factors. Reported weak relationship between money growth and inflation in low inflation economies (e.g. European countries or the US) in the form of non-significant influence of two mentioned variables comes from not accounting for movements in equilibrium velocity due to Fisherian movements of interest rates[3]**.** Most of arguments against Quantity Theory of Money are based on unstable and unpredictable velocity of money as was mentioned earlier. But Samuel Reynard in his paper (Reynard, 2012) argues that if **movements in equilibrium velocity** due to money demand adjustments are accounted for, we can develop model based on equation 10. **That**

[2] Studies are (Reynard, 2012), (Reynard, 2007) and (Reynard, 2006), there will be reference to last one only. All publications are within one research. They basically develop the same model and precede each other.

[3] Easiest explanation of Fisherian movements is through example. Let's have economy with 2% inflation and 3% interest in one year. In case inflation rises by 1%, interest rates should rise as well by 1% in order to preserve same time value of money. Given change in interest rates due to inflation change is then called **Fisherian movement of interest rate**.

basically means we consider interest rate – opportunity cost of money as main driver for velocity (argument why opportunity cost of money should represent equilibrium velocity will be explained in section 2.2.2). Total methodology is based on long-run equilibrium analysis involving money, prices, real GDP and previously stated interest rates.

2.2.1. Monetary aggregate selection

Firstly, most important question is: "What is money?" Where to draw a line in very complicated financial markets to separate money from others assets. From definition, each individual or company can decide between holding money – more liquid asset which can be turned into good very quickly and, on the other hand, holding some investments, bonds, higher interest earning accounts. Drawing a line between these two categories is not easy at all. Even most liquid asset, cash equivalent current account can earn interest while even most illiquid financial asset as bond with long maturity can be turned into cash quite easily on money market. Let's take an example. If somebody asks you, "How much money do you have", what should be your answer? Should you come up with your wallet and count coins and paper notes? Or should you tell them balance at your current account if you have a credit card with you? And what if don't? Or should you tell them all information above with actual price of your car, house and all properties because you are able to sell everything within 3 months? Answer is neither straightforward nor clear…

Given question can be basically transformed into monetary aggregate selection. In two given categories, money vs. investments, we should be able to measure some opportunity cost to have an idea what we are losing if we are not holding investment positions, bonds, etc. We can do that only if we decide what we still consider as money.

Therefore, there can be several different "lines drawn" to separate money from investment

Liabilities (1)	M1	M2	M3
Currency in circulation	X	X	X
Overnight deposits	X	X	X
Deposits with an agreed maturity up to 2 years		X	X
Deposits redeemable at a period of notice up to 3 months		X	X
Repurchase agreements			X
Money market fund (MMF) shares/units			X
Debt securities up to 2 years			X

(1) Liabilities of the money-issuing sector and central government liabilities with a monetary character held by the money-holding sector.

Figure 3– Definitions of euro area monetary aggregates, source (ECB, European Central Bank)

assets. For better imagination, we will use definition of monetary aggregates from European Central Bank. You can see what each aggregate includes in Figure 3.

In very strict understanding, we would only consider M1 - currency in circulation with the shortest overnight deposits. Equivalent M1 aggregate was even used by Federal Reserve Board in the USA in 1980s as one of main driver of monetary policy. There was a consensus that money are useful indicator for monetary policy with assumption of constant velocity. However, it did not last long until the first breakdown of this consensus occurred. The velocity of M1 started to exhibit fluctuations, as can be seen from Figure 4, after having grown smoothly for the prior three decades. From observed fact, it was argued that demand for money in M1 become unstable. Most of the theories explained the velocity fluctuations by financial deregulation, introduction of new accounts providing transaction services and the disinflation.

Moreover (Reynard, 2012) describes main reason for fluctuation similarly. Properties of newly introduced accounts were not sufficiently recognized and difference between transaction and savings accounts become difficult to find. Checking (transaction) accounts began to earn interest in period of 1980s. Several arguments point out that the problem can be in aggregate M1 during given period. Furthermore, it is often suggested, that an explanation for the increasing trend in M1 velocity from post-war period comes from innovations in field of banking, introducing of credit and debit cards, ATMs etc. Individuals are allowed to economize on money balances and justify income elasticity below unity.

In conclusion, monetary aggregates, especially M1, became less trustworthy. Change in trend of M1 velocity was main driver for the argument that monetary aggregates should not be considered as indicators for monetary policy. However, let’s consider the model where people trade off real resources with monetary assets. Funds necessary for consumption and some reserve are held in transaction account and the rest in saving account. Balances should be proportional to interest rates. Consequently, the question should rather be the smooth behaviour of the velocity during 1970s. The velocity did not drop with falls of interest rates in 1970 and 1974 and also increased faster than interest rates over 1950s, 60s, and 70s. There is also significant instability in 1990s caused by further financial innovations and sweep programs (see (Reynard, 2012)).

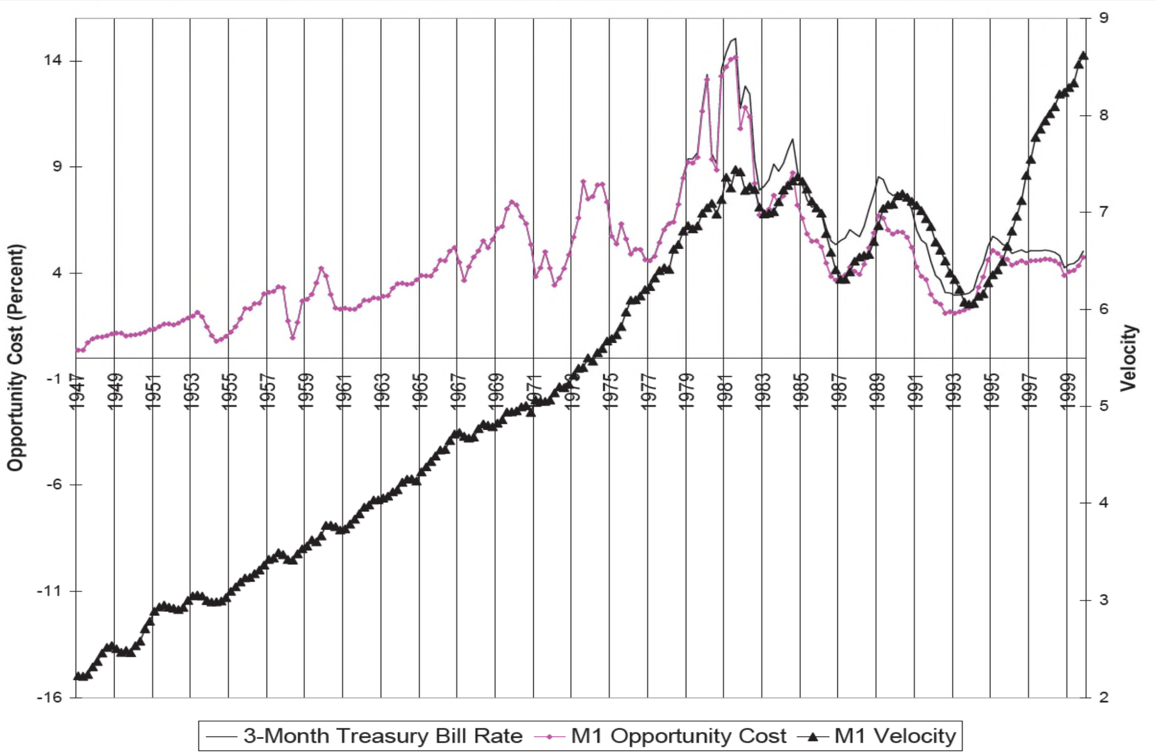

Figure 4- M1 the US velocity of money and opportunity costs, (Reynard, 2006)

Introduction of financial innovations and increase in liquidity of markets caused new behaviours of monetary aggregates. E.g. if you look at Figure 3 you can see what M3 contains. There are money market funds, shares and other securities. It is the broadest understanding of money and it definitely earns "interest". M3 is rather investment than money for transaction. But you can turn them into cash in matter of seconds with minimal loss. It shows that definition for "What is money?" is not so easy any more.

Consequently, in our study it is more reasonable to analyse money in broader understanding. M2 and M3 have much better properties for research and are not so dependent on technical innovations. Velocity is influenced by opportunity costs much more than by other factors. Their behaviour can be much better predicted and understood. But we still have to consider the other side. If we consider assets like time deposits with longer maturities, we should include securities with similar properties such as government or even corporate bonds as well. Not including them is likely to generate money demand instability (they have similar characteristics and e.g. investor's shift from bonds to time deposits can cause major discrepancy of studies). But on the other hand, logical link of e.g. corporate bonds to transaction concept is not clear at all.

Last but not least fact to mention is that assets included in those aggregates yield rates equal to or above the 3-month market rate, those aggregates are sometimes positively correlated with the 3-month or even higher rate, thus making their policy stance interpretation difficult. As we want to consider saving/investment assets as an opposite of money, thus we cannot consider "very broad" aggregate and we need at least 3M or higher maturity interest rate to be considered as opportunity cost.

In conclusion, distinguish between money and investments is not easy at all. Simplified, we can say that very narrow understanding of money is not reasonable because it is not representing transaction and subsequently consumption motive. On the other hand very broad definition contains accounts which are typical for investing and saving rather than transaction, even though sometimes they are used for consumption purposes as well. Taking the "middle way" seems to be the most reasonable. Moreover, we perform further analysis and choose aggregate with decent support also from data and observations. Based on work (Reynard, 2012) and also in our study best attributes for this study has aggregate M2. Reynard used slightly adjusted M2 in the USA (included small time deposits) and called it M^{US} (or MUS). However in EU area, Switzerland or Japan, M2 was used as it is defined. You can see support from data in Figure 5. In our study we take over M2 aggregate as main driver. On top of that, in several studies focused on Euro area, broader understanding of money (M3) is being used. As a consequence, for comparison with other studies and for more stable results it is reasonable to consider several different definitions of money and perform our analysis in different parallel lines (with M2, M3 and even check M1).

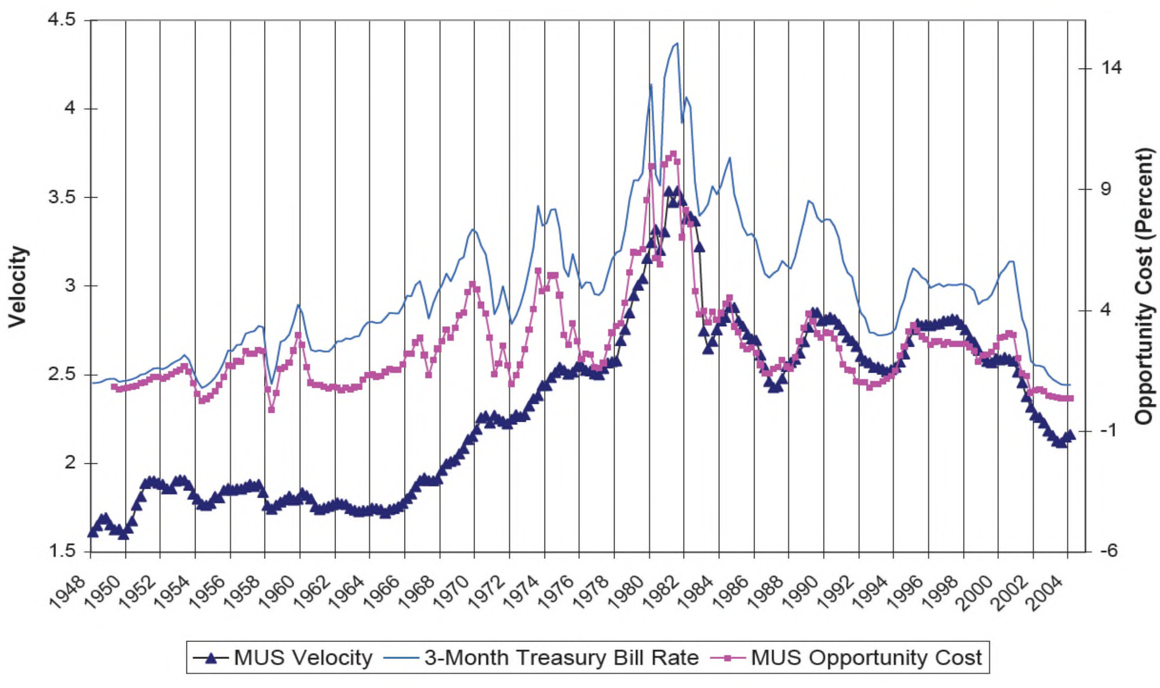

Figure 5 - M^{US} velocity of money and opportunity costs, (Reynard, 2006)

2.2.2. Opportunity cost of money

In following subsection, we are going to explain the main and most important assumption of model introduced in (Reynard, 2012). Opportunity cost of money should be understood as something “we need to pay” if we are holding money or other low interest earning balances instead of long maturity deposits, bond etc. You can easily have a feeling that it should be some interest rate what you are NOT earning while your funds are on your current account. It is exact definition of opportunity cost in economic meaning. Money, same as any other goods, has this characteristic.

Let’s take an example. Imagine the situation where each individual has to make following decision. He determines how much money he wants to hold and put rest of earning to saving account. If opportunity cost is high, he will invest everything what is not necessary to hold for consumption. It is very costly to hold any money “just in case something happens”. Then total amount of money in circulation should be based on Quantity theory of money influenced by total output of economy as goods which are bought and by prices which are paid for those goods. Now consider the situation when interest rates and total opportunity cost starts to fall in given economy. Holding money balances instead of investments is not so costly any more. People have less intension to invest and total demand for money rise, especially due to precautionary and speculative motives, not only for transactions. Equation 10 cannot describe given state with assumption of constant velocity. Neither output, nor prices are rising but total money demand is growing, equilibrium is higher, monetary aggregate is bigger.

In contrast, if we consider there is a linear link between velocity and interest rates, Quantity theory of money holds. It is further argued in (Reynard, 2012) that opportunity cost can be considered as velocity of money in this sense. S. Reynard explains: “...monetary aggregates must be adjusted by equilibrium velocity changes, which can be approximated with a backward-looking filter, to account for the fact that people decrease their real money balances when inflation and interest rates increase, and vice versa, inducing money movements without corresponding effects on subsequent inflation...”

In this point, you can still ask why interest rate on long term deposit is considered as velocity. We would like to emphasize the fact from given model situation, that if rates are low, an individual is willing to have much higher short term deposits because it is less risky and rates for short and long term are close. He will have much higher current account balances or he can even have more money in his wallet or “under the pillow” because it is not such a loss. As a logical consequence, all individuals have similar behaviour and at the end of the day, total amount of money (defined as current accounts or currency) is rising. Still not all the money are used for transactions. Individual are not paying with them for goods. Let's analyse equation 10 again. We have the environment where monetary aggregate is growing much faster than prices or real output. Money growth must be compensated by decrease in velocity. Therefore, theoretical connection between money and opportunity cost is reasonable and should be clear by now. Although opportunity cost explains our situation, we still need to find solid support in real data and decide which interest rates take into account as our main driver for velocity.

As a consequence of previous subsection, we should be able see difference between money for transaction and money as an investment. Despite the decision for "separating line" between transaction money and investments, definition of opportunity cost is still tricky. This can be considered as long term deposits with maturities up to 10 years, but on the other hand, even 3M rate can be seen as an investment.

Furthermore, we cannot just take long-short term interest rate spread. There is one other factor we need to account for. As we have already stated, we need to consider Fisherian moves of interest rates, somehow implicitly consider effect of inflation. S. Reynard explains it as follows: "*An important feature of that spread is that it does not exhibit a downward trend over the past 25 years. A major conceptual issue in using that spread as the opportunity cost is that the 3-month rate, supposed to reflect the own rate, is in fact the alternative rate of large parts of* M^{2EA} *and* M^{3EA}*.*"[4]

It explains why we want to see nominal interest rates, not adjusted by inflation in any sense in our model. Nominal interest rates express people's expectations about future inflation together with current real rate. In fact, variable with this information needs to be considered as a main driver for velocity. Considering opportunity cost as spread between long and short term rates would cancel effect of expected Inflation. We would be neglecting disinflation of recent decades.

In conclusion, arguments above implicitly account for two most important facts:

- If there is "one-to-one" relationship between opportunity costs and velocity, we can represent equilibrium velocity change by observed opportunity cost, having all needed information for equation 10

[4] M^{2EA} and M^{3EA} represent given monetary aggregates in Euro area, in line with definition from Figure 3– Definitions of euro area monetary aggregates, source

- Asymmetric price behaviour can be explained. If money levels are rising faster than price or output levels, money-price relationship still stays in place because of decreasing opportunity cost.

In addition, described idea is supported by data as you can see in Reynard's study in Figure 6 (example chosen in Euro area) and in our study in Figure 7 (example on data from Switzerland).

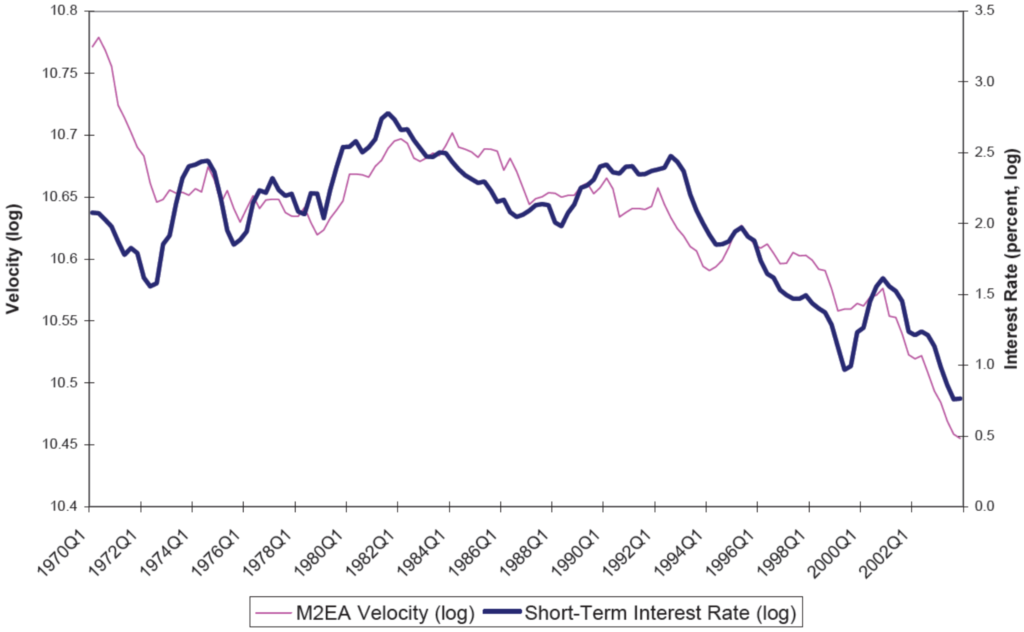

Figure 6 – Velocity vs. Short-term interest rate relationship, (Reynard, 2006)

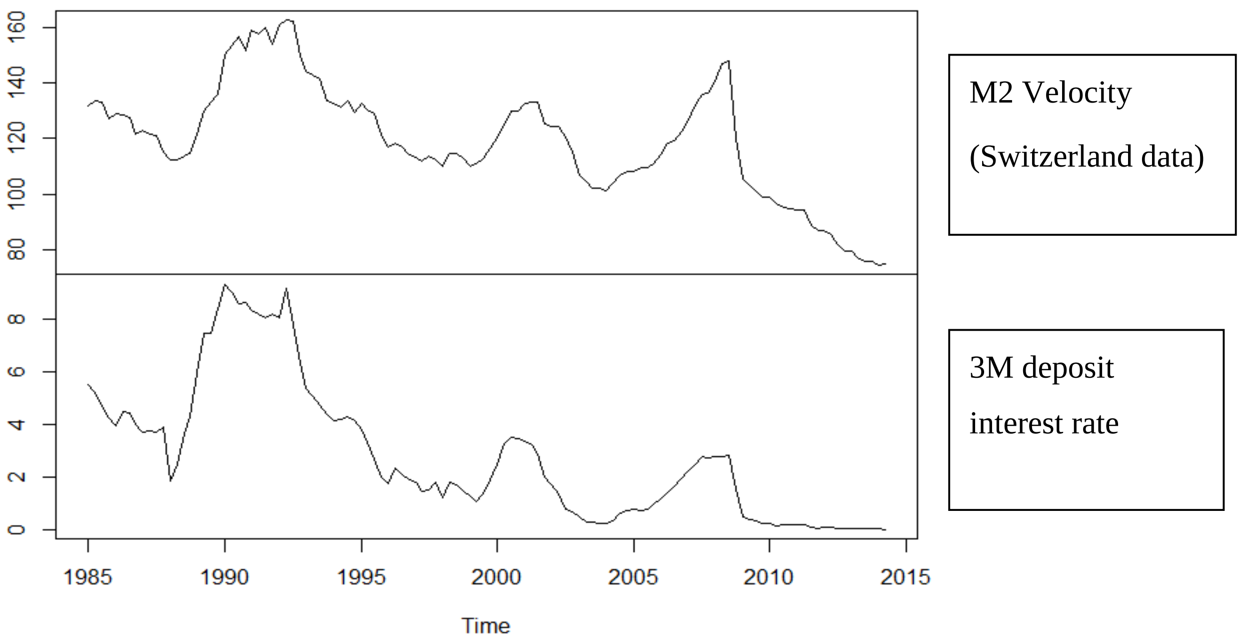

Figure 7 – Velocity vs. Short-term interest rate (3M deposit rate), own analysis, data (International Monetary Fund)

Nevertheless, one could argue that observed correlation between interest rates and velocity of money does not imply causality. That is a fair fact but arguments introduced above and further developed in Reynard's studies seems to be giving to this observed correlation quite reasonable explanation of causality. However, our objective will be investigate this relationship and prove it with data and observations.

2.3. Data selection

Based on theory introduced above, we will develop and apply our model in several environments. In advanced economies, we can observe time series of several most important macroeconomic variables since 70s or at least 80s onwards. But firstly, our task is to define geographical scope. Main drivers for country selection were:

- **Economies had to be comparable to each other**. We wanted to get at least similar results and bring up some general conclusions. It is important to have comparable methodology also from "data quality" point of view.
- They should be **relevant for our study of Quantitative Easing impact in Europe and the USA**. We want to find and subsequently apply general patterns. Find behaviour of price level, its reaction on money growth or changes in interest rates and these patterns can be different in more exotic countries.
- A third criterion was to have **major economies included.** Our scope is mostly European Union. We are taking all key countries from EU together with some of minor ones from Central Europe. We wanted to perform study on separate countries but on the other hand get result for our region – Slovakia.

Consequently, we took developed countries of Europe together with the USA. The first point will be to start with the United States. Historical reasons, good data quality and character of economy made the United States best starting point to test given assumptions of our model. Furthermore, it is our intension to get comparable results with Reynard study. Secondly, our scope will be EU economies such as the United Kingdom, Germany, France, Switzerland, Italy and several others from the Central Europe.

Also due to data completeness and data quality, we were struggling sometimes and we had to make few adjustments in the data selection. Country selection and time window were needed to be adjusted in order to perform reasonable analysis.

As previously indicated, we will need to observe four time series in each country on quarterly basis.

i. **Gross Domestic Product (GDP, Y) –** Number representing total output of economy. Represented as real output in fixed prices, estimated in expenditure approach. We used basis year 2010 and worked with indexes. Source was (International Monetary Fund eLibrary)

ii. **Price level (P) –** Price level in given country. It is possible to use several price indexes. Either we can take consumer prices, indexes CPI or HICP (both have similar properties). This was our preferred approach. Or it is possible to take GDP deflator[5], indexed as well. We will point out each time if we use this alternative. However, our results are often very close in both measures. Source was (International Monetary Fund eLibrary)

iii. **Money level, monetary aggregate (M#, M) –** Selection of monetary aggregate was described above. From technical point of view, we will use quarterly observation, total amount at the end of period in USD, for the US, EUR for EMU countries and national currency for others. Source was (International Monetary Fund eLibrary) for the US and Switzerland and (NBS database) for EU and other countries.

iv. **Interest rates (r, R) –** Variable relating to the **velocity of money**. In different environments it is hard to say what kind of interest rate represent opportunity cost and velocity most precisely. We used the safest rates from credit risk point of view, officially announced LIBOR rates or government bonds yields. Concerning the maturities, we used short term rates (3-month) as well as long term up to 10 years. Source (International Monetary Fund eLibrary) or (OECD)

Finally, time series start as early as 1970 onwards, if possible. It covers several interesting periods, mostly great disinflation (decline of inflation from higher values ~5-10% to broadly considered optimal level around 2%) but also great fluctuations in velocity and some breakdowns and fluctuation in other variables separately. Even though, it is desirable to have longest possible history, in some countries we had data only from 1980 or even later. Time windows selection is then case dependant, but in most cases it was as large as possible.

[5]**GDP deflator** is number representing difference between real and nominal GDP. It is computed as $GDP_{def} = \frac{GDP_{nominal}}{GDP_{real}} * 100$. So it has value of 100 at GDP index year.

2.4. Final model

2.4.1. Velocity estimate

Under proposed assumptions, we can develop the model now. We have our starting point from Quantity Theory of Money. Equilibrium equation 10 once again (labelled with the same number as before):

$$M_{(t)} \times V_{(t)} = Y_{(t)} \times P_{(t)} \qquad \textbf{10}$$

It should hold in each time period t, each year or each quarter separately. In this form we have variables in absolute form and they are multiplied. It makes perfect sense, but it would be very difficult to make estimates of given equation. Let's consider some data transformation. In our case, we decide for equation in additive form with logarithmic transformation. Equation has the form

$$\ln(M_t) + \ln(V_t) = \ln(Y_t) + \ln(P_t)$$

Our final goal will be to get relation between money growth and prices. So we would like to firstly see impact of total real output and velocity (represented by interest rates). So we can rearrange variables in following way.

$$\ln\left(\frac{P_t}{M_t}\right) = \ln(V_t) - \ln(Y_t) \qquad \textbf{11}$$

$$(p/m)_t \sim c + \beta r_t + \alpha \mathrm{y_t} + \epsilon_t \qquad \textbf{12}$$

For regression purposes, we need to estimate so-called interest rate (velocity) elasticity **β** as well as income elasticity **α**. In addition, very important remark here is that **β** should be negative and **α** positive in order to be consistent with theory. It is also reasonable to add constant, in the meaning of time-independent effect. All estimated coefficients should be constant in time. We marked logarithm of our variables with lower case letters in regression model, representing $x_t = \ln(X_t)$. Only exception here is velocity. Estimate is done in a following form:

$$\ln(V_t) \sim c + \beta r_t$$

Model is more reliable if interest rates are not transformed. Also evidence for interrelationship was much higher in presented form.

Let's have a brief discussion about estimation procedure in practise. The results from study were in line with theory as well. When we approximated velocity with interest rates, we got $\alpha < 0$ and $\beta > 0$. Moreover, when we look closer at the original equation 10 and our linear model in the equation 12, we can see that the income elasticity α should equal to 1 exactly. Consequently we restricted $\alpha = 1$ wherever it was possible in order to have the approach more consistent with theory.

In addition, if we treat variables as time series observations, we can experience some difficulties. Based on time series theory, our variables should fulfil several properties. Autocorrelation of residuals have to be dealt with in some way because linear model expect independent observations. Our assumption is following. Opportunity costs adjusted with elasticity β and constant c are considered to be true velocity. All deviations of computed velocity $Y * P/M$ from true equilibrium are considered to be price shocks. We will analyse its autocorrelation later in auto regressive way.

Consequently, we can make new monetary variable. We start once again from basic equation in log form.

$$\ln(M_t) + \ln(V_t) = \ln(Y_t) + \ln(P_t)$$
$$\ln(M_t) + \ln(V_t) - \ln(Y_t) = \ln(P_t) \qquad \mathbf{13}$$

We have money, opportunity costs and output on the left hand side of equation 13. We can take RHS variables as one time series. They represent money growth cleaned for the effect of output growth and the equilibrium velocity. This “Net Money” (NM), in a terminology of (Reynard, 2012), can be computed from original time series as follows.

$$m_t^* = m_t - |\alpha|\mathrm{y_t} + (c + |\beta| r_t) \qquad \mathbf{14}$$

Where all considered variables, except interest rates, are in logarithms. Coefficients α and β are estimates from previously discussed equation. Restriction $\alpha = 1$ is applied if possible.

2.4.2. Data preparation

In order to get more stable results and represent equilibrium changes of velocity and potential product, we filtered interest rates and GDP with Hodrick-Prescott (HP) filter[6].Our goal was to investigate Money – Prices relationship. To extract necessary information, we have to omit short term shock in V and Y sequences. Conventional parameter for quarterly data, λ=1600, was used for GDP. Exception was interest rates, which was low-frequency filtered with λ around 200 (resulting in higher effect of short term movement than long term trend). Moreover, during the prior observation of NM level vs. Prices, we filtered all of variables. Our goal was to detect rather longer term changes.

[6]See R, stats function *hpfilter* in package mFilter, (Balcilar, 2007)

In practice we did the following. Firstly we used original M and P with filtered Y and R for linear estimate, retrieving elasticities and construction of NM from equation 14. Consequently we filtered M and P as well for observation purposes. However in next steps we came back to original Money and Price levels.

Let's illustrate technical framework of HP filter. Figure 8 represents HP filter applied for the US short term interest rates. In upper part of the figure you can see original data with shock and fitted trend estimated with HP filter. In lower part, you can find deviations from trend which should be called short term shocks rather than cyclical component.

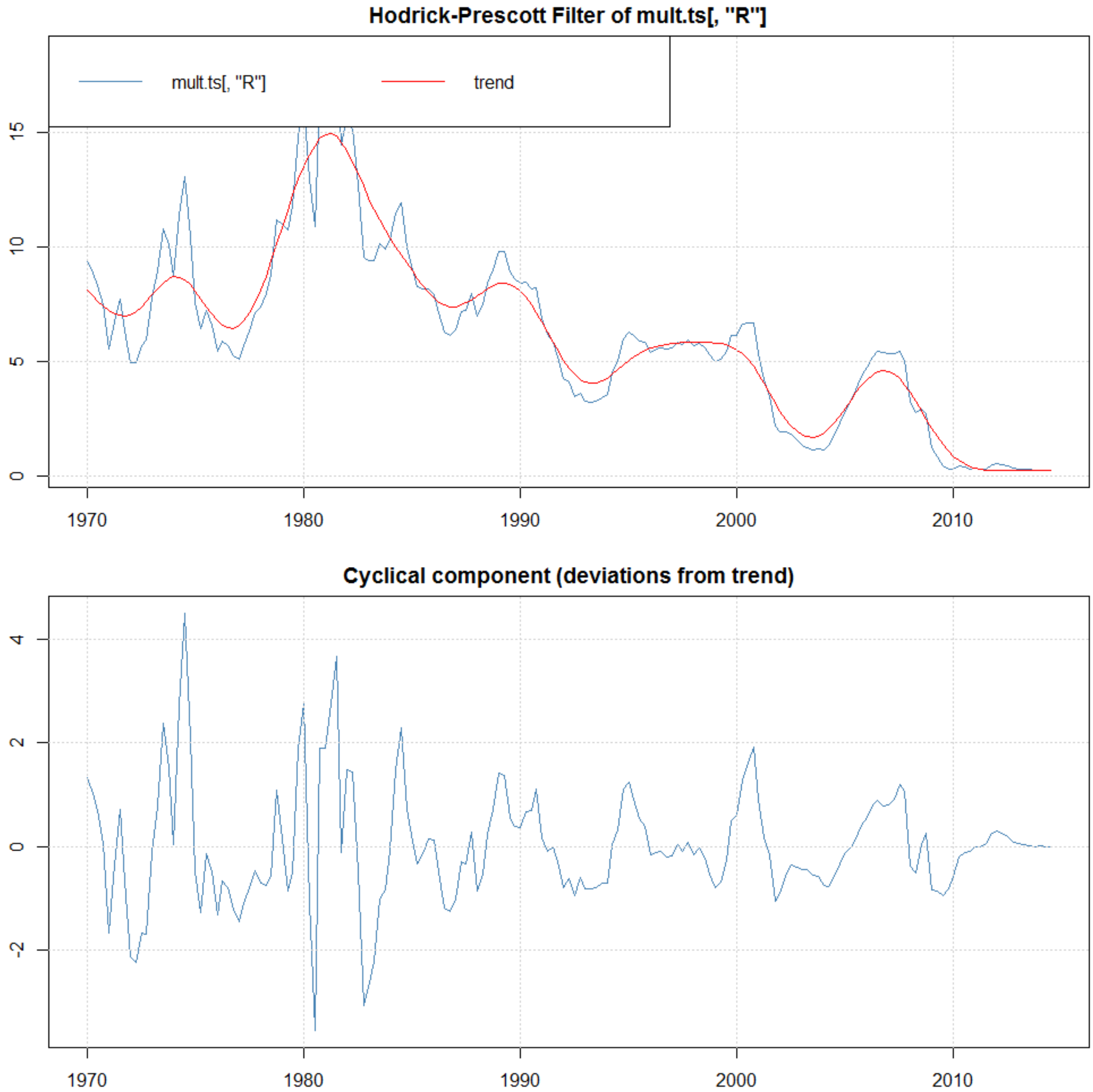

Figure 8– HP filtered short term (3M) interest rates, the US

The logic behind this adjustment was to remove cyclical components and short term shocks. Our objective is to find long term relationships and dependencies of variables. However, one has to be careful especially in analysis of beginning or end of observed time series. Given filter uses values of t-i as well as t+i to construct observation in time t. Intension is to obtain a smoothed representation of a time series, one that is more responsive to long-term than to short-term fluctuations. It is great tool to find interdependencies and trend of variables in longer term horizon. However, trend at the end (or beginning) of our series can be unstable, because we do not know the values of t+i (t-i respectively). As a consequence, forecasting ability of filtered arrays is much weakened. For predicting purposes we used original data.

2.4.3. VAR analysis

To summarise inputs, we have prepared variable called Net Money Growth from equation 14 as one driver for later analysis (basically differentiated m_t^*). It is assumed to be more or less exogenous in our study because it should be in hands of monetary policy. Secondly, we have taken differentiated logarithm of price level, representing Inflation.

First, rather straightforward, approach for Structural Vector Auto Regression model have form of

$$\begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \Delta m_t^* \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix} + \begin{pmatrix} {\gamma_{11}}^1 & {\gamma_{12}}^1 \\ {\gamma_{21}}^1 & {\gamma_{22}}^1 \end{pmatrix} \begin{pmatrix} \Delta m_{t-1}^* \\ \Delta p_{t-1} \end{pmatrix} + \ \dots \begin{pmatrix} {\gamma_{11}}^p & {\gamma_{12}}^p \\ {\gamma_{21}}^p & {\gamma_{22}}^p \end{pmatrix} \begin{pmatrix} \Delta m_{t-p}^* \\ \Delta p_{t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t \end{pmatrix},$$

in a 2 dimensional case, drift considered. This is the most general form of model we can work with. As you can see we ordered variables in a way, where NM Growth is only influenced by historical values of both sequences and Inflation is instantaneously affected by NM Growth and then as well by both sequences of higher lag.

Secondly, we can observe absolute values (log of their index) for Net Money and Price level. For illustration, look at the Figure 9. Observed shocks do not have to be just errors. We can define the **difference between Net Money and Price** as new time series, which is in (Reynard, 2012) called **excess liquidity**. Consequently it is possible to observe, whether excess liquidity is positive, we have more money and there is a pressure on price level. Or if excess liquidity is negative, price level is high and there is rather a pressure on money growth, or decrease in inflation. Both effects with some time lag. VAR model contains in that case Excess Liquidity instead of NM Growth, and inflation as before. Given idea was implicitly included in previous approach as well. However, in the context of Excess Liquidity, it is clearer.

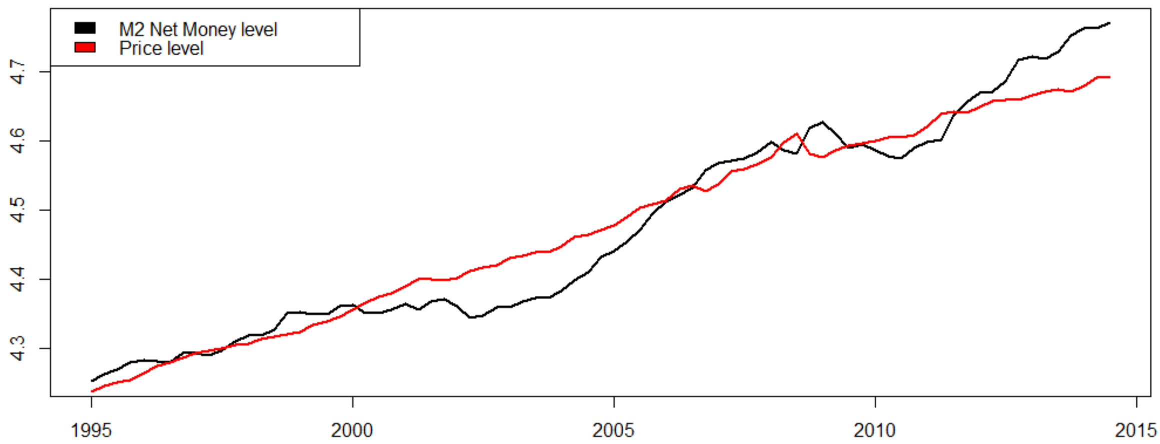

Figure 9 – M2 Net Money level vs. Price level, the US data

At the end, final analysis is performed with all assumptions above, looking for relationship from money level towards prices/inflation. Our study and final conclusions are always based on some version of introduced VAR(p) models. Software R (stat) was used in version 2.15.1. Package for VAR modelling was **vars**, (Pfaff & Stigler, 2013).

2.5. Quantitative Easing

The situation of zero interest rates and money supply growth is called **Quantitative Easing (QE)**. It is a policy, when bank purchases government securities or other securities from the market in order to lower interest rates and increase the money supply. Typically, central banks target the supply of money by operating in government bonds market. It directly lowers short term, but in fact also medium and long term interest rates (yields). This stimulates financial sector because there is central bank in position of additional buyer who has indefinite funds to buy anything, total demand for all securities

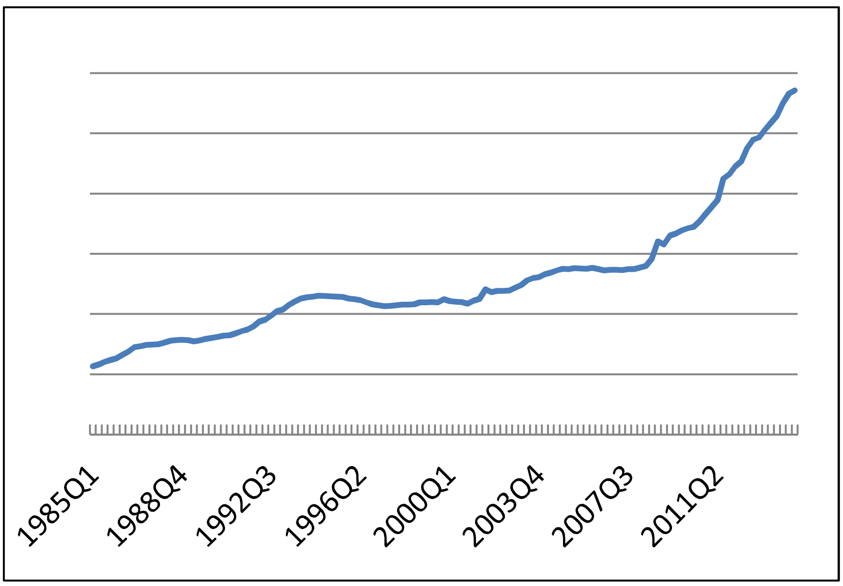

Figure10 - Monetary aggregate M1 the United States, SA 1985-2014

rise. With enough liquidity[7] in financial sector, through transmission mechanism, we can expect rise of credits and loans to real economy and total growth in aggregate demand and total production.

On the other hand, we can look at given situation differently. If the money supply increases too quickly, quantitative easing can lead to higher rates of inflation. This is due to the following fact. There is still a fixed amount of goods for sale or at least growing only with limited speed but more money is now available in the economy. If money are lent through banks to individuals and companies, the only logical consequence is rise of prices for products.

For illustration, you can see in Figure10 how QE can be spotted in money growth. In the US, growth in volume of aggregate M1 was more or less stable. However, from year 2008 upwards we can clearly see the difference in slope caused by interest rates policy of Federal Reserve System.

Whole situation of low interest rates is rather new. We cannot predict with certainty what happens in such a situation. It can stimulate total output and economy or cause undesirable inflation. On top of all that, in the history of major central banks were only a few situations with interest rates so low or when money growth was so high. Right now short term interest rates reached zero lower bound (in some occasional cases even overcame it leading to negative rates). Central banks have limited experience in this environment and as we have seen above, economic research was not so focused on analysis of money-price relationship. Quantity theory was not used due to beliefs of unstable and unpredictable velocity. S. Reynard (Reynard, 2012) explains that historical episodes of financial crises have been accompanied by different monetary stimulus. They were function of different monetary policy and different financial sector transmission mechanisms, what resulted in different inflation paths after the crises. Study is focused on Argentina, where consequence of crisis was high inflation while in Japan we observed deflation, although severe quantitative easing was introduced in the1990s and 2000s.

[7] Liquidity in this meaning can be understood as money, fund of banks and other financial institutions

3. Research findings

In the following subsections, we will present results for key countries considered in our sample. First subsection (3.1 United States) will be more detailed and explanatory. Other subsections will be rather plain in methodology description, rather focused on findings and special behaviour.

Short conclusion can be found at the end of every subsection. However, more complex findings and properties of model as a whole are discussed at the very end.

3.1. United States

United States is the country where financial breakdown started in 2008 and it was the very first economy where aggressive simulative facilities were introduced afterwards. For example, the short term interest rate dropped from ~5% in 2007 below 1% in the beginning of the 2009. It is reasonable to start our study here as it is representative and quite stable economy. First step of our analysis is to check for relationship between velocity of money and interest rates. Velocity is computed through Prices, GDP and selected money aggregate. You can see the result in the Figure 11.

We can clearly see the correlation of these two variables in time horizon from 1970 to 1990 and then also in 1995-2014. But there is a break of trend, positive shift in velocity in the beginning of the 90s. It was the time of prosperity and expansion of the US economy. Money growth in M2 was not following growth in prices and GDP what all lead to positive shift in velocity. We would like to emphasize that 1992 breakdown is rather the issue of money selection and M2 definition (several shifts among accounts, high rate of investment into real economy rather than into debt securities). If we want to fit absolute value of velocity, it is obvious we have to adjust our dataset somehow. In our case, we analysed only data series from 1994 onwards, due to our main goal – analyse latest observations and QE.

In selected data series, we are going to estimate elasticities for income and interest rate. Regression model with absolute variables have following form, presented in Table 1.

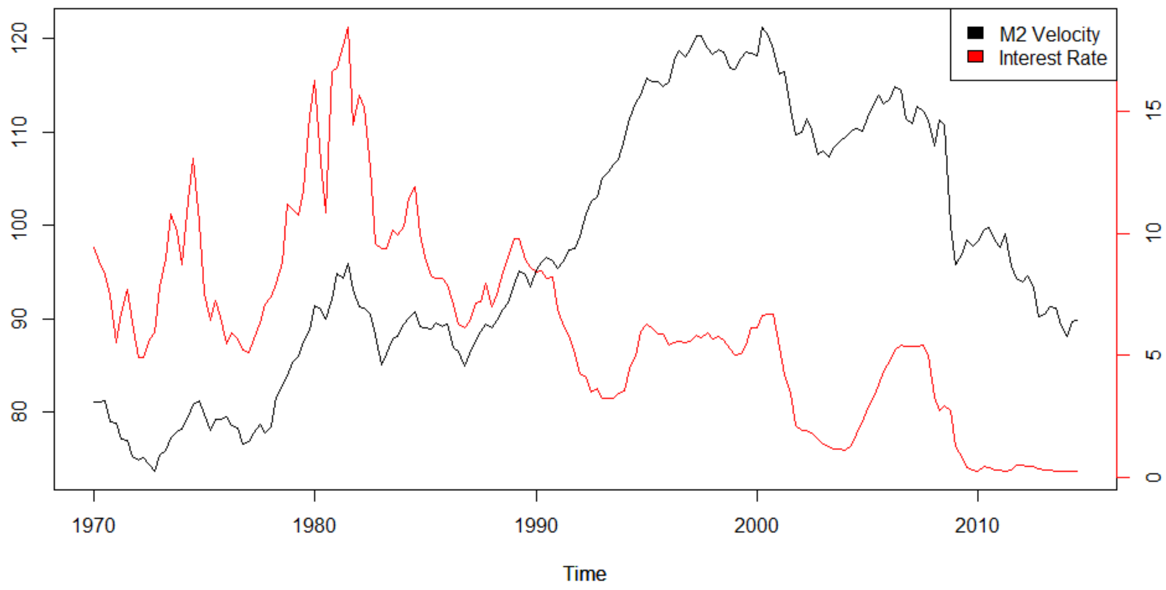

Figure 11 - M2 Velocity (lhs) vs. Interest rates (10Y money market rate,rhs in %), the US, source (International Monetary Fund)

Formula: ln(P/M) ~ 1+ (R) + ln(Y) + ε

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.590227   0.246009   18.66   <2e-16 ***
x            0.039334   0.003698   10.64   <2e-16 ***
x2          -1.007874   0.052542  -19.18   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03824 on 80 degrees of freedom
Multiple R-squared: 0.9704, Adjusted R-squared: 0.9697
F-statistic:  1313 on 2 and 80 DF,  p-value: < 2.2e-16
```

Table 1 – M2 Velocity estimate, HP filtered interest rates, US data 1994-2014

We can clearly see that explanatory power of model determined by R^2 is very high. It supports our belief that interest rate is a good driver for velocity. Another important fact is, that estimated coefficient for GDP (marked as x2 in Table 1) is almost -1. After running linear hypothesis test, we confirmed that income elasticity is not significantly different from unity. We can restrict introduced model and proceed with form ln(P/M) ~ 1+ (R) + 1*ln(Y) + ε = ln(Y*P/M) ~ 1+ (R) + ε, what is the direct estimate of velocity. Given result states that Quantity Theory of Money holds for the US economy under given assumptions. Let's do subsequent analysis.

Second important step is to construct Net Money cleaned for equilibrium income and velocity movements. After HP filtering and differentiation, we can get 4 stationary time series. Using equation 14 from section 2.4 leave us 2 time series (completely HP filtered), equilibrium Net Money growth and equilibrium Inflation. Both plotted in Figure 12. At this point, we should be able to see some relationship. NM growth should have some predicting power for inflation. We can observe comovements and inflation reaction in 95-98 and peak around 2005.

However, further analysis proved this relationship to be unstable. Both variables satisfied theoretical tests, Augmented Dickey-Fuller test for unit root, they appear to be stationary and not integrated. We created VAR model with 3 year (12 quarters) lag. Even Granger Causality test shows that there can be impact from NM growth towards inflation (p-value with Granger causality as alternative hypothesis is 0.3%). Moreover, inflation does not Granger cause NM growth in given dataset (p-value for Causality test did not cross 5% threshold). But major difficulty of this VAR regression is Impulse Response analysis. We uncovered negative effect on Inflation in case of positive shock in NM growth. Moreover results are unstable and bootstrapped confidence intervals for IRF are very broad.

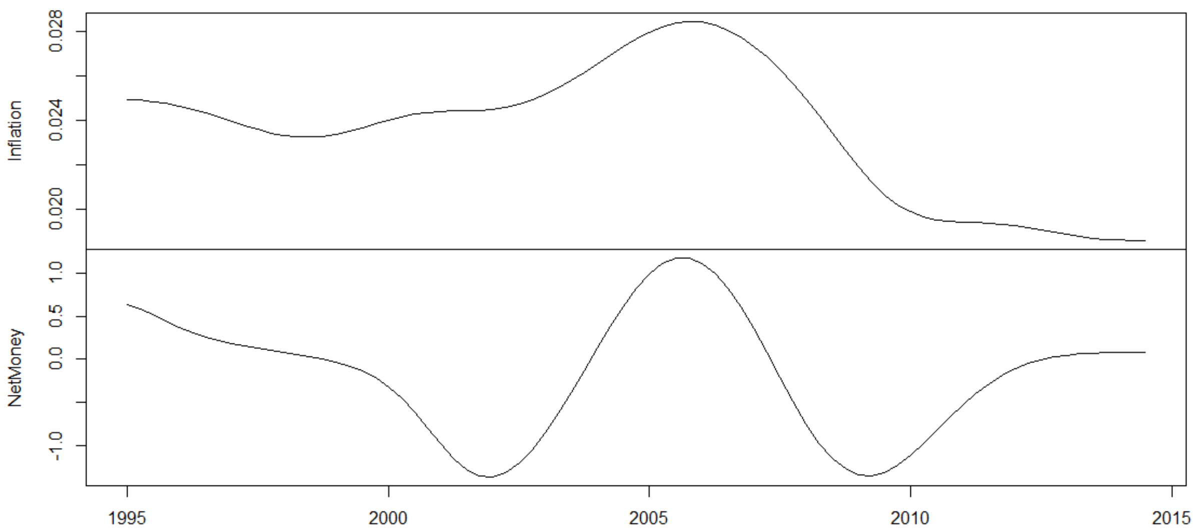

Figure 12 – Inflation vs. M2 Net Money growth, HP filtered, US data 1994-2014

If we look closer, observed comovements holds in environment of positive NM growth. However, we can see that Inflation does not follow NM growth when it turns negative. Consequently, periods around 2002 and 2008 are characteristic of decrease in NM together with slightly increasing inflation. It can be the main reason for apparently negative relationship using wild quantitative approach with least squares estimate in VAR. On the other hand, this fact is quite well explainable by nature of Inflation. NM growth, even M2 growth itself is not regulated so closely. Inflation, on the other hand, is main target in monetary policy. All reports and statements from FED are presenting Inflation changes. Deflation does not happen so easily. Expectations of growing prices and high regulation go against it.

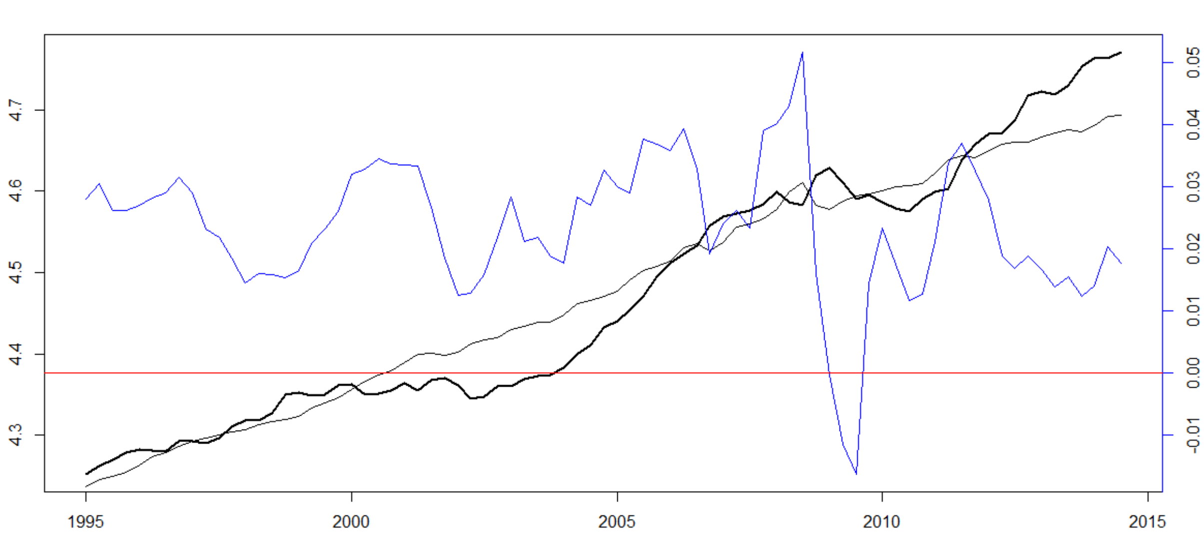

Figure 13 – Money (large black) and Prices (thin black), Inflation (blue), US data 1994-2014

Let’s do the whole process after velocity estimation differently. Instead of NM growth we can create log Net Money index (using velocity, income and M2 index). This time, we used unfiltered money and price level to study real rather than equilibrium movements. Income and velocity corrections stayed as before. We analysed the relationship between (log) Prices and estimated NM level. You can see a plot of these two variables, together with differentiated price level- inflation in Figure 13. Look at the data first, before doing any harder quantitative analysis. Let’s consider the measure of Excess Liquidity (EL), defined as difference of NM vs. Price[8]. Given difference is slightly positive in the beginning of the observed period, where Inflation is quite high. Small turmoil in Excess Liquidity around 96-97 is reflected in Inflation decrease with lag of 2-3 years. In later period, this relationship more or less stays in place. However, periods of decrease in Excess Liquidity are not followed by inflation as closely as positive changes. It basically means that when our measure of EL is positive, we can observe higher levels of Inflation after some lag. On the other hand when EL shifts to negative numbers, Inflation does not turn negative, just goes towards 0% border instead (without crossing it, except 2009 turbulent period). In order to observe this fact, look at the Figure 14 where we plotted

[8] One has to be careful using this measure. Based on velocity estimate, EL has zero mean (residuals from linear model) and inflation is positive most of the time.

Excess Liquidity (forward shifted) against Inflation. Or more clearly written, effect of variable $EL_{t-shift} = m^*_{t-shift} - p_{t-shift}$ on Inflation $\Delta p_t = p_t - p_{t-4}$.

Presented result gives us only partial support of our theory. We can observe periods when there is obvious relationship but there are also some years (e.g. 2005) when given arguments are not so obvious. Doing VAR analysis in second approach, theoretical assumptions are again fulfilled quite reasonably. There is also slight evidence in IRF (Figure 15) of positive reaction after 1.5 year from inflation to Excess Liquidity shock. However, evidence is not significant and we are still dealing with issues of negative instant response. Moreover, even Granger causality test are not rejecting the null hypothesis so there is no evidence of this kind of relationship.

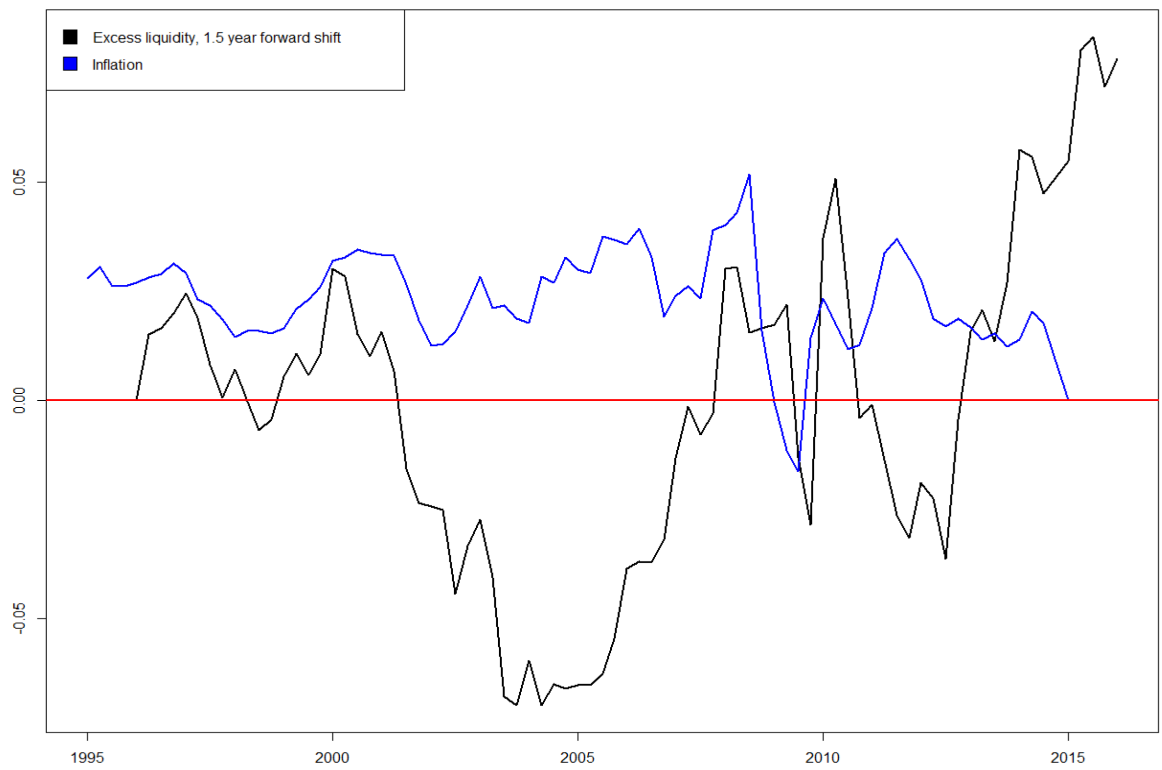

Figure 14 – EL (forward shift) vs. Inflation[9]

[9]Due to technical reasons of plotting we artificially added 0 as first data point for EL and last for inflation

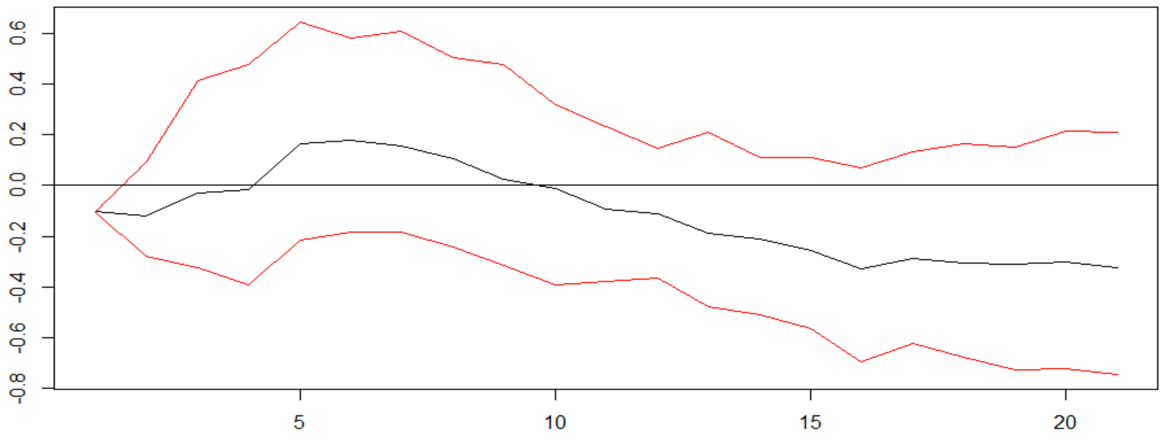

Figure 15–Impulse Response of Inflation from EL shock

To conclude, we can tell from observation and VAR analysis of NM that there can be a link from EL lagged values towards Inflation. In other words, Inflation in time t can be predicted using 1-3 year lagged values of its own series and series of money growth. As a result, high growth in NM level can indicate higher rate of Inflation in the US environment. But low interest rates and decent GDP growth keeps NM level lower despite the fact of severe Quantitative Easing. Therefore risk of inflation is low.

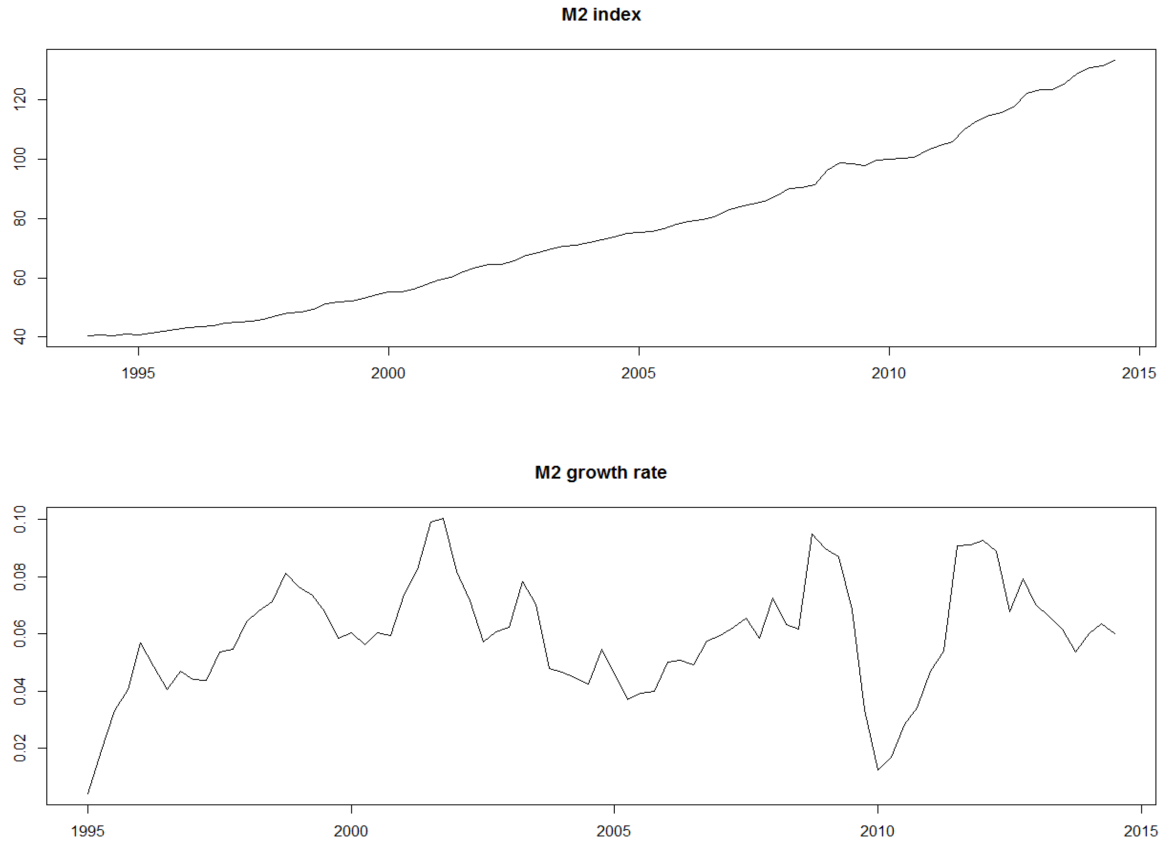

Figure 16 – M2 growth, considered period 1995-2014

Moreover, we uncovered that QE does not have so strong effect in the US economy. Considered measure of NM did not growth so strongly since 2008. Actually slight decrease is observable around 2008 in the Figure 13. NM starts growing only in the end of the period. Further analysis of original Money aggregates is even more interesting. M1 aggregate was affected by QE obviously as we have seen in the Figure10. However, if we look at M2 aggregate in the Figure 16 which contains more stable assets, QE effect is negligible. Created liquidity stays in banks and do not have serious consequence on real economy. On the other hand, liquidity trap of zero interest rates and possible breakdown in GDP growth can cause inflation shocks in the future.

3.2. Switzerland

In work of (Reynard, 2012) is Switzerland considered as fundamental economy for further studies. Observed relationship between M2 Velocity and interest rates is strong (Figure 17). Two time series are in line most of the time, except significant shock in 2008. Positive increase in velocity is explained by decrease in M2 aggregate. Switzerland is rather small and open economy, thus unexpected shifts in our variables can occur. This outlier observation is affecting some of the results and fits. In order to have more reliable analysis, we analysed the aggregate M3 and compared results. Breakdown in M3 is not so severe because aggregate is more stable in general. It also proves that 2009 shock can be taken as random shock rather than significant discrepancy. However, presentation of final results here is done on M2 in the interest of more stable approach across all countries.

Figure 17 - Switzerland, M2 Velocity (lhs) vs. Interest rates (10Y money market rate,rhs in %), source (International Monetary Fund)

Linear model for velocity prediction is capped in Table 2. Our R^2 measure has higher level what points out that our model should be good. However, when you notice coefficient we get income elasticity (x2) significantly lower than unity. Unexpected discrepancy is caused by discussed shock in 2008. Cutting dataset for period 85-05 gives more reasonable result. Moreover M3 analysis leads towards estimate of -1,04, satisfying unity restriction. So we decided for application of model ln(Y*P/M) ~ 1+ ln(R) + ε for whole dataset with M2 aggregate. It still leads to reasonable R^2 of 0,77.

Formula: ln(P/M) ~ 1+ (R) + ln(Y) + ε

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.40583    0.48605   4.950 2.52e-06 ***
x            0.14648    0.01064  13.772  < 2e-16 ***
x2          -0.58012    0.10285  -5.641 1.20e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08164 on 117 degrees of freedom
Multiple R-squared: 0.9306, Adjusted R-squared: 0.9294
F-statistic: 784.3 on 2 and 117 DF,  p-value: < 2.2e-16
```

Table 2 - M2 Velocity estimate, HP filtered interest rates, Swiss data 1985-2014

Following step is again the construction of Net Mooney series together with Inflation, using corrections for velocity and income. Comparably with previous case, we are going to perform analysis of resulting sequences in three different ways.

Firstly, all underlying data series are HP filtered and differentiated. Observations are put together in Figure 18. We can observe slight predicting power of NM growth in the beginning period until 93-94. However, later periods shows very weak evidence of interrelationship. Time frame between 95-05 is characteristic for fluctuations in NM growth and stable Inflation. End of the dataset shows rather negative link from NM towards Inflation.

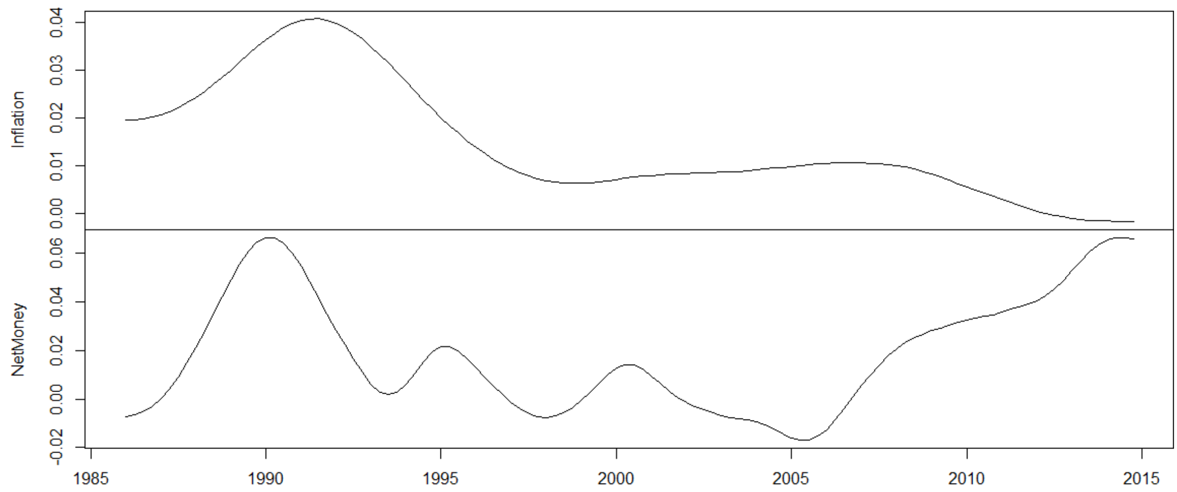

Figure 18 – Inflation vs. M2 Net Money growth, HP filtered, Swiss data 1985-2014

Our attempts to fit reasonable Vector Autoregressive model in given dataset were only partially successful. Granger causality and IRF analysis was insignificant with negative correlation. However, there was again slight evidence of Inflation reaction on NM growth with lag 6-8 (very similar to Figure 15).

Secondly, we applied measure of Excess Liquidity. Fitted NM level is compared to Price level. Final result is shown in Figure 19 together with Inflation levels and in Figure 20. It is rather clear from first observation, that there can be some dependency in several periods. Shock of EL in the beginning is reflected in inflation around 1990. EL fluctuations in 90-05 period are only partially followed by Inflation. Relationship, comparably to the US case, is not so clearly supported by data.

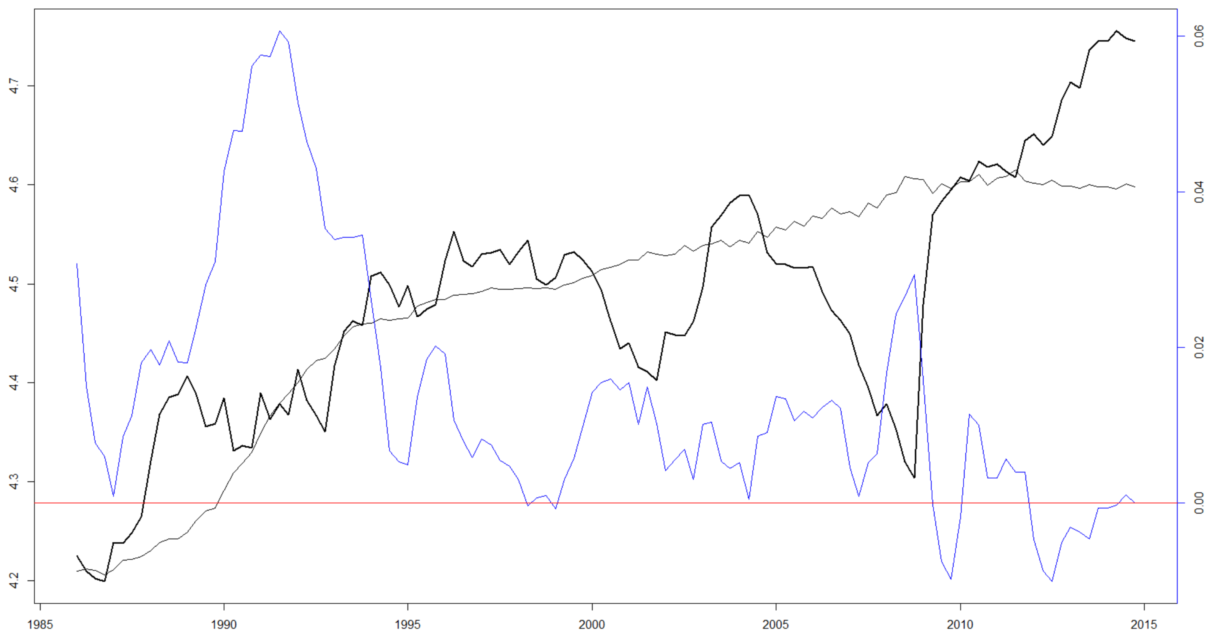

Figure 19– Money (large black) and Prices (thin black), Inflation (blue), Swiss data 1985-2014

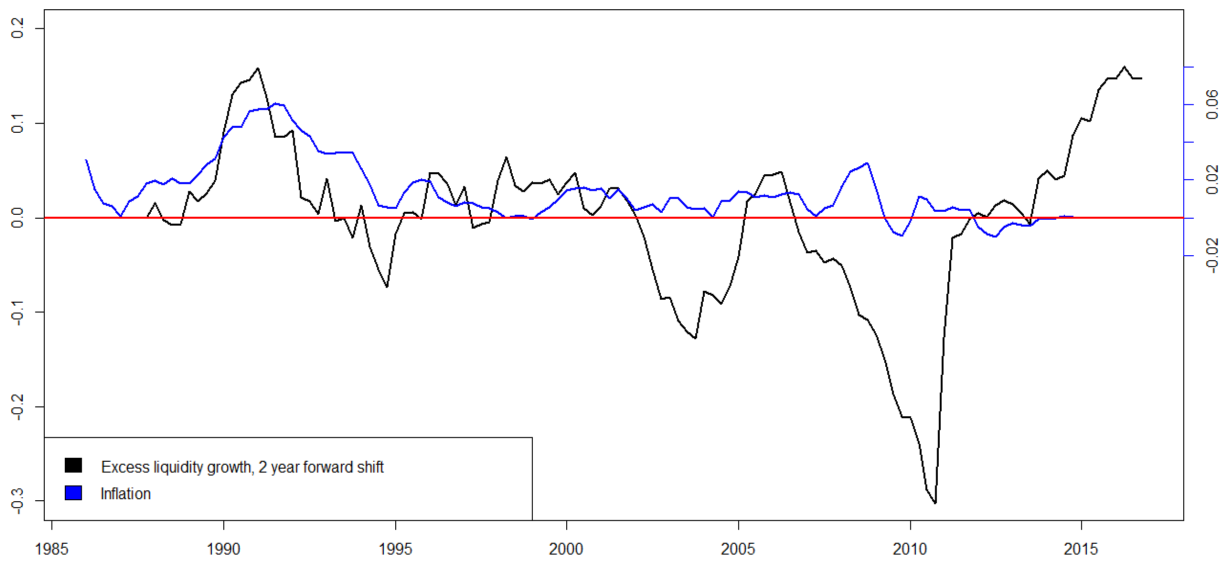

Figure 20 -EL (forward shift, lhs) vs. Inflation (rhs)

However, last part of analysis, using EL growth still appears to be the most reasonable. We can spot much stronger evidence of comovements than in the previous case. But, we are still experiencing troubles caused by positive values of Inflation despite NM growth turns negative often.

In conclusion, our results are very similar to the US case. Velocity estimates with interest rates are stable and reasonable. Consequent analysis using VAR framework is on the other hand unreliable. Assumed relationship of money and prices has not significant evidence. However, simple observation of the data sequences reveal possibility of link from NM growth and NM-Price difference towards Inflation.

3.3. Germany

Our research is focused on analysing the QE effect especially in the area of Europe. Germany, is a dominant economy in European Union and in whole Europe. Therefore it cannot be excluded from our scope. Result are representative for several other countries e.g. Poland, Denmark or several other northern economies. Moreover, compared with the US, Germany is well known for its conservative opinion on QE and expansive monetary policy. Massive quantitative easing and hyperinflation in 20s and 30s lead to the strong depression and made German Bundesbank much more careful in recent days.

We had access to data series in 1980-2014. In order to have observation long enough, we decided to use GDP deflator instead of CPI. Results were very similar in both cases but we had access to CPI only since 1991.Evidence for the first model assumption is reasonably sound. We can observe comovements of computed M2 velocity and market observed interest rates (opportunity cost) in Figure 21.

Common downward trend is obvious and even some fluctuations are in line, yet there is slight discrepancy around year 1989. Period of Germany merger caused positive shift in GDP, Price and interest rates, but did not affect money level which remained stable. All resulted in velocity breakdown and interest rates increase. In subsequent years, relationship turns to be reasonably stable. We can run linear model.

Upcoming analysis has rather stable result even with merger period included (see Table 3). $R^2$is 0.98 for the fit in whole period visible in Figure 21. Even income elasticity is near -1. Direct restriction leads to the model with determination around 0.9. Consequent outcome of the linear model is stable and velocity correction to the money level can be applied here.

Consequently, Net Money aggregate was created in same logic as before. It was followed by NM growth computation with lag of 4 quarters. Result is presented in the Figure 22.

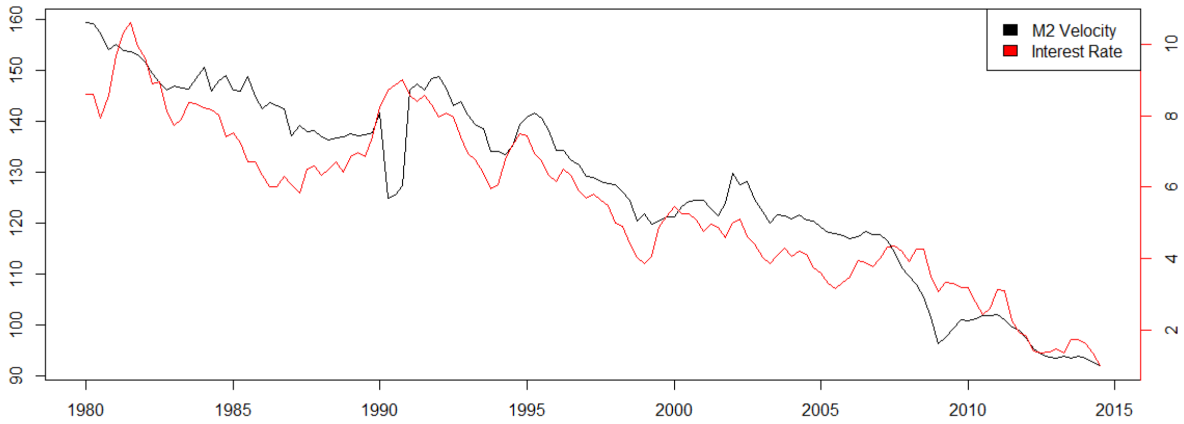

Figure 21 - Germany, M2 Velocity (lhs) vs. Interest rates (10Y money market rate, rhs in %), source (International Monetary Fund)

Formula: ln(P/M) ~ 1+ (R) + ln(Y) + ε

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.907688   0.192790   25.46   <2e-16 ***
x            0.055424   0.003959   14.00   <2e-16 ***
x2          -1.086413   0.039446  -27.54   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.04717 on 136 degrees of freedom
Multiple R-squared: 0.9827, Adjusted R-squared: 0.9824
F-statistic:  3853 on 2 and 136 DF,  p-value: < 2.2e-16
```

Table 3 - M2 Velocity estimate, HP filtered interest rates, Germany data 1980-2014

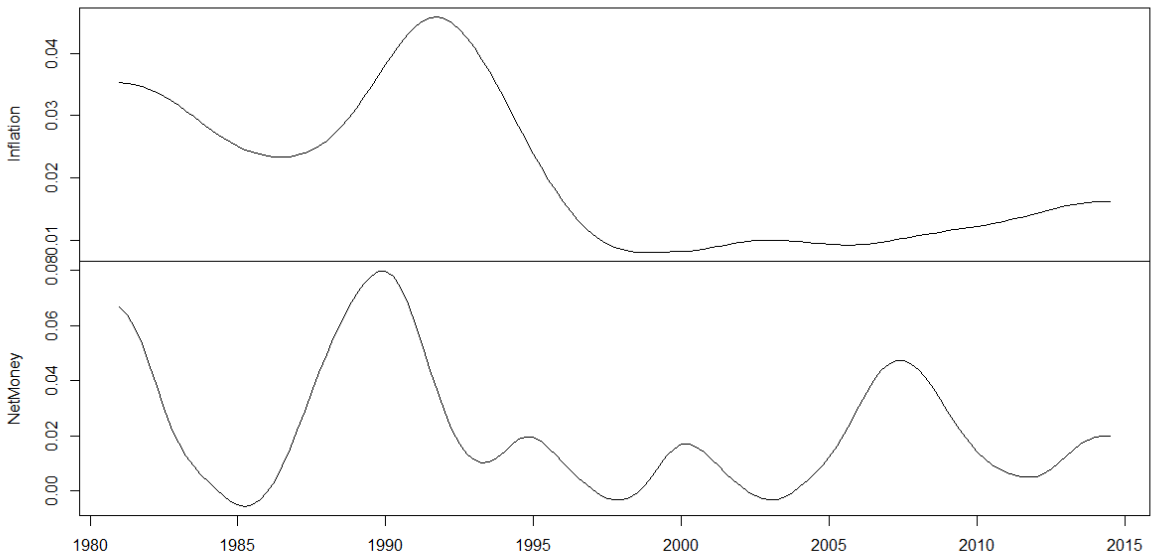

Figure 22 - Inflation vs. M2 Net Money growth, HP filtered, Germany data 1985-2014

Possible causality is much clearer this time. Fluctuations in NM growth are followed by same movements in Inflation with approximately 2 year lag (even if we exclude the peak around 89). Effect is especially strong in the beginning of our time frame and weakens in time. On the other hand, even though reactions from inflations are not so strong, they can be observed. E.g. peak in NM in 2000 can correspond with slight Inflation increase in 2002. Peak around 2008 can similarly correspond with slight upward trend since 2010 till the end of our time frame.

Slight evidence of price level reaction does not mean that this relationship is easy to prove using VAR framework. Autoregressive model of the data introduced in Figure 22 was not stable. However, if we took same dataset without filtration of Money and Price levels, we found interesting evidence. Granger causality test with H0: "Net Money Growth do not Granger-cause Inflation" had p-value of $4.9 * 10^{-8}$. Check for reversed causality ended up with p-value of 0.839. Moreover, impulse response function indicates possible effect of NM growth with time lag 6-8 (1.5-2 years).

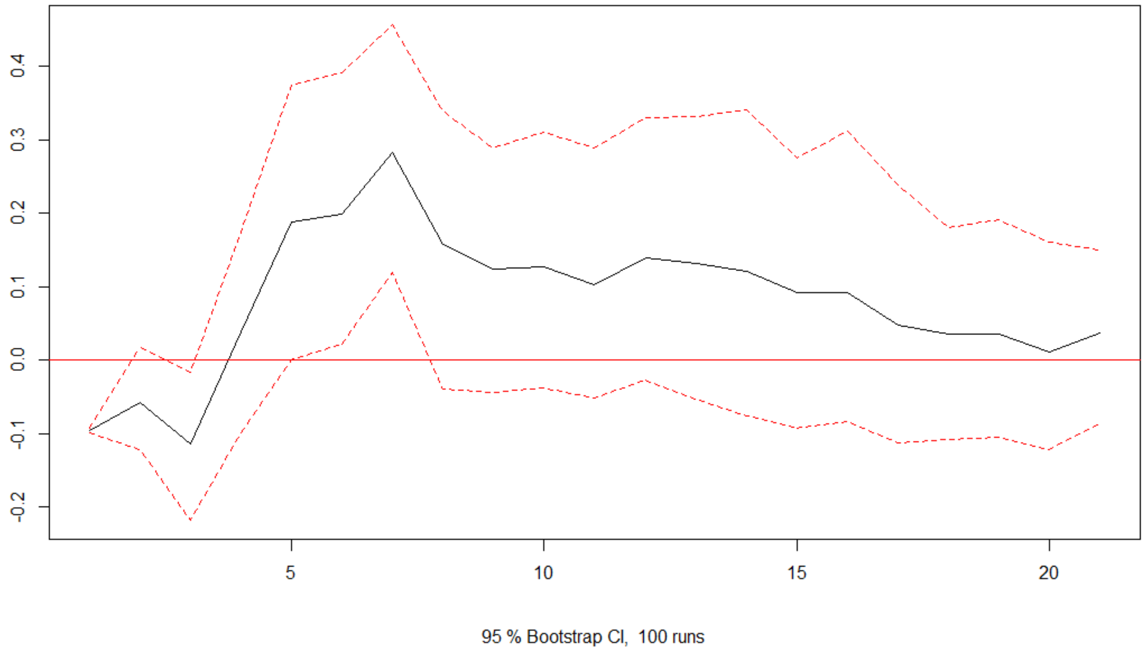

Figure 23 – Impulse response of Inflation from NM growth shock

Considering Excess Liquidity measure, slight evidence still holds. Results are in Figure 24. Positive shocks predict Inflation increase, the negative ones are predicting Inflation decrease. It holds even in the crisis period. Observed lag for this effect is in line with previous finding, around 1.5 year. Observation is more obvious in the next Figure 25. EL growth result is not presented this time. But it carries similar information as already displayed graphs.

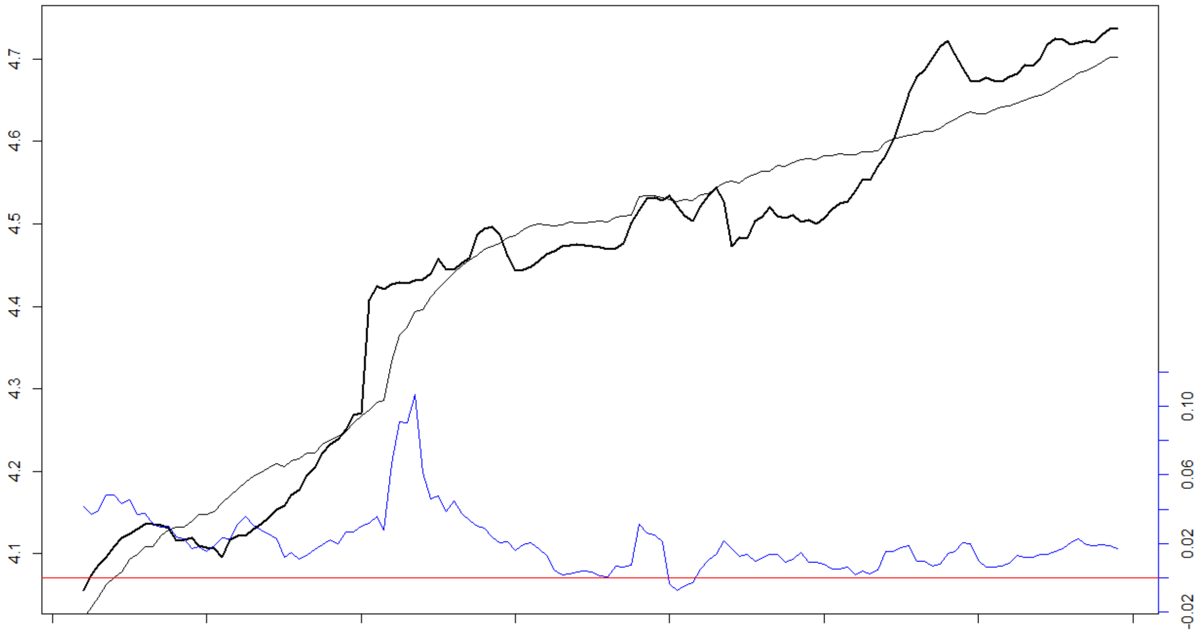

Figure 24 - Money (large black) and Prices (thin black), Inflation (blue), Germany data 1980-2014

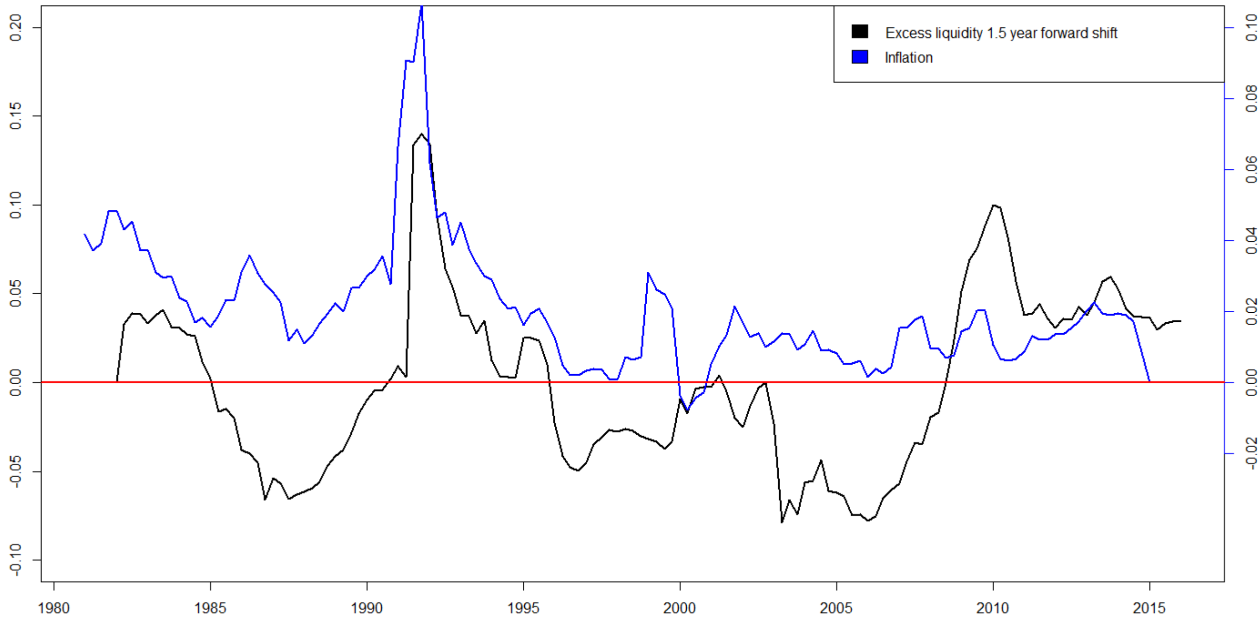

Figure 25 - EL (forward shift, lhs) vs. Inflation (rhs)

All results for Germany are in line with previously presented countries. Evidence for NM vs. price relationship is slight but observable. In this case, time lag for Inflation reaction is around 1.5-2 years. Effect is observable more or less throughout entire time window. In contrast, recent years are not so characteristic for quantitative easing as Switzerland or the US. Original M2 aggregate together with our adjusted NM criterion are growing much slower. Inflation remains low but positive EL can still be causing its slight increase.

3.4. France

Next country in our scope was the second obvious choice from Monetary Union after Germany. France is a stable strong economy with all properties for our analysis. Moreover, it is representative economy due to results comparable with several other south-western countries such as Spain, Italy or even UK. Dominating factor in indicated countries is high effect of 80s recession together with 90s recovery. This factor leads to discrepancy in velocity, due to M2 stagnation and decent growth in GDP and Price levels. It caused sharp increase in M2 velocity. Interest rates had decreasing trend instead. Evidence for opportunity cost model assumption is very small, as we can see in the Figure 26. Effect is similar to 90s expansion in the US case but this time is our dataset affected in much longer time frame. We accounted for this aspect with considering M3 monetary aggregate instead. Broader definition of money was much more stable during 80s and 90s. Interest rates vs. M3 velocity can be observed in the Figure 27.

Figure 26 is characteristic for much higher discrepancies and evidence for linear relationship between opportunity cost and velocity is not so obvious. Considered sequences with M3 were used for linear model in velocity estimate. We performed estimate similar to the previous cases. You can see the result in Table 4. Resulting R^2 of 0.96 is very reasonable. All higher differences were explained by GDP with income elasticity much higher than unity (usual outcome for M3 aggregate study). Despite previous result, we restricted income elasticity again, in order to be consistent with already presented methodology. Process lead us to R^2 at the level of only 0.53 with much higher $\alpha = 0.0327$. Despite worst fit of linear model, resulting data series of Net Money were very similar and had same properties.

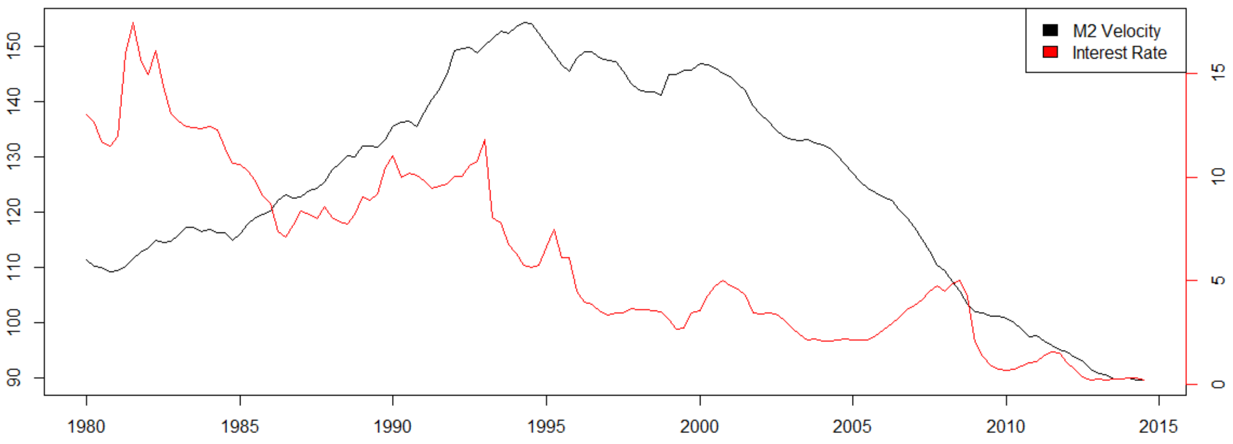

Figure 26 - France, M2 Velocity (lhs) vs. Interest rates (10Y money market rate,rhs in %), source (International Monetary Fund)

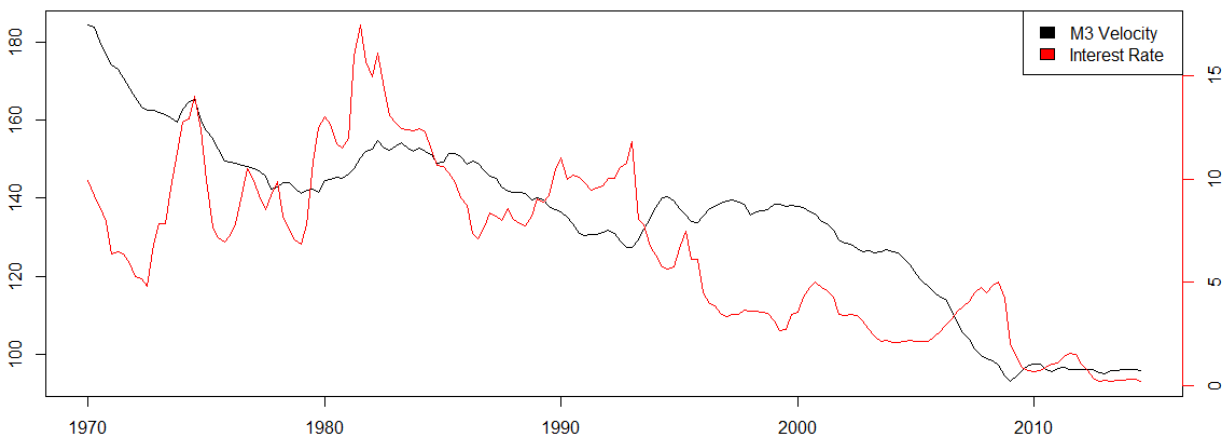

Figure 27 – France, M3 Velocity (lhs) vs. Interest rates (10Y money market rate, rhs in %), source (International Monetary Fund)

Formula: ln(P/M) ~ 1+ (R) + ln(Y) + ε

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6.925760   0.161675  42.838   <2e-16 ***
x            0.006357   0.002471   2.573   0.0109 *  
x2          -1.485951   0.034776 -42.729   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08103 on 176 degrees of freedom
Multiple R-squared: 0.9646, Adjusted R-squared: 0.9642
F-statistic:  2398 on 2 and 176 DF,  p-value: < 2.2e-16
```

Table 4 – M3 Velocity estimate, HP filtered interest rates, France data 1970-2014

Observation of produced NM level and inflation is similar and does not reveal results different from other countries. Both graph for growth rates are in the Figure 28. VAR model leads to partially questionable output again. Granger causality has conservative outcome with p-value of 0.1 in case of NM growth effect on Inflations and around 0.23 in case of reversed hypothesis. Both Impulse response function of NM growth and Excess Liquidity analysis point out possible positive reaction of Inflation. See Figure 29 and Figure 30.

Figure 28 - Inflation vs. M3 Net Money growth, HP filtered, France data 1970-2014

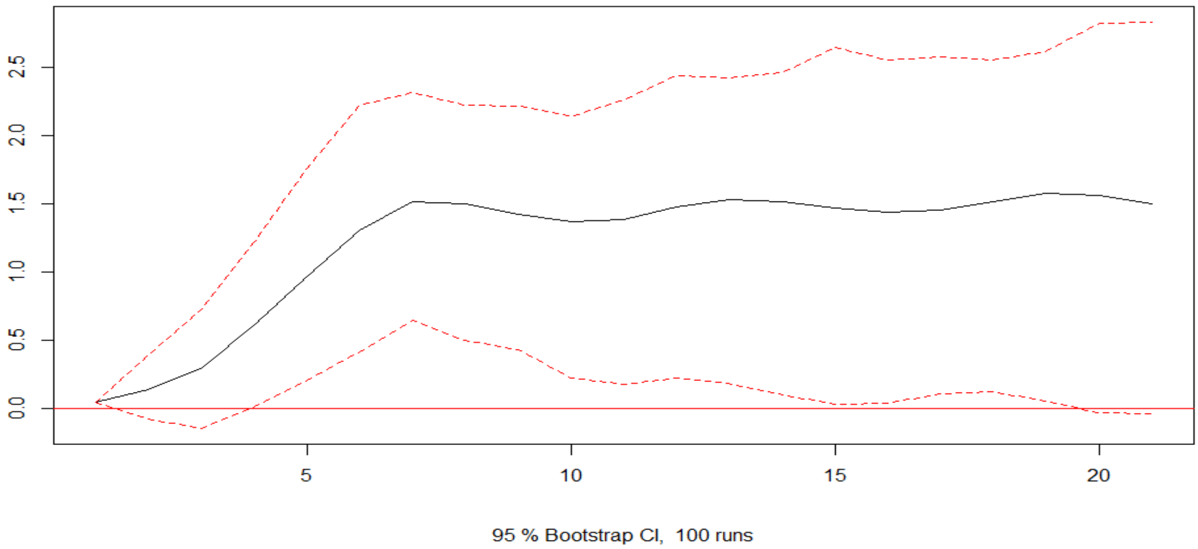

Figure 29 – Impulse response of Inflation from NM growth shock

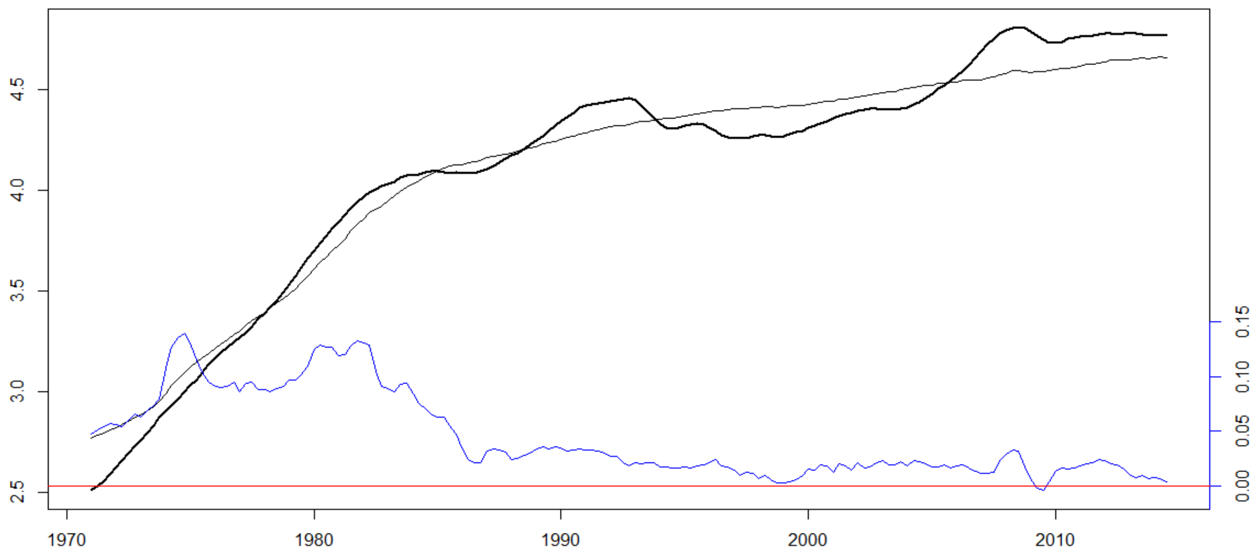

Figure 30 - Net Money (large black) and Prices (thin black), Inflation (blue), France data 1970-2014

At the end, despite much worse explanatory power of interest rates in velocity estimate, we had still reasonable evidence for Money price relationship. Even IR function is much more stable with more significant evidence than previously discussed cases. Observed inflation answer to Money level has lag of 2 years in case of France.

3.5. The Economic and Monetary Union (EMU)

Investigation of Monetary Union as a whole will round up the picture of real data support for our model. We can find here countries with different policy transmission channels. In past, we could observe various monetary policies. However, since 1995 integration of European Union started to be inevitable. Convergence led to common currency EURO since 1999 for first 11 countries, with common monetary policy steered from European Central Bank. Today, EMU consist of 19 countries.

Our first model assumption is velocity fit with opportunity costs. Evidence for this hypothesis is quite reasonable. However we experience same troubles as in the analysis of France economy. Several major economies of EMU were affected by 80s crisis and 90s strong recovery period in similar way as France. It can weaken our observed relationship because interest rates are decreasing much faster than velocity. See results in Figure 31. Moreover, we can observe 2 strong shocks causing discrepancy of our assumption. Turbulent period is window between years 1999 and 2002 (integration of euro Euro currency in countries) and especially steeper decrease till 1999. Second strong shock of interest rates is increase since 2006 until the drop in 2009. This period can be again explained by acceptance of new countries in European Union in 2004, their common convergence and promised enlargement of EMU. Situation changed since 2009, when union became rather stable and its expansion was no longer priority. Interest rates started to be perfectly in line with computed velocity.

Figure 31 - EMU, M2 Velocity (lhs) vs. Interest rates (3M money market rate, rhs in %), source (International Monetary Fund)

Even though, we would like to emphasize following fact. We approach HP filtered interest rates as equilibrium velocity. "Computed" velocity shocks are then considered as price shocks. However, to correct 2 major shocks discussed above, we decided to use bigger "lambda" in HP filter (value of 1300).

Let's look at the linear model now. We again performed fitting with income elasticity estimate first. This time, model ended up with α around -2 instead of unity and reasonable R^2 of 0.93. Despite credible result, previously discussed theoretical properties of estimated equation and consistency with other datasets we made final estimate with model constrained for unity again. Resulting interest rate elasticity β is 0.09 with still acceptable $R^2$of 0.77 (see Table 5).

Formula: ln(Y*P/M) ~ 1+ (R) + ε

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.629059   0.018716  247.34   <2e-16 ***
x           0.092867   0.005669   16.38   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0789 on 78 degrees of freedom
Multiple R-squared: 0.7748, Adjusted R-squared: 0.7719
F-statistic: 268.3 on 1 and 78 DF,  p-value: < 2.2e-16
```

Table 5 - M2 Velocity estimate, HP filtered interest rates, EMU data 1995-2014

Resulting series of Net Money growth and Inflation are in the Figure 32, HP filtered again. Slight dependency can be observed from NM levels towards Inflation. We performed usual tests for causality. Granger causality for dependence towards Inflation ended up with p-value of 0.11. It is conservative value and we cannot argue for causality in data, however it once again indicate possibility of some link here. Moreover, reversed hypothesis had p-value 0.74. It is in favour of H0 that there is not any relationship.

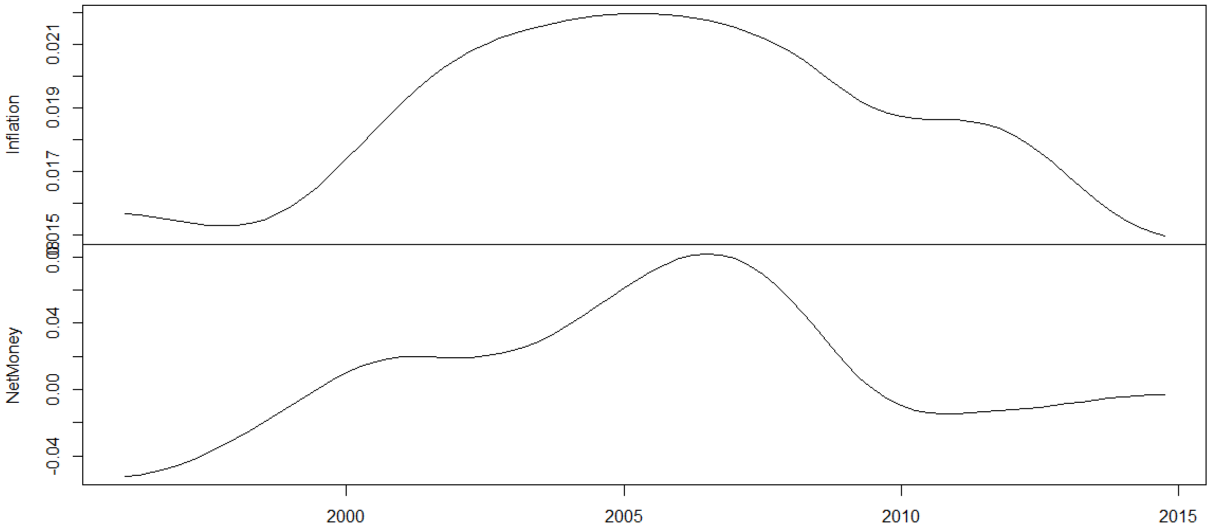

Figure 32 - Inflation vs. M2 Net Money growth, HP filtered, EMU data 1995-2014

Next step was computation of Impulse Response function based on presented model. In our interest is only response of Inflation level from NM growth shock. Outcome is in Figure 33. We can observe evidence similar to previous cases. Shock in NM growth is affecting Inflation with time lag of 5-6 periods.

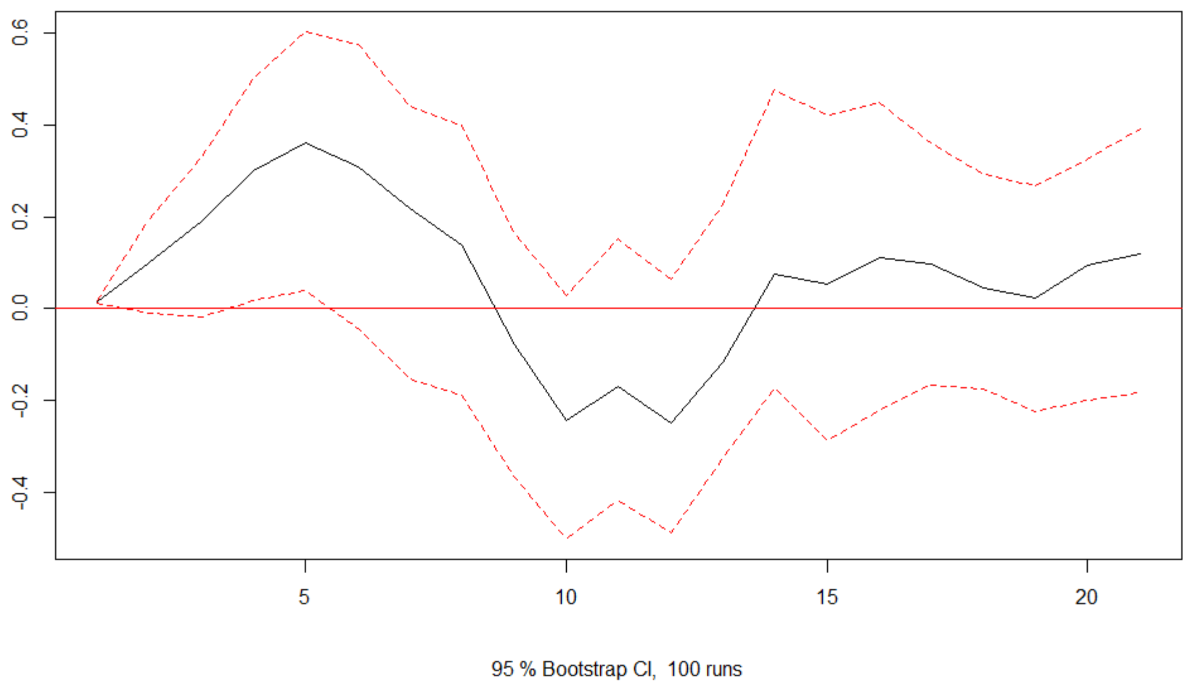

Figure 33 - Impulse response of Inflation from NM growth shock

Our second approach, the measure of Excess Liquidity is this time little bit less explanatory. Observation of the Figure 34 is dominated by 2 major discussed shocks. They are not affecting only interest rates but Money level as well.

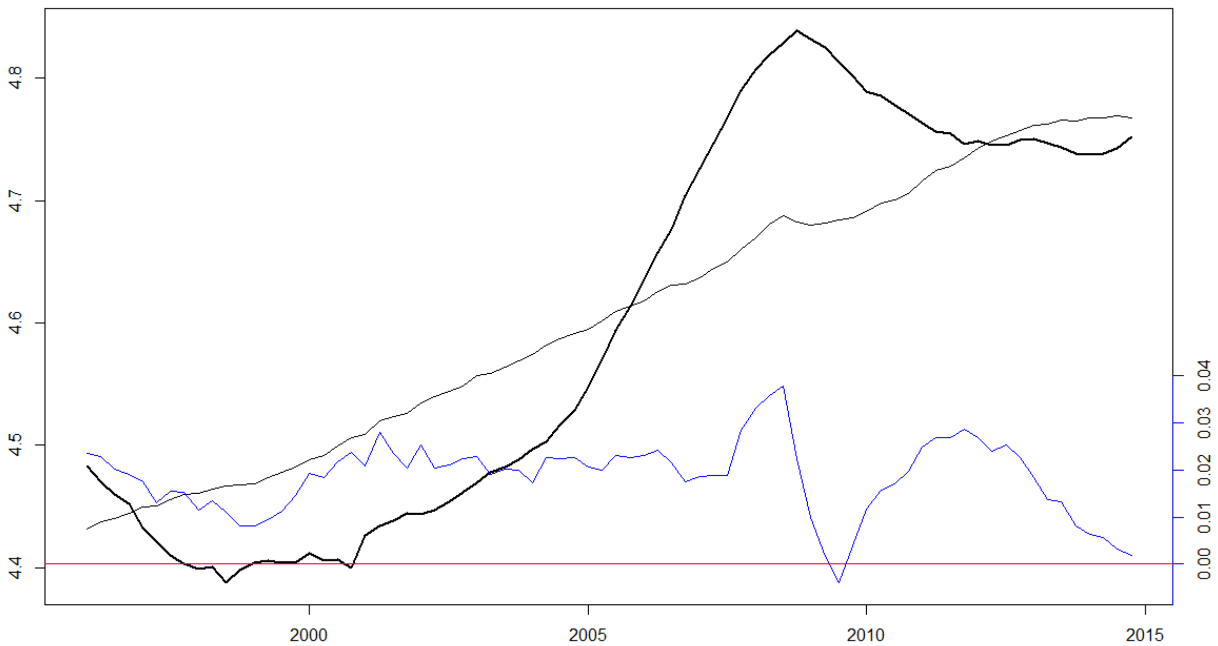

Figure 34 - Net Money (large black) and Prices (thin black), Inflation (blue), EMU data 1995-2014

Period of 2000-2005 is characteristic for negative, but stable EL. It should leads towards lower rate of inflation, but we can observe more or less constant level. Both of them being constant is not particularly contradicting, it is reasonable. But period, when EL turns positive without effect on Inflation, is problematic. On the other hand, there is observable relationship until the year 1999. Moreover, end of period have negative EL and later decrease in Inflation as well. In general, slight evidence in particular time windows is observable, but total explanatory power of EL is small.

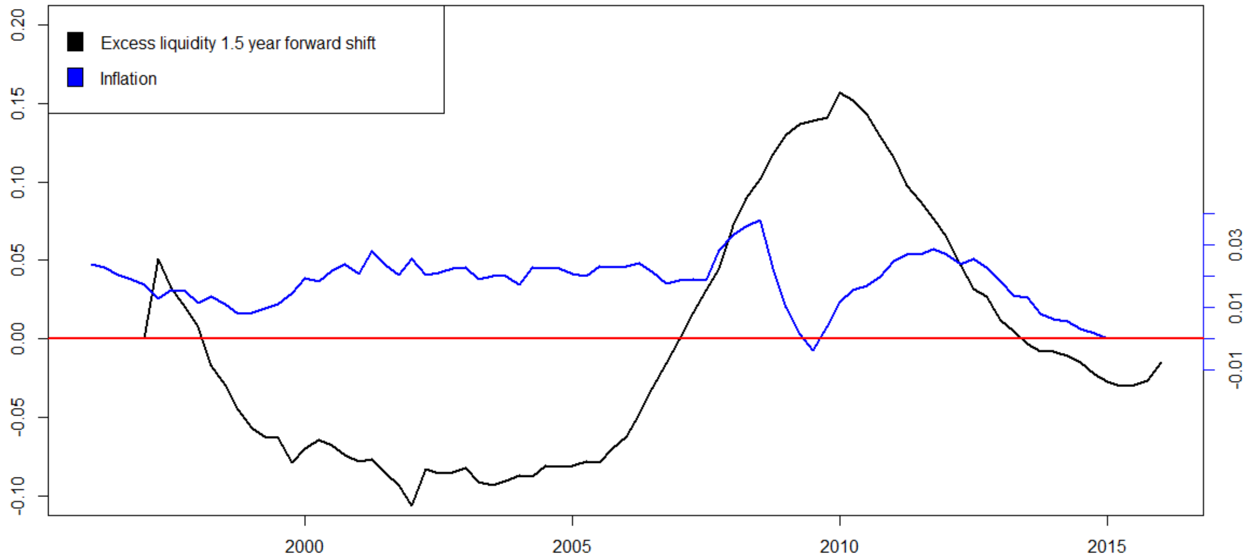

Figure 35 - EL (forward shift, lhs) vs. Inflation (rhs)

Final conclusion for The Economic and Monetary Union is little bit more unstable than previous results. Interest rates shocks caused discrepancy of our study and relationship being proved is weaker than expected. However, differentiated data series were not affected so hard. Analysis with our first approach led us to better results and minor evidence for our hypothesis of dependency. If we take into account results for Germany, France and several other not presented countries (Italy, Spain, ...), we can say that link from Money to Inflation is possible but not confirmed on EU level. In order to expand our analysis, it would be useful to have observation longer than current lifetime of EMU. Moreover it is possible to take into account more stable interest rates cleaned for discussed external shocks and broader definition of money.

Nevertheless, we can make several final statements. Undoubtedly there is an evidence for NM decrease at the end of the period together with very low Inflation. Furthermore if we consider better support for our model at the end of the period, we can say the risk of Inflation is very low. More probable situation is possibility of deflation. The answer from ECB side was rather quick. In the beginning of 2015 ECB announced Expanded Asset Purchase Programme. Details can be found in (ECB, Press Relase - ECB announces expanded asset purchase programme, 2015). Given response to current environment can be "translated" as strong Quantitative Easing to support and boost real economy in short run, consequently to avoid deflation. Based on our results, we consider this step as reasonable and well-timed. Effect will be probably comparable to the US situation with only minor effect on Inflation. On the other hand QE has to be performed very carefully considering possible shocks in GDP. If transmission mechanism does not hold, GDP growth can stay low and Inflation can rise. QE will have to be stopped in such situation.

3.6. Slovakia

Last country in our scope does not fulfil all conditions stated in section 2.3. Slovak economy is much more open with much more different balance of trade. On the other hand, one of our primary goals is to estimate the impact of QE on our national economy, to see how money growth can affect Inflation in country such is ours. Moreover, Slovakia is still reasonably stable to run and test our model, thus we decided so.

First step is again to check for velocity-interest rate relation, in the Figure 36. To be consistent with methodology presented for Germany or France, we used long term interest rate. However, in this case we can observe quite strong predicting power of interest rates even with shorter maturities for velocity.

We can recognize two major shocks in data. First one is in 2004, positive shock in velocity (more detailed analysis points out it is a price shock) probably caused by obtaining the European Union membership. Second major discrepancy is in 2009, positive money shock causing decrease in velocity. However, total final evidence in whole observation period is still reasonable. With unity restriction, we reach the model from the Table 6. Model is comparable with ones, we have seen before, despite the R^2 value which is smaller. Remember that, weaker evidence can be caused by much smaller observation period. We start in year 2000 and still have 2 major shocks. However, when we performed our analysis with different (3M maturity) interest rates, it led to model with determination of 0.79. Moreover, both models without unity restrictions have R^2 above 0.90 with income elasticity still relatively close to unity. All given findings approve us to take into consideration model in Table 6.

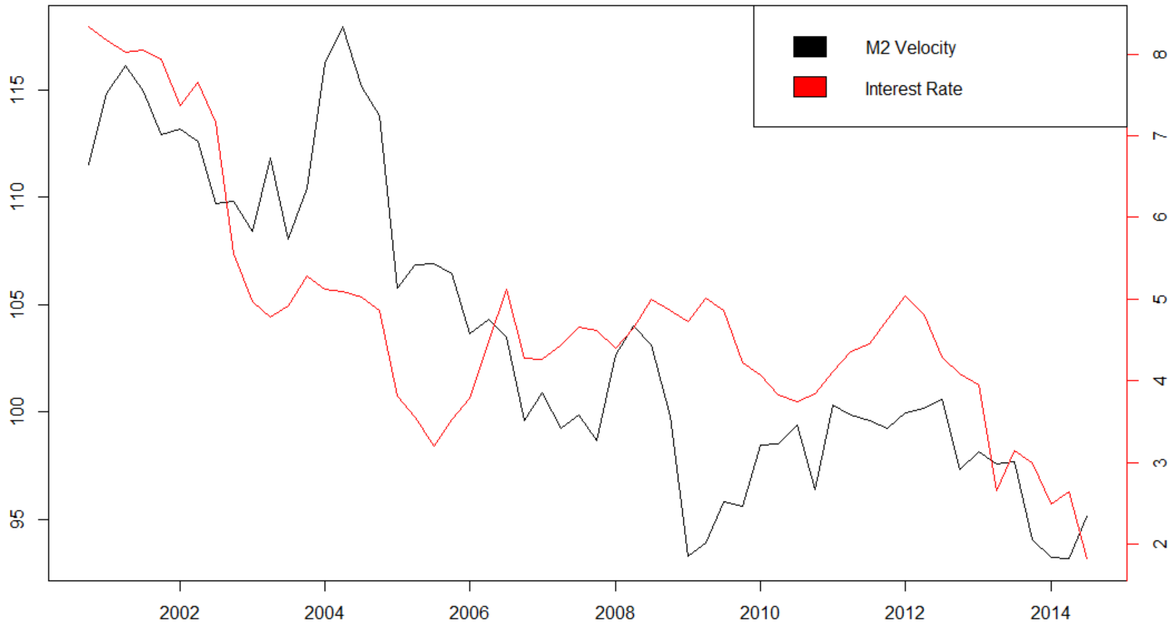

Figure 36 - Slovakia, M2 Velocity (lhs) vs. Interest rates (10Y money market rate,rhs in %), source (International Monetary Fund)

Formula: ln(Y*P/M) ~ 1+ (R) + ε

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.443158   0.026546 167.378  < 2e-16 ***
x           0.040910   0.005387   7.595 4.46e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.05089 on 54 degrees of freedom
Multiple R-squared: 0.5165, Adjusted R-squared: 0.5075
F-statistic: 57.68 on 1 and 54 DF,  p-value: 4.463e-10
```

Table 6 - M2 Velocity estimate, HP filtered interest rates, Slovakia data 2000-2014

After setting up NM level, our first approach with NM growth vs. Inflation suffers from short observation period again. Filtered data does not contain reasonable information so this time we made the Figure 37 from both, HP filtered and original datasets (only GDP and interest rates were filtered).

It is clear from observation, that stationarity assumption in case of Inflation appears to be broken. We have to consider created model very carefully and result are not so trustworthy as before. Stationarity ADF[10] test for differentiated price level confirms our observation. There was quite strong disinflation in this relatively short period of time. However, with

[10]Augmented-Dickey-Fuller Unit Root Test

outcome from other countries, we can make an assumption that price level is stationary here in long term as well. It is appealed to see the results, even though they are not very credible. Granger causality had high p-value in both directions. Impulse response functions were insignificant as well. However, we can once again observe slight reaction of Inflation in the Figure 38. Although IRF is insignificant and confidence interval contains 0 along all x values, the shape of function is similar to previously presented cases in other countries. Minor positive reaction after lag of 5 values is noticeable.

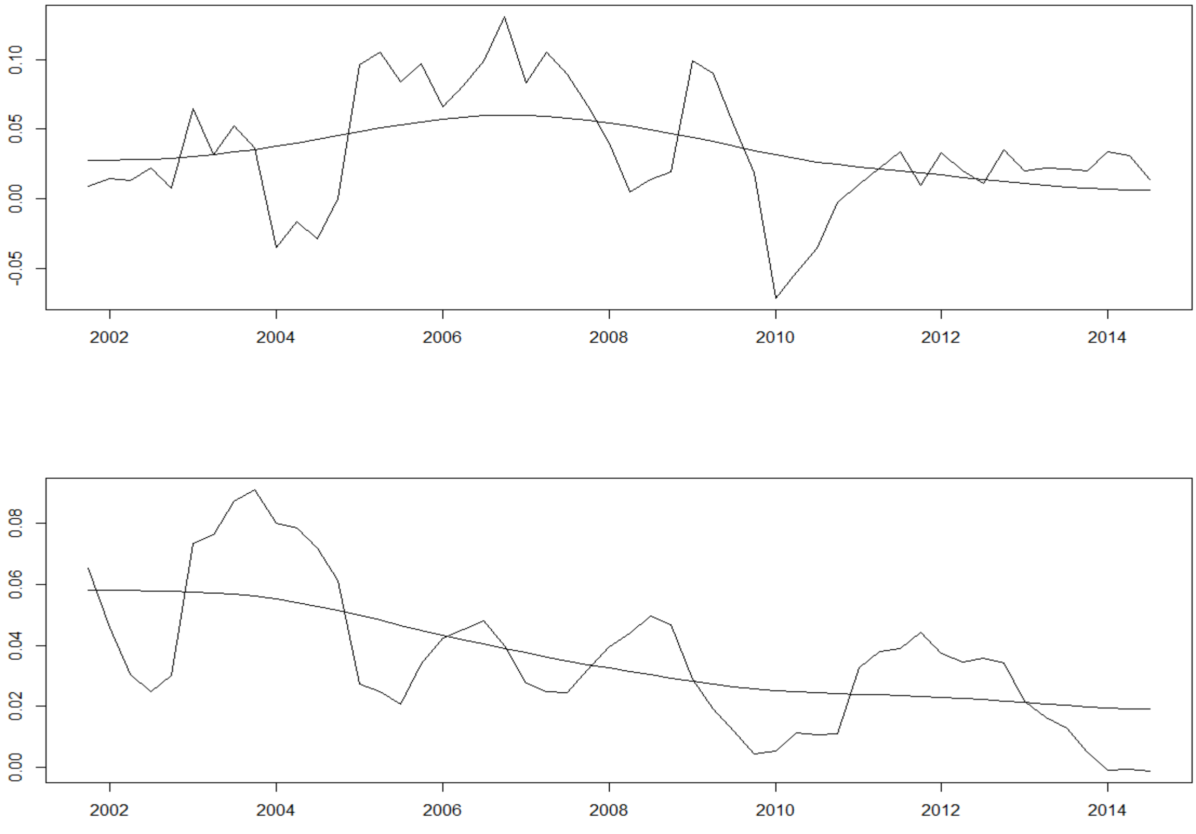

Figure 37 - - Inflation vs. M2 Net Money growth, HP filtered, Slovakia data 2000-2014

To round up a picture for Slovakia, we tried second approach with Excess Liquidity again. Figure 39 represents fitted NM level together with Price level. Inflation is drawn in the bottom part of same figure. Outcome is not so contradicting and is analogous to other datasets. For example, we can see NM level turning negative in beginning of period and consequent decrease in inflation year after. Growth in NM (even though EL is still negative) is reflected in higher inflation around 2006. Moreover NM peak and positive EL in 2009 can be reflected in Inflation growth from 2011.

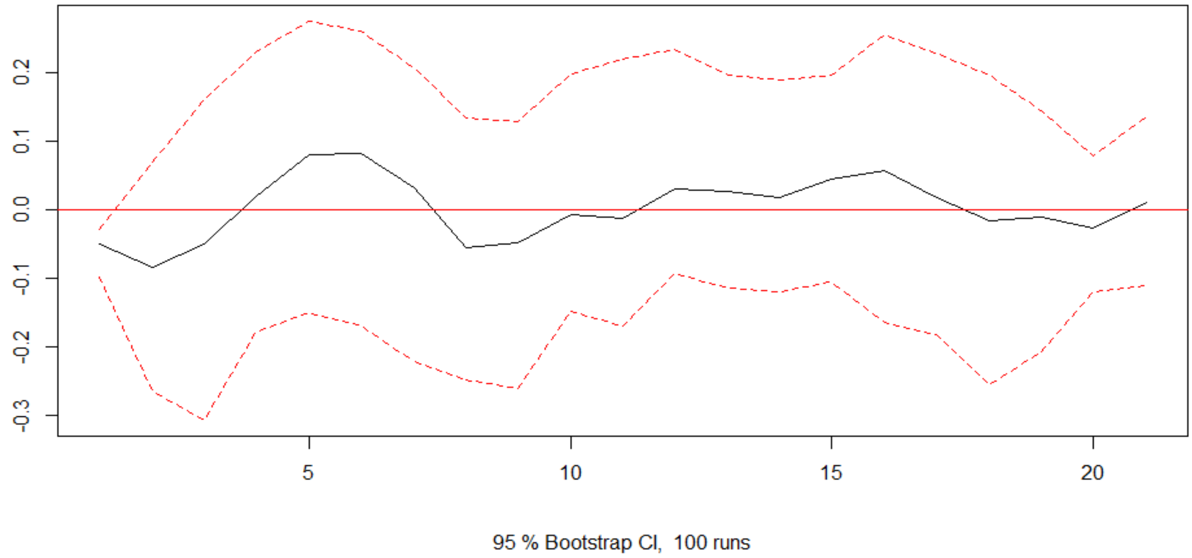

Figure 38 - Impulse response of Inflation from NM growth shock

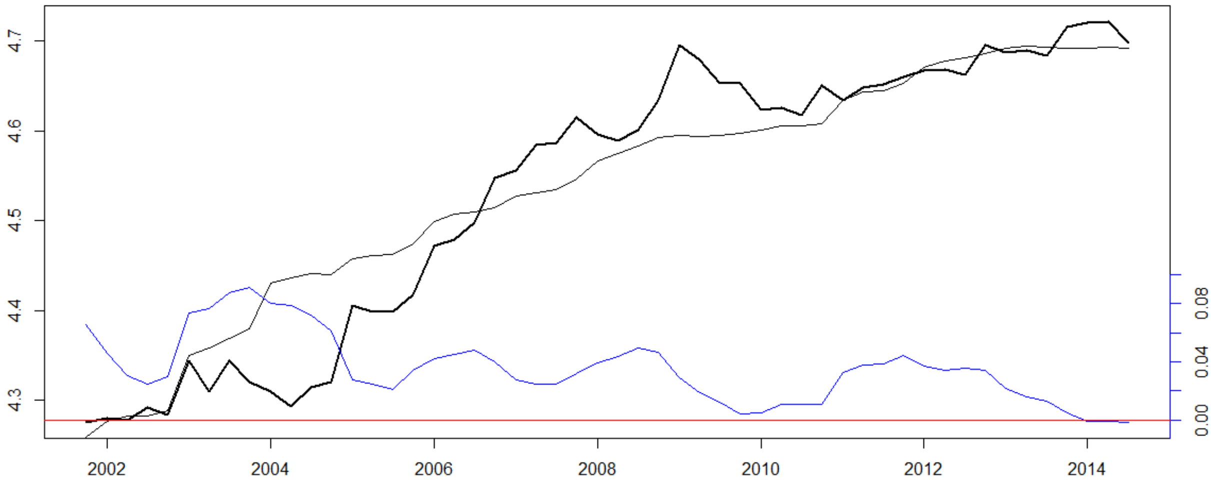

Figure 39 - EL (forward shift, lhs) vs. Inflation (rhs)

Conclusion in case of Slovakia, taking into account previous results is following. Due to short time period and several external shocks, analysis ended up to be unstable and led to insignificant results. On the other hand, slight evidence for our model was still observable. If we extrapolate result from other countries, we can say that there can be a reaction of Inflation on NM shocks. Time lag is probably shorter this time. Observed dependency have approximately 1-2 year delay. Since we have short observation period, we cannot tell exactly.

Conclusion

Taking into account all results presented before, we are able to derive several general findings. Study was mainly focused on Europe and the US. Study started with the United States in order to see the impact of QE and stimulative financial programs introduced in this economy. Then we analysed several European countries separately, to see the effect of individual monetary policies and behaviour of Quantity Theory of Money in different environments. Final outcomes are more or less equal across all analysed countries.

In details, we can apply Quantity Theory of Money considering interest rates as velocity. In bigger entities, it seems to be more reliable to use short term interest rates. On the other hand, in smaller economies, long-term rates are used. However in practise, interest rates of all types are usually strongly correlated so it did not cause big difference in whole process whether we used short term or long term rates. Subsequently, observed velocity, computed from Nominal GDP divided by Money level, can be estimated by equilibrium interest rates and holds very well. Determination of linear model measured by R^2 was always above 0.9 with income elasticity estimate, and between 0.5 - 0.8 when we restricted income elasticity for unity. Therefore, we argue that Quantity Theory of Money still holds today. Essential idea of contradicting theories is that velocity is not constant (or constantly increasing) and prices are affected by other shocks and are sticky in short term. Basically our research proved that velocity is not constant. In addition, our research proved more. Velocity can be observed and measured by interest rates. Equation $M * V = P * Y$ holds, even in short term.

On the other hand, it is a bit harder to deal with sticky prices. Introduced model basically assumes, that the difference between Net Money level (part $M * V/Y$ from equation) and Price level is some shock with zero mean. They should be stationary in long term, so for example if Net Money grows, Price should react after some time. So shocks are auto correlated and can be handled by AR models. Across all countries, fitted vector AR models were rather insignificant but we discovered partial evidence for Price reaction. In case of NM growth, Inflation appears to be growing after 1.5-3 year lag in all datasets. Especially in the case of The Economic and Monetary Union and countries such as Germany, France, Italy, Spain or Slovakia was evidence for this reaction even bigger and more significant.

When one looks closer at reasons of instability, it can be observed that all our variables except Prices can decrease. It even happened several times (minor or major crises periods can be recognized from data). On the contrary, Price level is the main target in monetary policy and is undesirable to decrease. Moreover, we found that dominant part of Inflation is autocorrelated. In practice, Inflation expectations and observed historical values have serious impact on future Price growth. As the target is set to be positive, all reports and statements from national banks are making positive expectations. Therefore deflation is really rare and very low if occurs. In this part, simple observation of data leads to better results than analytical quantitative approach with regression. One can notice NM decrease followed by drop of Inflation several times in different datasets.

Our detailed study uncovered similar results as those in (Reynard, 2012). Assumption of velocity approximation with interest rates equilibrium movements has proven to be reasonable. However, Reynard argued for more stable VAR model. Predicting power of Money level on Inflation appears to be higher in his work.

In conclusion, we uncovered very interesting results. However, they are only partially significant in some countries. That is why our model should be used for qualitative purposes rather than quantitative ones. Particular and accurate prediction of inflation level cannot be done. On the other hand, we can say that NM growth (increase of equilibrium interest rates, monetary aggregate or decrease of potential GDP) usually affect inflation with lag of 1-3 years. In theory, if we observed interest rates decreasing towards 0 and stable growth of money and income, it would result in disinflation. Another example can be from crisis times. If we saw constant velocity with 0 interest rates and breakdown in GDP, stable or even higher money growth could be compensated only by Price increase. But how it really is in practice?

Considering current environment, QE appears to be stable without significant impact on Inflation. Firstly, we would expect effect on Money. But we showed in section 3.1 on example of the United States that impact of “Money printing” is small. M1 aggregate have noticeable change in trend since 2008. However impact on M2 aggregate is negligible. Trend is stable and growth did not change much. Similar result can be observed in other economies as well. We did not discover substantial impact on M2 in any of considered economies. Secondly, drop of velocity can be causing slightly lower Inflation and GDP growth. Overall situation appears to be sustainable with only small risk of Price increase in the US and Switzerland. Moreover, Inflation risk in other European countries seems to be negligible. On the other hand, rise in equilibrium velocity and/or stagnating GDP can be considered as potential risks to high Inflation in the future.

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