COMENIUS UNIVERSITY IN BRATISLAVA

FACULTY OF MATHEMATICS, PHISICS AND INFORMATICS

EXPERIMENTAL STUDY AND MATHEMATICAL MODELLING OF HUMAN WALKING WITH INTERACTIONS IN THE ABSENCE OF GEOGRAPHICAL CUES

MASTERS THESIS

2015 **Mário Miglierini**

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MASTER THESIS

Study programme:	Economic and financial mathematics
Study field:	1114 Applied mathematics
Department:	Department of applied mathematics and statistics
Supervisor:	Mgr. Katarína Boďová, PhD.

Bratislava 2015

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UNIVERZITA KOMENSKÉHO V BRATISLAVE
FAKULTA MATEMATIKY, FYZIKY A INFORMATIKY

EXPERIMENTÁLNA ŠTÚDIA A MATEMATICKÉ MODELOVANIE ĽUDSKEJ CHÔDZE DVOJÍC S INTERAKCIOU BEZ ORIENTAČNÝCH VNEMOV.

DIPLOMOVÁ PRÁCA

Študijný program:	Ekonomická a finančná matematika
Študijný odbor:	1114 Aplikovaná matematika
Školiace pracovisko:	Katedra aplikovanej matematiky a štatistiky
Vedúci práce:	Mgr. Katarína Boďová, PhD.

Bratislava 2015

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77600875

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ZADANIE ZÁVEREČNEJ PRÁCE

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Študijný program:	ekonomická a finančná matematika (Jednoodborové štúdium, magisterský II. st., denná forma)
Študijný odbor:	9.1.9. aplikovaná matematika
Typ záverečnej práce:	diplomová
Jazyk záverečnej práce:	slovenský

Názov: Experimentálna štúdia a matematické modelovanie ľudskej chôdze so vzájomnou komunikáciou bez orientálnych pomôcok. / *Experimental study and mathematical modelling of human walking with interactions in the absence of geographical cues.*

Cieľ: 1. Previesť dôkladné experimenty v teréne s použitím GPS na zaznamenanie trajektórii kráčajúcich párov. 2. Skúmať, ako vplýva interakcia medzi kráčajúcimi na ich trajektórie. 3. Na základe získaných výsledkov zostaviť matematicky model ľudskej chôdze s interakciou. 4. Zistiť za akých podmienok je spoločná chôdza výhodnejšia oproti chôdzi jednotlivcov.

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Dátum zadania:	29.01.2014

Dátum schválenia: 10.02.2014

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Acknowledgements

This thesis is made possible through the help of the company Geotronics Slovakia that lent us GPS devices for the purpose of this research. My special gratitude belongs to its employer Dominik Laufík who witness every experiment and who helped us with obtaining needful data and operating devices. My gratitude also belongs to the company Agropartner, which allowed us to conduct our first experiment on their estate. This thesis could not have been finish without measured volunteers, who were willing to undergo long and sometimes uncomfortable measurements. The same thanks is dedicated to members of the staff team who voluntary devoted their time and energy to data measuring. I would like to express my thanks for the help and support from Dominika Svetlíková who helped me with editing my thesis. Special thanks belongs to my supervisor Mgr. Katarína Boďová, PhD who leaded my research with her expert advices and suggestions. I would like also thank to my family and friends who supported me during writing the thesis.

Abstrakt v štátnom jazyku

MIGLIERINI, Mário*: Experimentálna štúdia a matematické modelovanie ľudskej chôdze dvojíc s interakciou bez orientačných vnemov.* [Diplomová práca], Univerzita Komenského v Bratislave, Fakulta matematiky, fyziky a informatiky, Katedra aplikovanej matematiky a štatistiky; školiteľ: Mgr. Katarína Boďová, PhD., Bratislava, 2015, 55s.

V našej práci študujeme chôdzu jednotlivcov a párov ľudí v teréne, ktorý sa snažia udržať priamy smer chôdze. Ľudia pohybujúci sa v teréne kde nie sú dobré možnosti orientácie sa väčšinou snažia udržať priamy smer. Cieľom práce je previesť dôkladnú experimentálnu štúdiu ľudskej chôdze za takýchto podmienok, spracovať získané dáta a vybudovať základný matematický model pre tento typ chôdze. Na základe odhadnutých parametrov pre modely jednotlivcov a dvojíc sa snažíme porovnať ich schopnosť udržať priamy smer chôdze. V práci študujeme stratégie dvojíc ako najmenších možných skupín ľudí. Z predošlého výskumu vyplýva a aj v našej práci sa ukázalo, že chôdza dvojice je silno ovplyvnená tým, či sa v nej nachádza dominantný člen, ktorý sa snaží udávať smer celej dvojice. V práci preto študujeme, či sa takýto člen v našich dvojiciach nachádzal a ako jeho prítomnosť ovplyvnila chôdzu dvojice. Dva samostatné experimenty boli vykonané na rozsiahlej rovnej lúke, kde boli meraní jednotlivci a dvojice. Meraní dobrovoľníci sa pohybovali so zaviazanými očami vo vopred určenom smere. Interakcia v dvojici bola tvorená elastickým lanom, ktoré držal každý z dvojice. Pomocou geodetických GPS prístrojov bola každú sekundu zmeraná poloha dobrovoľníka. Na základe získaných polôh v práci vypočítame uhľovú výchylku od pôvodného smeru a prejdenú vzdialenosť za sekundu. Pre tieto veličiny hľadáme model, ktorý by dobre vystihoval ich vývoj v čase. Ukázalo sa, že dáta a teda aj uhlové výchylky obsahujú veľké množstvo šumu, ktorý bol spôsobený hlavne pohybmi meracieho prístroja pri chôdzi meraných, nerovnosťami terénu a vysokou frekvenciou merania. Tento šum sa snažíme odstrániť vynechaním časti dát teda vypočítaním uhľových výchyliek za viac sekúnd. Na základe pozorovania viacsekundových výchyliek zistíme, že dáta sa dajú modelovať aj jednoduchším modelom, o ktorom sa ukáže, že nie je až tak ovplyvnený šumom. V poslednej časti práce uvádzame dva alternatívne prístupy k odstráneniu šumu ktoré nám dávajú možnosť odhadnúť pôvodný model. Prvým je modelovanie vzdialenejších (v zmysle poradia dát) uhlových výchyliek, než dvoch po sebe nasledujúcich. Druhým prístupom je použitie jednoduchého matematického filtra na zmerané pozície, čím sa odstráni veľké množstvo šumu ale aj variability uhľových výchyliek. Na základne porovnania koeficientov pre všetky tieto tri prístupy zistíme, že všetky sú použiteľné na spracovanie našich dát, pretože dávajú relatívne rovnaké výsledky, aj keď hodnoty parametrov nie sú rovnaké.

Kľúčové slová: Kruhové trajektórie, chôdza človeka, orientácia v priestore, uhlové výchylky, lineárny model, spracovanie šumu, matematické modelovanie

Abstrakt v cudzom jazyku

MIGLIERINI, Mário*: Experimental study and mathematical modelling of human walking with interactions in the absence of geographical cues*. [Master Thesis], Comenius University in Bratislava, Faculty of Mathematics, Physics and Informatics, Department of Applied Mathematics and Statistics; Supervisor: Mgr. Katarína Boďová, PhD., Bratislava, 2015, 55p.

In our work we study walking individuals and pairs who are trying to maintain the straight direction of walk. People who are walking in a terrain where orientation is difficult often tries just to walk in a straight direction. The aim of our work is to conduct an experimental study of walking in such conditions, process the acquired data and build a basic mathematical model for this type of walking. On the basis of the estimated parameters of the models for individuals and pairs trajectories, we are trying to compare their ability to keep straight direction while walking. In this work we study the strategy of pairs as the smallest possible group of people. The previous research shows that a walking pair is strongly affected by the presence of a dominant member who is trying to determine their direction. Therefore in this work we study whether such members in our pairs are and how their presence affects them. Two separate experiments were conducted on a large flat meadow where individuals and pairs were measured. Measured volunteers walked blindfolded in a predetermined direction. The interaction of the pair was established by an elastic rope which each pair was holding. Via geodetic GPS devices we measured their position each second. Based on the positions we calculate the angular deviation from the original direction and distance traveled per second. For these values we developed a mathematical model that describes their evolution in time well. It turned out that the data and thus the angle deviations contain a lot of noise. The noise could be produced mainly by moves of the measuring instrument head while measuring the walking volunteers, by rough terrains and high frequency of measuring. We are trying to remove the noise by deleting a part of the data and thus calculating angle deviations per more seconds. Based on the observation of angle deviations per several seconds we found that the data can be modeled by a simpler constant model which demonstrated to be not influenced by noise in such extent. We estimate parameters of the model and compare their values for individuals and pair. We study whether the parameter that represents systematical bias of an individual increased or decreased. In the last part of the work we present two alternative approaches to elimination of the noise which give us the opportunity to estimate the original model. The first is a simple mathematical filter. Each filtered position is calculated as a mean of three successive positions. In this approach we significantly reduce the noise as well as the real variability of the trajectory. Although the estimated parameters from this method have a similar to the first approach, we haven't proved that use of this method is legitimate. In the second approach we modelled dependence between angle deviations φ_{n+k} and φ_n instead of successive angles. From this models we are able to estimate the parameters of the original linear model from such From these models we were able to estimate.

Keywords: Circular trajectory, human gait, navigational ability, resampling, noise reduction, mathematical modeling, angle deviations

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Introduction

This thesis is studying people walking without orientation cues. It is a common belief that people walking in unfamiliar terrain in fog or during the night often end up walking in circles by which we understand curved intersecting trajectories and loops. The inability to walk straight was tested also in a commercial TV spot called Mythbusters [1]. They observed that people are unable to walk, swim or drive a car in a straight line while blindfolded. They haven't found an answer why people are unable to make such things. The scientific research about this phenomenon was made by Souman, et al. [2]. They let six participants walk in a large flat forest and instructed to walk as straight as possible in a given direction. Four of them walked on a cloudy day and all of them walked in the loops repeatedly crossing their trajectories without noticing. In contrast when the sun was visible the participants followed almost perfectly straight course. These results suggest that men need external directional reference to recalibrate their sense of direction [2]. With no such reference people often lose right direction which can produce mentioned curved intersecting trajectories.

Can more people in a group by mutual communication overcome this individual's inability to walk straight? Knowing an answer to this question can help people to make a decisions while they are lost in unfamiliar terrain whether to split up or find a way together. We tried to examine this in our previous work [3]. We conducted blindfolded experiments with six volunteers who were measured as individuals and as pairs. The measured volunteers (or pairs) stood in the middle of the 30m long corridor and were instructed to maintain a fixed course at normal walking speed. They checked the direction and then walked blindfolded until they left the corridor and the exit distance was measured afterwards. The comparison between individuals showed that performance of a pair in such a task depends on qualities of individuals that formed the pair. There are two main individual characteristics that influence the exit distance of a pair. First is the ability to maintain the fixed direction of each member of a pair as an individual. Second is whether there is a dominant member in the group or not. Without a dominant member a resulting exit distance of a pair is close to the average of exit distances of both individuals. If there is a dominant member the result of the pair is close the one of dominant individual. This "compromise and leadership" behavior was observed on the homing pigeons by D. Biro, et al [4]. They tracked trajectories of homing pigeons (both individuals and pairs) via GPS system. The pairs navigated more efficiently than did the individuals of which they were composed, even though leadership was not necessarily assumed by the more efficient bird. The goal of our work is to perform a field study with individuals and pairs in a flat terrain using GPS system to track the trajectories. This approach can provide more information than just the exit distance, from our experiments in 2013.

In addition to comparison of individuals and pairs the second objective of this thesis is to build a mathematical model that describes a walking of a group. The work includes data processing and model calibration. We have chosen to study a pair as the smallest possible group. Knowing strategy and behavior of pairs can be easily extended to the larger groups of people. If they can estimate their abilities correctly they can choose the two most efficient members to navigate the group. While building the model we use a concept of correlated random walk. This approach has been also used to model the trajectories of animals. P.M. Kareiva and N. Shigesada in their work [5] studied observations of *Pieris rapae* (cabbage white butterfly) flight and *Battus philenor* (pipe-vine swallowtail) crawling as a correlated random walk.

Correlated random walk is a motion where in each discrete time interval the subject deflects from the previous direction by a random angle with some probability of a given distribution and then moves by a fixed or random step. The average angle between the direction of a motion at certain step and previous step represents a bias [3]. Let f be a response function, i.e. the deterministic component of displacement function and g be the amplitude of a random component. Then ξ determines the randomness of the motion and has a selected probability distribution. Then the motion can be described by the difference equation (1):

$$\varphi_{\mathrm{n+1}} = f(\varphi_n) + \mathrm{g}(\varphi_n)\xi_{\mathrm{n}} \tag{1}$$

Different choices of functions *f, g* and probability distribution of ξ produces different trajectories and for each person a unique set of parameters could be estimated. J. Dzúrik [6] in his thesis showed that simple linear and constant functions *f* and *g* produce similar trajectories to those which were measured by Souman et al. [2]. Under these assumptions M. Jánoši has studied data from work [2] in his thesis [7]. He tested several extensions of models from J.Dzúrik’s [6] work and found that the simplest linear model with a constant magnitude of noise describes the data very precisely. In our work we will estimate the parameters using the class of linear functions. We will test whether the data behave according to the same model as data from in [2][7]. During the estimation procedure we find that there is a strong noise in the data. The last part of the work is devoted to three approaches how to deal with such a problem. In that part only trajectories of individuals are analyzed. Firs approach is that we find the model that is not so strongly affected by such type of noise. Second is that we use a mathematical filter to smooth data and the third is that we analyze at angle deviations *k* steps away instead of successive as it is in equation (1). The comparison of the results of the all three approaches shows that they provide same relative results. It follows that they are all usable to analyze our data.

1 Experimental part

1.1.1 Theoretical introduction to experiment design

A primary goal of our experiments was to obtain detailed trajectories of individuals and pairs with communication. Spatial navigation is a complex process that is affected by many factors. Therefore designing such an experiment is extremely difficult and it is still impossible to exclude all external factors. Some of them can be eliminated by conducting experiments indoors. This was made in our previous experiments [3] in 2013. Also Souman et al. [8],[9] developed omnidirectional treadmill to allow people to walk in a virtual environment. Problem with blindfolded walking experiments conducted indoors is that people have a good perceptions of walls and volunteers can also feel a fear of colliding with them while walking blindfolded which result in unwanted slowing in the motion as observed in our previous experiments [3]. This chapter will first provide the overview of factors affecting our navigational ability and summarize previous experiments made by Souman et al. [2] and M. Miglierini [3].

Orientation in space is often based on external sources of information such as maps and GPS, so in our experiments volunteers were not allowed to use them. Without navigational equipment people should rely on their senses. The most useful senses to orientation are evidently vision and hearing, but there are other perceptions that can be used for estimation and recalibration of position and direction. For the visual orientation people often use salient landmarks and also visibility of the sun (or moon) can be used as shown in the experiments of Souman *et al.* [2]. Souman *et al.* let 6 volunteers to walk in a vast forest, without significant landmark for several hours. Their task was to walk as straight as possible in a specified direction without any use of navigational equipment. Four individuals walked during a cloudy day. They walked in loops, repeatedly crossing their trajectories without noticing it. On the contrary, the individuals who were walking on a sunny day were able to almost accurately maintain the course. These results suggest that the availability of a reliable external source of information about the direction of locomotion is crucial for maintaining one's course through unfamiliar terrain [2]. Souman *et al.* [2] conducted the same experiments at the Sahara desert and obtain the similar results. The two participants who walked during the heat of the day veered from the direction of motion that they were instructed to follow but did not walked in circular trajectories. The third participant walked during the night with the full moon initially visible. After the moon disappeared behind the clouds, he made several sharp turns, bringing him back in the direction from which he came [2]. To prevent the visual orientation during our experiment done in 2013, our volunteers checked the proper direction and then walked blindfolded. Souman *et al.* in their work [2] did also blindfolded experiments where they observed that volunteers are able to maintain fixed course for a short time (about 20m) but in the longer walks their trajectories are highly random. Three out of 15 volunteers showed a strong bias in one side which resulted in walking in circles for the most of the time.

Even though the participants were blindfolded the sun could give a sense of direction if they felt it on their skin. It was impossible to completely avoid this effect. Participants were supposed to wore a hat during the experiments to prevent feeling the sun at least on their heads and faces.

Hearing cannot provide the same quality of information (in terms of orientation) as the sight [10]. Nevertheless, by hearing one is able to obtain information on the nature of the space in which he is present [11]. Moreover if there is a permanent source of noise coming from one side, people are able to use it to estimate their position even if they are blindfolded. For this reason participants wore headphones with music to be soundproof during the individual measurements in 2013 [3].

Ability to find out how far and which direction is the sound coming from, was the key factor in experiments of pairs in 2013. The participants were acquiring the information where their partner is situated by hearing. Participants were equipped with a beeping device. Both volunteers of a pair held a mobile phone in their hand. The phone played (at a loud voice) MP3 recording consisting of a sequence of 0.5s of 1 kHz (sine) tone and then 2.5s of silence throughout the whole experiment. Signal "beep" was played asynchronously to allow participants to estimate the approximate position of their partner on the basis of auditory orientation in space [3]. Problem with communication based on hearing was, that it was impossible to make participants soundproof during the experiments of pairs same way that it was in the experiments of individuals. They wore headphones but without music to be less effected by ambient sounds.

Bredin *et al.* [12] in their work tested how accurately a blindfolded person can reach a target at a distance of 10 meters. Experiments were carried out at three different walking speeds. At different speeds, different results were obtained, indicating that a person's ability to maintain a fixed course, without visual orientation, is also affected by the moving speed [3],[7]. This indicates that walkers may be greatly affected by a moment of inertia, therefore an individual who is moving fast had a greater momentum and therefore a smaller tendency to deviate from the direction of motion. The concept of moment of inertia was also used in a thesis of M. Jánoši [7] as an assumption of the mathematical model. In order to reduce the impact of speed in experiments of Souman et al. [2] and M. Miglierini in 2013 [3], participants were asked to walk at their natural walking speed and if one of the participants visibly accelerated it was noticed as an unusual situation.

One possible explanation of the circular trajectories in human motion are the biomechanical asymmetries of the human body such as different leg length or strength. In studies of Souman *et al.* [2] this issue was examined in a separate experiment. Fifteen volunteers have followed a given direction without the possibility of auditory and visual orientation for 50 min. Measured individuals either walked 10 periods of length 5min, or 5 periods of length 10min. Their trajectories were recorded by GPS system. Three (out of 15) participants showed a strong bias to one side, which resulted in walking in circular trajectories. Effect of biomechanical asymmetries of the human body could not be excluded from our measurements.

It is clear that that people feel the gradient of slope and thus can orientate by it, so experiments should be carried out on the flat plane. Also a weather, especially a constant wind (or sun) from one side wind can help to recalibrate the sense of direction. To prevent the impact of this factors, our previous experiments in 2013 [3] were conducted indoors on the experiment area which was 33m long. Problem of indoor experiment is that it is impossible to find an indoor area which is large enough to conduct longer experiments both in terms of space and time. We have decided to carry out experiments outdoors and searched for flat plane.

1.2 Experiment design

The experiment was designed with respect to all the knowledge and experience we had from previous experiments and also with respect to our time and financial possibilities. There were several issues that should be resolved during the experiment design.

1. Exclude external factors that affect the walking as much as possible.
2. How to establish the communication in the pair.
3. How to place GPS devices on volunteers to affect them as little as possible.

1.2.1 Communication in pairs

There were two main ways how one can establish communication in pairs. First was to use the same concept that was used in our previous work where communication in pair was based on hearing “beeping” device that both volunteers in pair wore. Volunteers were able to estimate positions of their partners and decide to follow them or increase or decrease distance from each other. As it was stated in previous chapter there was a risk that volunteers would hear ambient voices during the experiment. Since the experiments were carried out indoors it was possible to minimize this risk. In this work we wanted to conduct experiments outdoors where the noises of environment are stronger. Therefore we decided for the second option. During our experiments the communication was provided by elastic rope. The elastic rope was 2,5m long with diameter of 6mm, each participant held one end of the rope. Maximum possible stretch of the rope was 5,5m. The volunteers felt the direction where their partners were and also could estimate distances between them by increased tension as the distance between participants grew.

1.2.2 GPS devices

We used the geodetic GNSS devices of type Trimble R4-3 and Trimble R6-4 to track the measured volunteers. The devices were lend from the company Geotronics Slovakia. The trajectory was measured in S-JTSK03 JTSK system to gain output in X-Y-Z coordinates instead of latitude and longitude. The coordinates were translated after measurements so that each trajectory began in point [0 0 0]. Each of the devices consisted of two parts. The GNSS receiver and control device are showed on the figure 1.

Fig. 1. GPS devices

This figure shows the device Trimble R4-3 used during the experiments. It consists of two parts the GNSS receiver and a control device. The GNSS receiver should be placed on the measured volunteer. The control device was operated by a member of staff team.

The receiver is receiving signal from satellites to calculate its accurate position. The positions are sent to the control device via bluetooth so the staff team can follow participant at distance less than 30m and he could see the results instantly. Since the receiver is a bigger device and shouldn't be surrounded by any other objects we needed to place the receiver so that it would not affect the results of experiments. Before the main experiment we conducted device-testing experiment to test the best placement of the receiver. We observed that it is impossible for a participant to hold receiver in their hand because it is too heavy (1.5-2.5kg). Also we found that holding such the device in the hand biases the participant from a straight direction. Based on this we used following method to track the trajectory of volunteers.

The measured individuals wore a large bag well stabilized on their back. The GNSS signal receiver was on the metal stand that was fixed in the bag. We tested that the head of the measured volunteer does not hinder the GNSS signal during the device-testing experiment. Since the receiver sent measured data to a controller device instantly via bluetooth, we were able to match the current situation with data points during the experiments. This was helpful to record all unusual situations because experimenters saw the current sequence number and just wrote a comment about what happened. Completely equipped volunteer is pictured on the figure 2:

Fig. 2. Equipped volunteer

The volunteer with hat, headphones, scarf and bag with GNSS receiver.

1.3 First experiment

1.3.1 Description of the experiment

Six volunteers aged 22-24 years who agreed to participate in the experiment were measured. They were the same volunteers who were measured in our previous experiments in 2013. Four of them were men (A, C, E, F) and two were women (B, D). All of them were right-handed. The experiment was carried out on June 8^{th}, 2014 from 9:15 am to 5:00 pm on the field near Sološnica (Slovakia) with GPS coordinates: 48.4952128 and 17.2100124. The area looked like an equidistant triangle with a side of length 3km. The area was a flat field and the grass was cut one week before experiment so it was easy to walk on it. In some areas the grass was damaged by wild boars resulting in the rough surfaces with holes and lumps of earth. If volunteer entered such a location it was recorded. This happened often since small damaged areas were evenly distributed on the field. In the middle of the field there was a small building and some trees around it. There was almost no wind during the experiment but the sky was clear and it was extremely hot (32°C in shadow) so volunteers wore a hat to at least partially avoid the influence of the sun.

Before the experiment started we had performed a test with each participant to obtain his speed and average step length. Each volunteer walked straight (not blindfolded) for 60s. His trajectory was monitored by GPS system (to obtain the traveled distance) and the steps were counted.

The actual experiment was conducted in a way as in works [2] and [3]. Each measured volunteer was instructed to maintain a fixed direction and walk at his/her normal speed. One experimenter showed him a proper direction. The starting point and the direction changed in each measurement. The measured volunteer put a scarf on his eyes and headphones on his ears. Then he had to wait for a signal (pat on the shoulder), which came 10 seconds after he had put a scarf on his eyes. To avoid auditory orientation there was a classical music played in the headphones (Georg Friedrich: Händel Concerto grosso Op 6 No 11 and Music for the Royal Fireworks). The participant did not hear anything. One experimenter followed the participant at small distance during the experiment to maintain the bluetooth signal to recorded unusual situations and to ensure the participant’s safety. The experiment finished when the specified time passed or when the participant reached a border of the measured area. The participant was stopped by an experimenter who held his hand and led him in a random way for 10-20s. This way we achieved that participant didn't know the result from the finished experiment. Before starting next experiment one member of the stuff checked if the bag is properly fixed on the participant's back. Two same experiments carried out at the same time with second stuff team. There was no intersection between this two teams since the area was large enough.

The aim of the experiment of individuals was to measure three 6min trajectories and six 3min trajectories. Due to the extremely hot weather we had to finish the experiment before all the trajectories were measured for all individuals. Some trajectories were incomplete, because the devices run of batteries or they lost the bluetooth signal. The table 1 shows number of proper experiments per individual.

	3min exp.	6min exp.	Overall time
A	6	3	2686s
B	4	5	2853s
C	4	1	1097s
D	2	3	1540s
E	4	2	1615s
F	6	3	2230s

Tab. 1. Number of individual experiments

The table shows number of valid experiments and number of valid time points used for computing angle deviations (See chapter 2.2.1. Angle deviations).

We conducted also eight experiments of pairs during the first experiment. In contrast to the experiments of pairs the experiment of individuals always started at the same location with the same desired direction. The individuals were equipped the same way during the experiments of pairs. Communication between two measured was based on an elastic rope. Each of the measured volunteers hold one end of the rope and felt when the distance between them grew. The rope was 2.5m long in relaxed state and the maximal stretch of the rope was 5.5m. During our previous measurements in 2013 we found that it can affect the decisions in the pair when members know the results of their partner. They can consider the partner as "better in walking straight" and just follow him. We tried to avoid this by following procedure. At the beginning of each experiment all volunteers were asked to come near to the starting location and close their eyes. They were not allowed to open their eyes until the beginning of the experiment. Stuff team took two of them and leaded them to the starting line of the experiment. When they were fully equipped (with hat, headphones, scarf and bag with GNSS receiver) both got one end of the elastic rope. Afterwards two experimenters stood between them in a close distance and put their scarf down to show them the proper direction of walk (always the same direction) so the volunteers did not see each other while checking the direction of walk. This procedure was too time-consuming so the last two measurements were not measured this way. After the experiment ended both participants were disorientated the same way as it was during the individual's experiments.

All the experiments of pairs were carried out during the afternoon. Due to the hot weather the volunteers were more tired than during the measurements of individuals so we measured just one trajectory with the nine pairs. The table 2 shows lengths of experiments (in terms of time) for all the measured pairs.

Exp. Time [s]	202	206	236	239	251	199	173	190	118
Participants	CF	DF	EA	BD	BC	CA	AD	BD	EF

Tab. 2. First experiment of pairs
The table shows number of valid experiments and number of valid time points used for computing angle deviations (See chapter 4. Angle deviations).

In this part we examine impact of the surface to the speed of individuals. We will compare results obtained indoors in 2013 to the results from this experiment. The speed of pairs are compared in the section 1.4.2. table 5.

Data are in m/s	A	B	C	D	E	F
2013, 1st exp., initial test	1,19	1,26	1,50	1,08	1,29	1,12
2013, 2nd exp., initial test	1,43	1,25	1,56	1,17	1,21	n/a
2013 Experiment of individuals	1,039	1,115	1,308	1,054	1,067	1,120
2014, 1st exp., initial test	1,223	0,910	1,535	1,187	1,415	1,192
2014, Experiment of individuals	1,201	0,941	1,277	1,066	1,670	1,097

Tab. 3. Individual's speeds
The table shows speed of measured individuals. First two rows are measurements from the initial tests (not blindfolded) from the experiments in 2013. Third row is speed during blindfolded measurements in 2013. Other rows are displaying results from our experiments that we conducted for the purpose of this thesis.

The phenomenon that people walk slower while blindfolded has been observed in several studies (Bredin *et al.* [12], Nico *et al.* [13] and Glauser *el al.* [14]). In our work this was observed for four of our volunteers (A, C, D and F) but two walked faster during the experiments (B and E).

1.4 Second experiment

1.4.1 Description of the experiment

Second experiment was necessary because we did not have enough measurements of pairs from the first. The experiment was designed the same way as the first experiment but there were some small differences. Measurements were carried out on another place than the first experiment. They were conducted on a large cut wheat field near Bratislava GPS: 48.110527, 17.226347. The whole experiment were recorded by the mobile phone camera and unusual situations were commented during the recording. Volunteers have did know who their partners are during the measurements, since procedure to avoid it took too long time. The volunteer B was not present on the measurement. We measured 6 pairs each two times per 120 seconds. The table 4 shows more information:

Exp. Time [s] 1st measurement	129	123	122	121	130	127
Exp. Time [s] 2nd measurement	121	122	122	121	130	272
Participants	AF	EF	AD	AE	CE	CD

Tab. 4. Second experiment of pairs
The table shows number of valid experiments and number of valid time points used for computing angle deviations (See chapter 4. Angle deviations).

The experiment took place at 16:00 to 20:00 to avoid the strong sun around the noon. There was almost no wind during the experiment and the sky was almost clear. The sun shone from the behind of the starting line of measurements. All experiments started from the same location with the same target that was an electricity pole 304m away. Walking on the cut wheat field was more difficult than on the grass but there were no holes and lumps of earth so the volunteers could walk more fluently.

1.4.2 Strategy of pairs and unusual situations

If we look on the individual's strategy in pair, there are two main patterns. If a participant is confident that his direction is right and that his partner is heading wrong direction s/he tries to change it by pulling the rope. We call this state dominant. Second state is when a volunteer is following the direction of his partner and changes his own when his partner pulls the rope. We call this state following. Sometimes participants changed their behavior from dominant to following and vice versa during the measurement. The two triggers of the change from following to dominant were observed.

First is when one participant started to feel that his or her partner is varying from the direction which s/he subjectively considered correct. Second trigger is reaction to the tension of the partner. It was observed that when one of the volunteers started to pull the rope too often the second also started to enforce his/her direction. In this point dominance of one participant prevailed or one of the participants dropped the rope. This happened twice during the first experiment (BC, BD) and the measurements were terminated and restarted.

The dominance and overall bias of the pair is strongly affected by the speed of individuals in pair. Since volunteers in pair were connected by the rope, when one of them started to walk faster the tension of the rope increased. It caused two things. The faster volunteer was strongly deviated from his previous direction. The slower participant was pulled by the rope that had taken him/her the opportunity to continue in his previous direction and forced him to speed up. This acceleration caused that volunteers did not intersect their trajectory but made a great loop. Such situations occurred four times during the experiments for pairs AC and CF where the volunteer C was the faster one and for pair BE where the volunteer E was the faster one. This observation is visible on the figure 5 chapter 2.1. In this situation the faster volunteer is considered as dominant since such volunteer is leading the direction of pair. The trajectory of BC seems to look similar as AC but in that pair the B was dominant member. It is clearly visible from a video record. This observations indicated that speed is important factor that affect the walking, especially the trajectories of pairs. Speeds of both individuals and pairs are on the table 5.

Pairs	Speed volunteer 1	Speed volunteer 2	Individuals	Speed
CE	1.302	1.311	A	1.201
AE	1.255	1.287	B	0.941
BC	1.161	**1.274**	C	1.277
EF	1.254	1.214	D	1.066
CF	1.246	1.212	E	1.670
BE	1.159	**1.228**	F	1.097
CD	1.176	1.171		
AC	1.061	**1.129**		
BD	1.070	1.083		
AD	1.077	1.076		
AF	1.066	1.067		
DF	1.052	1.035		

Tab. 5. Comparison of speed of individuals and pairs

The table shows a comparison between speeds of individuals and pairs. In the left part there are speeds for pairs and the speeds for individuals are in the right part. The data were measured with frequency 1s so the step length (in meters) is also a speed in m/s. If there is relatively high difference between individuals and pairs the higher value is written in boldface.

This section is about some unusual situations which occurred during the experiments and interesting observations in strategies of some pairs.

The pair DF showed that cooperation in a pair can allow the pair to walk in a straight line even when each individual has a different speed. The volunteer F knew that D has a good ability to walking in a straight direction as individual. F walked much faster, passed D and continued at the straight direction. When the distance between D and F increased F felt tension of the rope from behind. F stopped and waited until they changed positions. So D was leading and F was walking behind D. D shortened the rope to have a better contact. Since F walked and showed strong bias to the left he passed D in a short time and started to pull D to the left. D did not deviated from the straight direction and forced F to change direction. For this reason we assume D as a dominant member despite the strong cooperation.

During one experiment of EF both participants wanted to draw their partner to their side. Volunteer F was stronger and forced E to follow his direction which resulted in strong bias to the left side. The same thing occurred during the experiment of AE. The A forced his direction which resulted in a sharp turn. It is visible on the figure 5 in the middle of the trajectory with the strongest bias to the right. During that experiment the maximum stretch of the rope was achieved.

The target location was reached only during the last experiment of C and D. During this measurement we found that volunteers are walking exceptionally straight so we extended the regular experiment time and surprisingly they walked the whole time almost without bias and came to the electricity pole that was the target.

During one experiment of AE an ambulance car passed by the near road. The volunteer heard the voice of the siren but could not determine the direction of the voice due to the music in the headphones.

From videos obtained during the experiments we can study which volunteers determined the direction of a pair. The dominance is not a relation of equivalence since it violates the transitivity (for example for pairs BC BE and EC). Thus we can’t sort participants by dominance but we can plot the dominance graph.

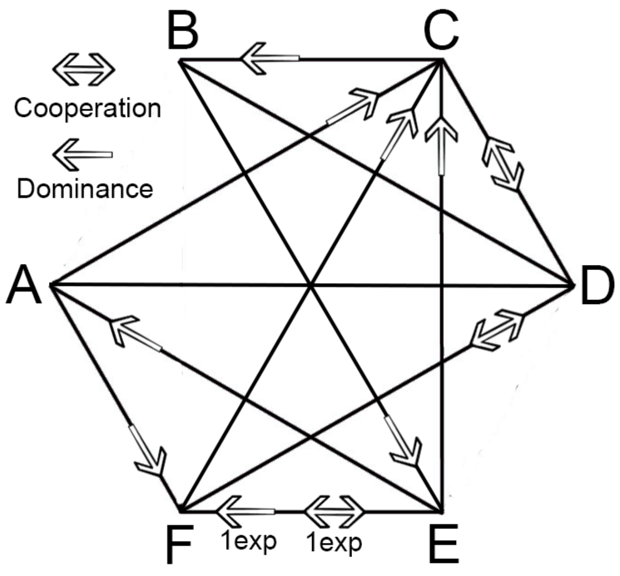

Fig. 3. Graph of dominances

The figure shows all experiments that we have done with pairs during the first and the second experiment.

It was impossible to judge the dominance for each pair because we have low number of measurements. Despite the dominance being the important factor that affects the trajectories of pairs it is not included in our model for trajectories. Development of the model that includes the dominance thus remains for the further research.

2 Data processing and model estimation

2.1 Data visualization

While visualizing data from experiments of individuals we connected all the measurements for each participant into one trajectory. Results are on the figure 4.

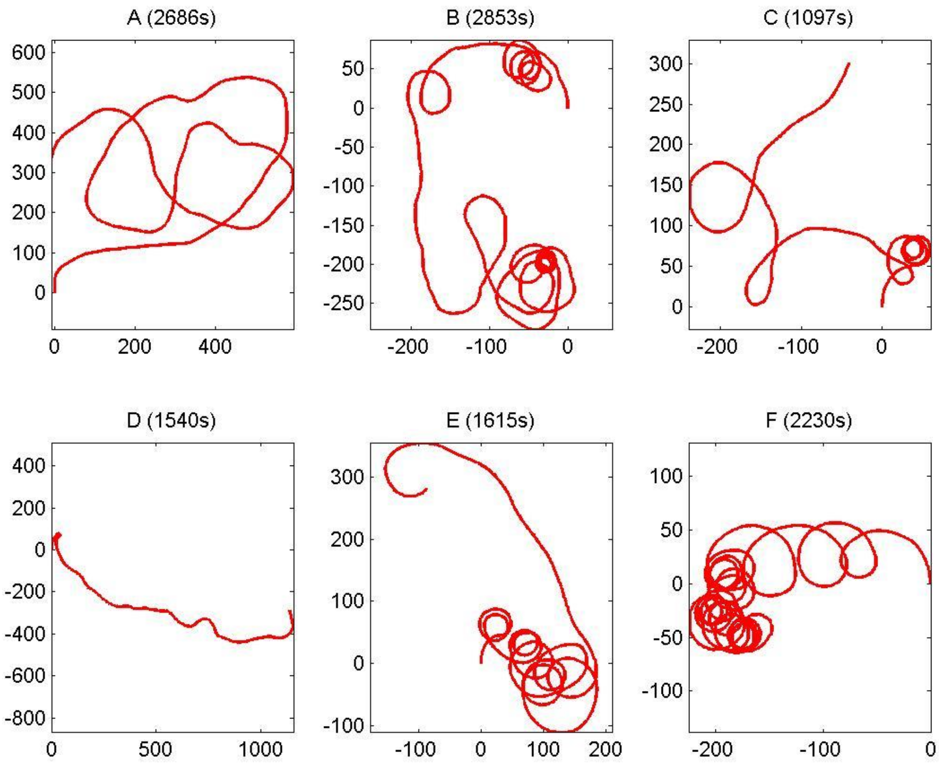

Fig. 4. Trajectories of individuals

Trajectories of six measured volunteers. All the experiments of each participant are connected into one trajectory. Distances are in meters. All trajectories are rotated so the initial direction is parallel to the y axis and they begin at point [0, 0].

The trajectories are translated and rotated so the trajectory of one individual begins at the point [0, 0] and the initial direction is parallel to the y axis.

Trajectories for one pair are displayed on one picture. The trajectories are rotated so that one member of pair begins the trajectory at point [0, 0] with initial direction parallel to the x-axis.

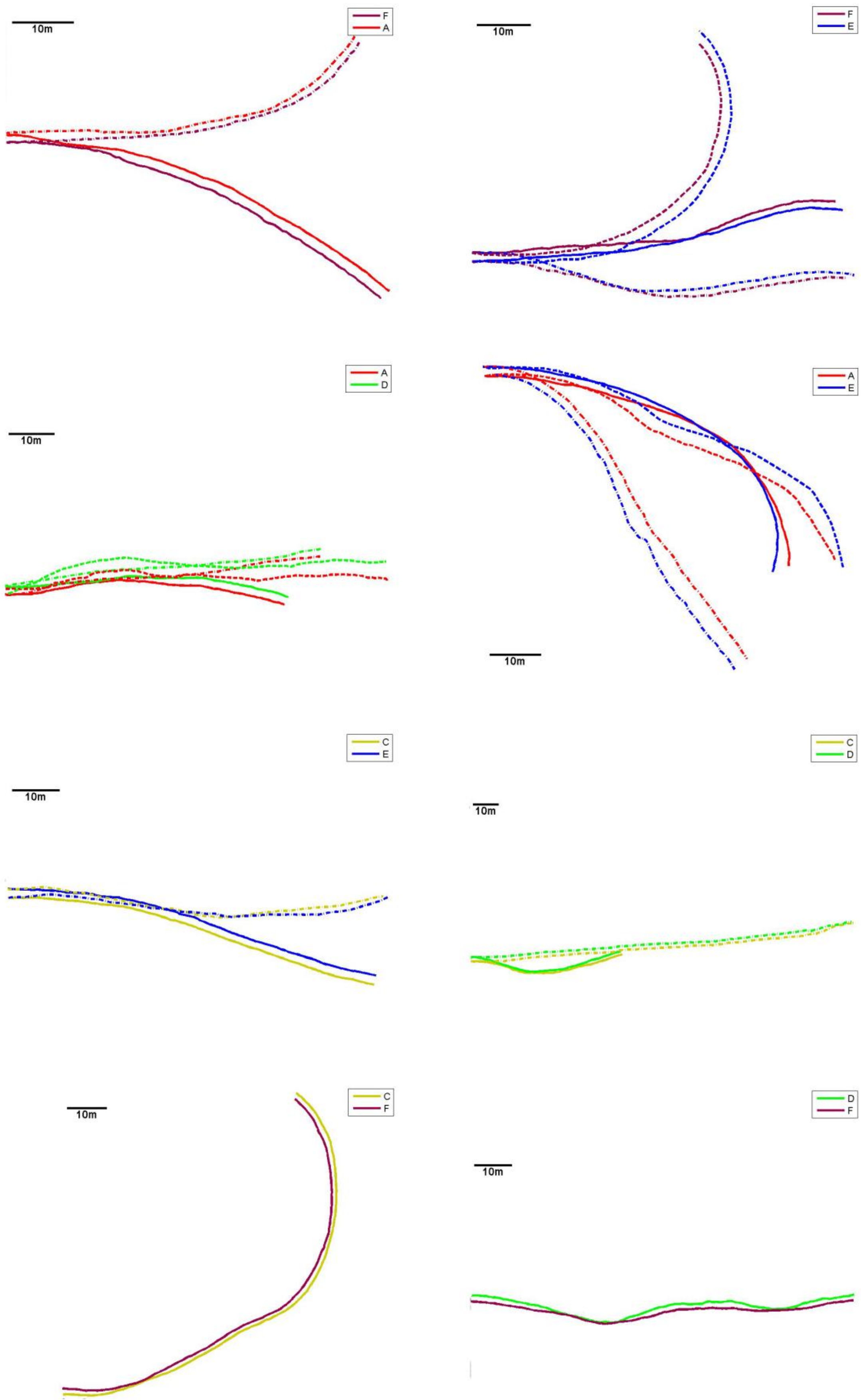
10m
F
A
10m
F
E
A
D
10m
A
E
10m
C
E
10m
C
D
10m
10m
C
F
D
F
10m

Fig. 5. Trajectories of pairs

Trajectories of measured pairs. The 12 different pairs were measured. All experiments of one pair are on the same plot. We measured from one (CF, DF, BC, AC, BE, BD) to three (EF, AE, AD) trajectories with single pair during two experiments.

Speed of volunteers affected their ability to walk straight direction. We calculated traveled distances per one second which is in fact speed for both individuals and pairs. The histograms of the traveled distances per seconds are showed on the figure pictures. The low values of volunteers B and D appears because both individuals had one measurement where they walked exceptionally slow. The step lengths are studied in the chapter 1.4.2 table 5.

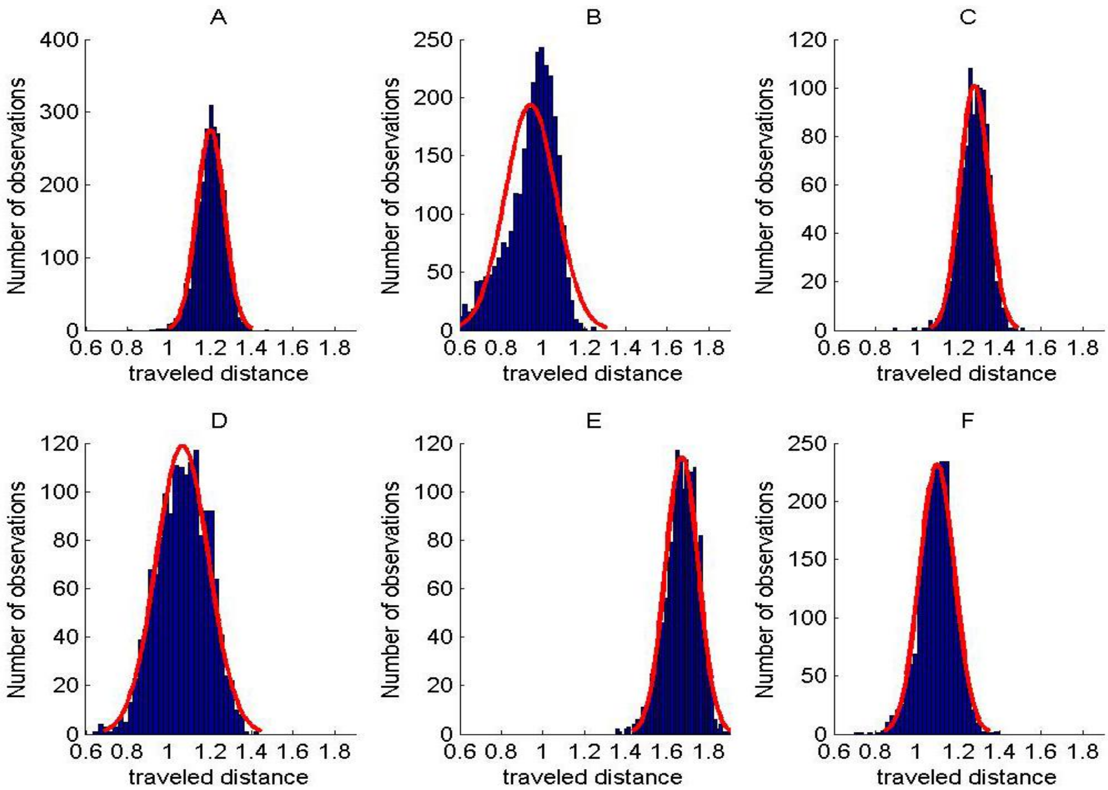

Fig. 6. Histograms of traveled distances per second individuals

The figure shows histograms of traveled distances for all 6 individuals. Red curve is a fit of normal distribution into the histogram. The mean values of step lengths are summarized in the chapter 1.4.2. table 5.

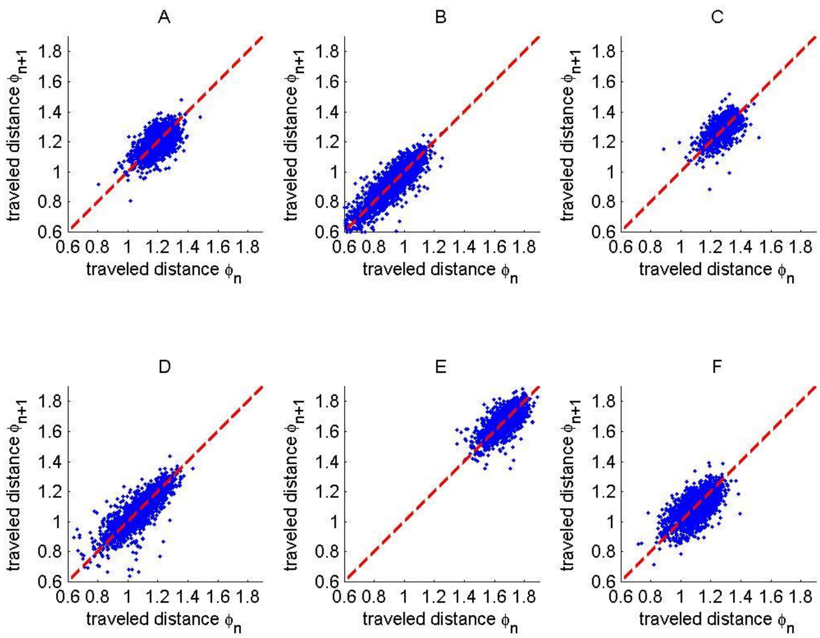

Fig. 7. Scatter plots of traveled distances per second individuals

Scatter plots of angle deviations for all six measured volunteers. Figure displays d_n against d_{n+1}. The rho is a correlation coefficient. The scale is in m on the both axes.

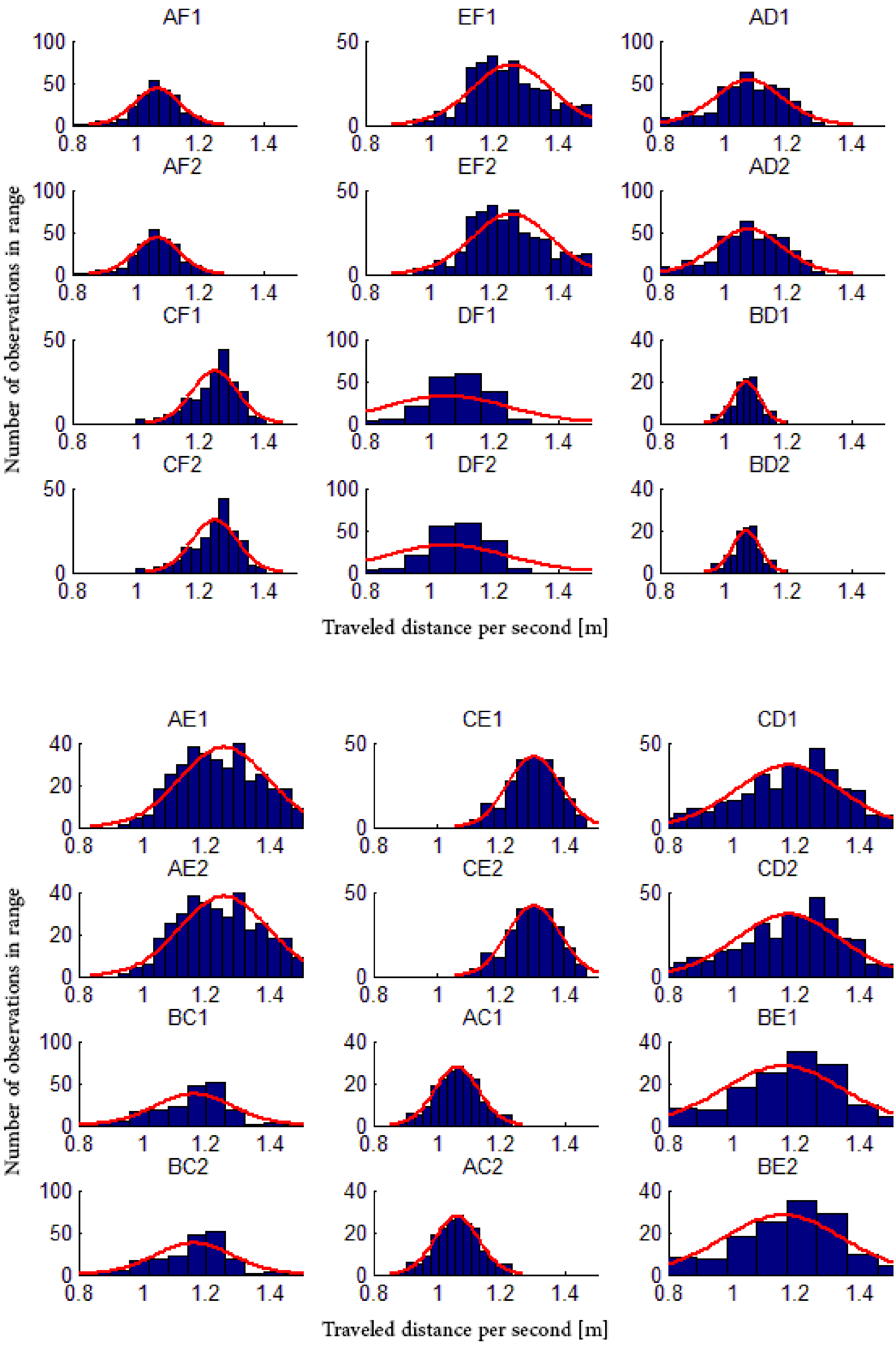

Fig. 8. Histograms of traveled distances per second pairs

There are two histograms for each of 12 pairs. Red curve is a fit of normal distribution into the histogram. The mean values of step lengths are summarized in the chapter 1.4.2 table 5.

2.2 First approach

2.2.1 Angle deviations

Studying of data is focused on processing angle deviations and traveled distances per second. Angle deviation is a change in volunteer's direction in one second. Output from device is the position of the volunteer in each second in X-Y-Z coordinates. In this part we are not analyzing the third coordinate (height) and considering just X-Y coordinates. From them we are calculating angle deviations and traveled distances. Let $V_i = [x_i\, y_i\,]$ be the measured position in time *i*. Let $d_i$be the distance traveled during the time interval *i* and *i+1*. To calculate the angle deviation φ we need three positions of the volunteer. The angle deviation φ_i is calculated from positions in time *i-1, i* and *i+1*. Where the vertex is the position in time *i*. All the variables are showed on the figure 9. Distances d_i is computed using Pythagorean Theorem. The angle deviation in time *i* is calculated as follows:

$$\varphi_i = acos\frac{|(V_i - V_{i-1}).(V_{i+1} - V_i)|}{\|(V_i - V_{i-1}).\|.\|(V_{i+1} - V_i)\|} \qquad (2)$$

Where acos is the inverse cosine function, $|.|$is a dot product and $\|.\|$ is a norm. From the formula (2) we get the magnitude of the angle deviation but we can't distinguish whether the deviation was made to the right or left side. Let positive angles indicate deflection to the left and negative angles deflection to the right. The method is also shown on the figure 9.

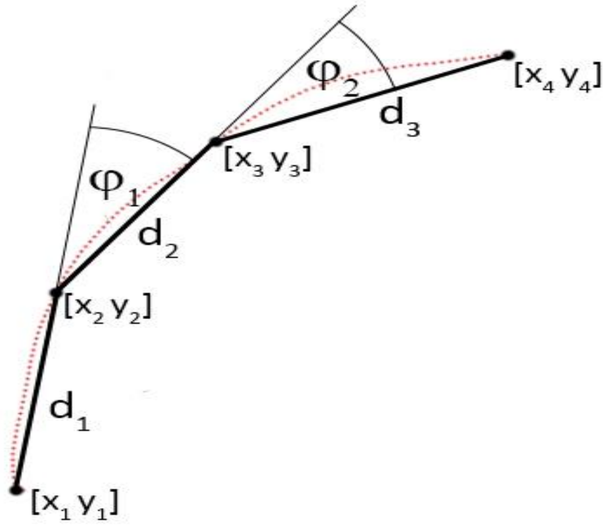

Fig. 9. Angle deviations calculation

Figure shows part of a trajectory, measured and calculated variables. Real trajectory is represented by red dots. Measured trajectory is a thick black line. Measured data points are x-y coordinated in the brackets and calculated variables are d and φ.

2.2.2 Finding the correct model

In this part, we are constructing the model for a trajectory of an individual. Let the one iteration of the model be one second. During the one step of the model, the angle deviation from the previous direction is $\varphi_n$and a distance traveled in this direction is l_n. As it was mentioned in the introduction we predict that angle deviations and traveled distances can be modeled by simple linear models with constant magnitude of noise. This prediction is based on the results of Jánoši [7] who modeled data from work of Souman *et al.* [2]. Jánoši observed that such a simple model describes Souman's data very well. Moves in the X-Y coordinates system can be calculated by equations 6 and 7.

$$\varphi_{n+1} = \varphi_n + \beta(\theta - \varphi_n) + \sigma\xi_n \tag{3}$$

$$l_{n+1} = l_n + B(L - l_n) + \tilde{\sigma}\tilde{\xi}_n \tag{4}$$

$$\omega_{n+1} = \omega_n + \varphi_{n+1} \tag{5}$$

$$X_{n+1} = X_n + l_{n+1}cos(\omega_{n+1}) \tag{6}$$

$$Y_{n+1} = Y_n + l_{n+1}sin(\omega_{n+1}) \tag{7}$$

The coefficient β and B are persistence coefficients. They are determining how the next angle deviation (or the next traveled distance) depends on the previous.

Parameter θ is a systematical bias of the volunteer and also the equilibrium state of the model. This equilibrium is asymptotically stable for $\beta \in (-1, 1)$. It means that if we omit the random element ξ and the current angle deviation is equal to θ the next angle deviation will not change. The same characteristics holds for the equilibrium parameter of traveled distance L. Errors ξ_n and $\tilde{\xi}_n$ (in equations 3 and 4) have standard normal distribution with standard deviation of σ and $\tilde{\sigma}$.

In the first part of this chapter we develop the model for angle deviations. If we want to find a systematic component of a movement we can plot φ_n against φ_{n+1}. The results are on the Fig 10.

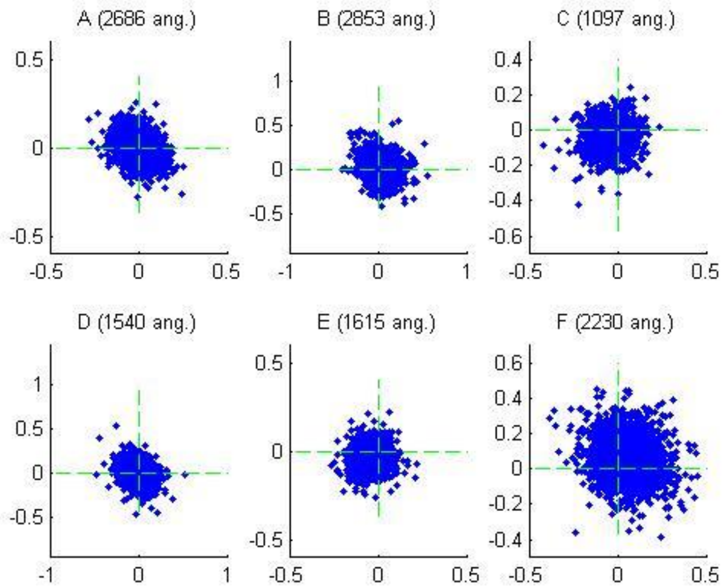

Fig. 10. Scatter plots step size 1s

Scatter plots of angle deviations for all six measured volunteers. Figure displays φ_n against φ_{n+1}. The scale is in rad for both axes.

It is clear from Fig. 8 that there is not a strong correlation between angle deviations in none of the plots. We expected evident positive correlation in the successive turning angles because people tend to turn step by step instead of making sharp turns. Also systematic bias to one side cause correlation between angle deviations. The bias is obvious on the trajectory of F (Fig. 4), BC and AC (Fig. 5). There are several explanations why this result doesn't match the previous research [2],[6],[7]. First explanation follows the work of Codling and Hill [15] who studied effects of the sampling rate on the parameter estimation. They observed that if a sampling interval is too small we are unable to observe any patterns as a correlation or other functional dependences. In our work the sampling interval is 1s so it may be such a too small sampling interval. Also roughness of the terrain could produce some noise in data, even though a significant change of participant's direction caused by terrain unevenness was observed only two times during the experiments. The noise in turning angles can be naturally produced by errors of measuring devices but we assume that this effect is low in our work since GPS devices measured with precision about ±1cm for a most of the time. Murray *et al.* [16] describes how is a walking person bending from side to side. We assume that such bends are main source of noise in our data.

The noise should decrease if we take larger time interval, that means skipping some data and looking at a larger time interval. In this part let call it step size. If the step size is k seconds, we take every *k-th* position of volunteer (in X-Y coordinates) and we calculate angle deviations from these positions. The different sampling rate influences the estimates of speeds as it was observed by Codling and Hill [15] in their work. Increasing a sampling interval will also significantly affects turning angles and traveled distances which results in the estimated parameters. Thus estimation of parameters per one second based on angle deviations obtained with larger step size is complicated and for some models impossible.

In our work the model contains three parameters that need to be estimated: β, θ and σ. The effect of sampling interval to the estimate of σ was studied in the work of Bovet and Benhamou [17] and Codling and Hill [15]. They derived relation between standard deviations estimated using different time step lengths. This relation contain a parameter that governs the effect of smoothing the trajectory and estimated its value empirically. By comparing equilibrium states of the models with step size one second and k seconds, formula for estimate of θ can be derived. The estimate of parameter β is more complicated and we were unable to find similar relation for estimates of the parameter as in works [15],[17]. Nevertheless angle deviations for larger step sizes are useful because they show interesting relations from which we are able to draw conclusions. The two successive angles are plotted against each other for the chosen step sizes on the figure 11:

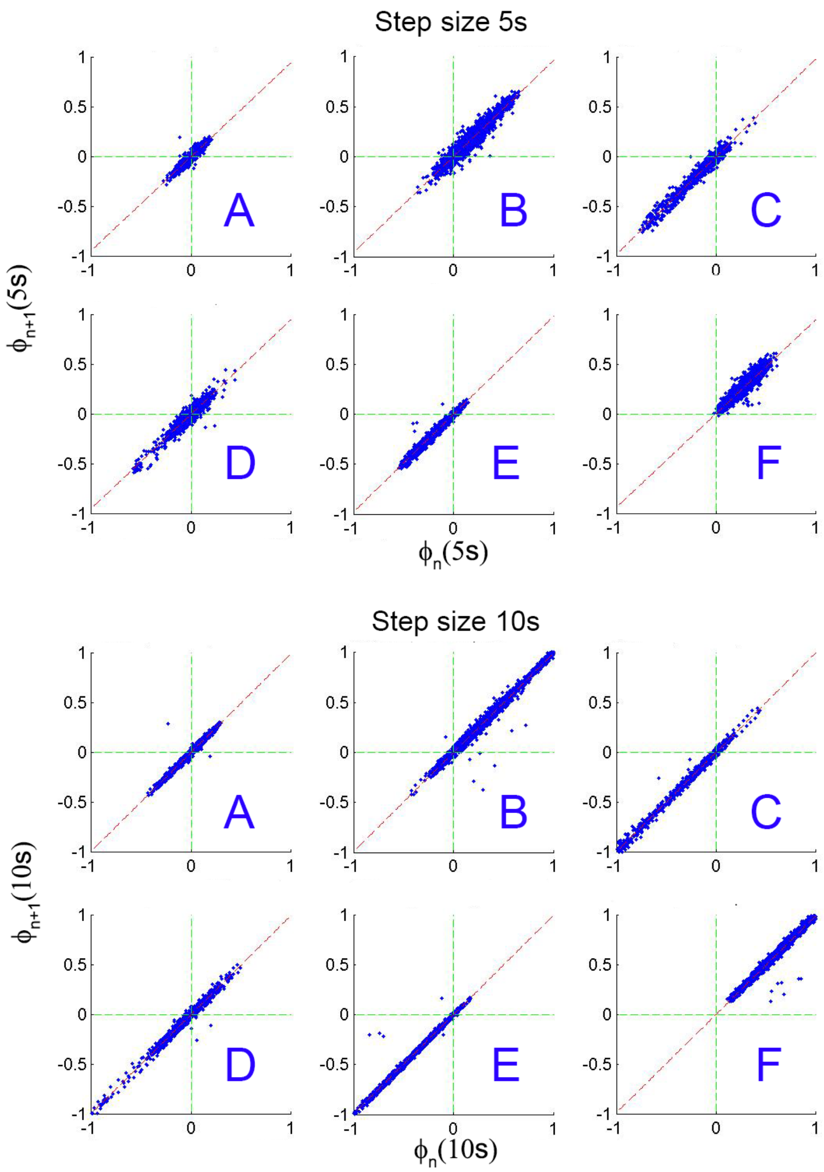

Fig. 11. Scatter plots step size 5s and 10s

Scatter plots of angle deviations for all six measured volunteers. Figure shows one-to-one correspondence between angle deviation and the same set lagged 5 and 10 seconds. (Step size: 5s and 10s).

The previous figure agrees with our assumption that the data for one second are dominated by noise. There are several approaches that can be used to reduce the noise. Two of them are described in chapter 2.3. The second thing which can be observed from figure 11 is, that there is a direct proportion on the plots (the slope of the regression line is almost equal to 1). It means that next angle deviation is almost equal to the previous. Such a perfect correlation can be caused by two things. Either the model have a perfect persistence or there is no persistence coefficient and the model for turning angles is constant. This leads to two possible models.

$$\varphi_{n+1} = \varphi_n + \sigma\xi \tag{8}$$

With β equal to 0.

$$\varphi_{n+1} = \theta + \sigma\xi \tag{9}$$

When β equal to 1.

So now we consider a model with either $\beta=0$ or $\beta=1$. In this part we examine which of these two models fits our data better by using moving averages of turning angles. We choose data window size w and starting with 1st angle deviation we calculate the average of 1st, 2nd…wth angle. Than we move the data window one angle forward. If there is a dependence between successive angles that causes cumulative increase or decrease in angles it supports the model without θ. If values vary around one constant it supports the model with θ. We used angle deviations calculated with different step sizes (1s, 5s and 10s). From them we calculated moving averages with different sizes of windows (10, 30, 50, 70 and 100). The plot for angles with step size 1s and data window of 10 angles was the most informative.

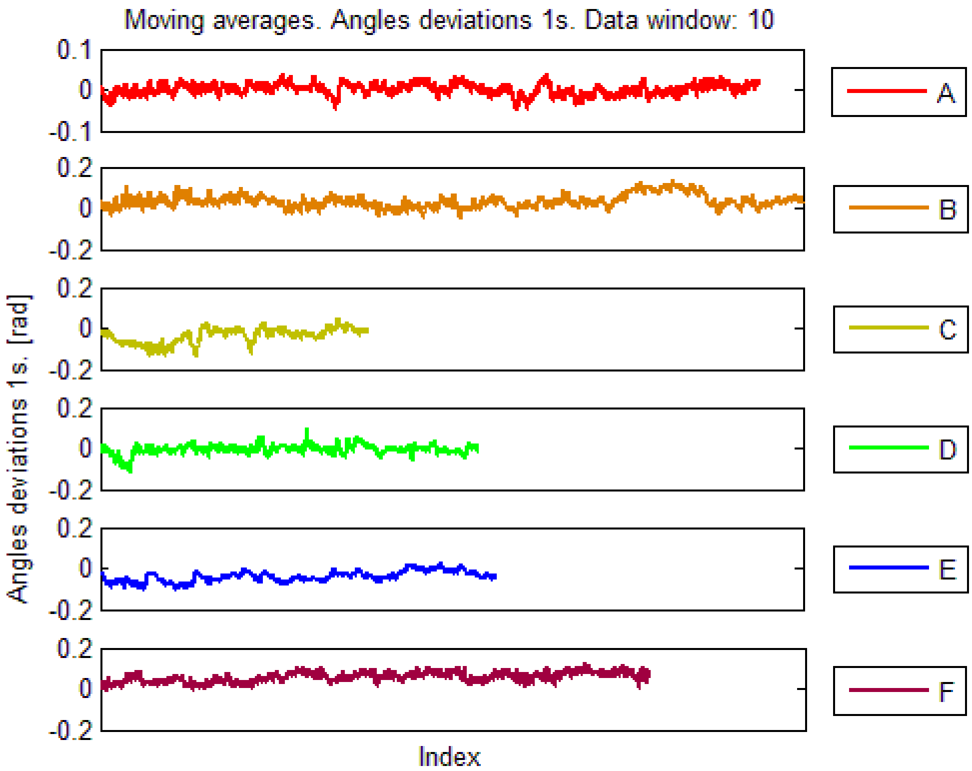

Fig. 12. Moving averages of angle deviations

The figure shows moving averages of angle deviations with window of averaging 10 angle deviations. The length of x axis is 2800 angles. Values are in rad.

The graphs fluctuate around constants which are unique for each volunteer. So we estimate a model of the form (9).

Parameters θ and σ are estimated by regression. We don't need to use larger steps since when we calculated θ for larger time steps and adjusted it to one second the estimates had almost the same value.

The table 6 shows calculated values for our final model for individual trajectories:

$$\varphi_{n+1} = \theta + \sigma\xi_n$$

	θ	σ
A	0.0009	0.0601
B	0.0330	0.0958
C	-0.0371	0.0774
D	-0.0061	0.0929
E	-0.0405	0.0560
F	0.0573	0.1076

Tab. 6. Parameters of model of individuals
The table shows parameters of the model for individual trajectories. Parameters are calculated for each volunteer A to F separately.

We can compare the ability of volunteers to walk straight direction by comparing the parameters of the model. Fist comparison can be based on the value of parameter theta which is systematic bias of the volunteer. It is relatively high for volunteer E and F, which is also visible from the trajectories on the Fig. 4. where the systematic bias results in circular patterns in participant's trajectories. Also low values of theta for volunteers A and D agree with their trajectories, where no systematic bias is visible.

Standard deviation of volunteers also refers to its accuracy in following straight direction but it is harder to find its effect on the plots of volunteers' trajectories. The general rule is that higher values of parameter sigma indicate higher randomness of the motion and thus lower ability to maintain the straight direction. Both theta and sigma can cause deviations from the straight line.

2.2.3 Residuals

In this section we study the residuals of the model for individual trajectories. We need to examine our assumption that random part of the model $\sigma\xi$ is independent normally distributed. The index plot of residuals is on the figure 13. It is good to mention that trajectories (and thus the residuals) are connected into one trajectory from more than one experiment. The different behavior in the experiments explains changes in residuals. For example the lower volatility in the last experiment in the residuals of volunteer D.

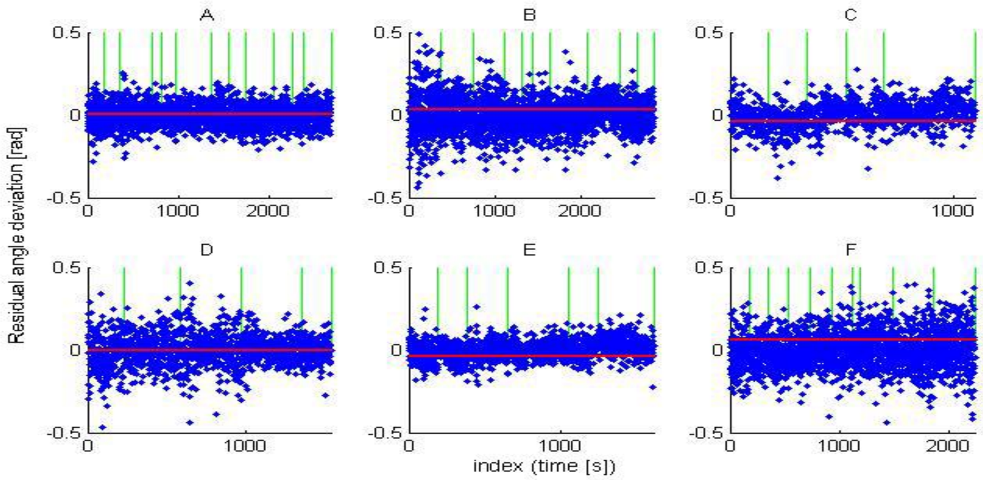

Fig. 13. Index plot of residuals (individuals)

The figure shows residuals of the models for each individual. Red line on the picture is the theta coefficient for the given volunteer. Green vertical lines divides residuals by experiments.

We assume that noise ξ in the model is normally distributed. To test this we plot the histograms of residuals for each volunteer. The normal distribution fits data well so we can assume the normality of residuals.

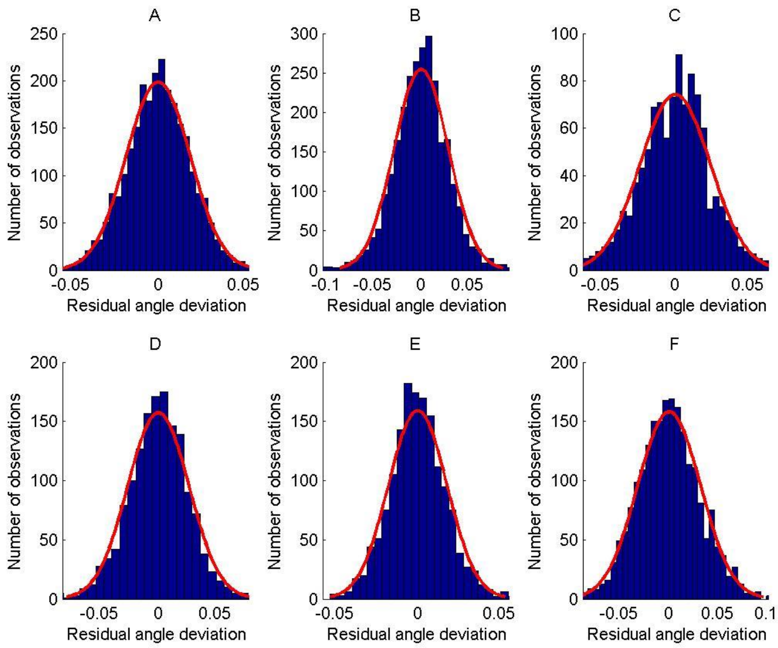

Fig. 14. Histogram plot of residuals individuals

The histograms of residuals for all 6 volunteers. Red curve is a fit of normal distribution to the histogram.

2.2.4 Model for trajectories of pairs

In this part we model the trajectory of pairs. We use two models. The model for angle deviations and the model for relative distance of volunteers in pair. We estimate the same model for angle deviations as was derived for individuals. These models are only a gross approximations and do not model the interactions in pair. They are developed to compare the estimates of systematic bias of individuals and pairs. For each pair we get two simple equations. The second index indicates whether we are modelling thefirst or the second volunteer in pair.

$$\varphi_{n+1,1} = \theta_1 + \sigma_1 \xi_{n,2} \tag{10}$$

$$\varphi_{n+1,2} = \theta_2 + \sigma_2 \xi_{n,2} \tag{11}$$

Although models (equations 10 and 11) do not model interaction in pairs explicitly we can see if there is change in individual's systematic bias while walking in pair. Results are summarized in the table 7.

Pairs	θ_1	θ_2	σ_1	σ_2	Individuals	θ	σ
CE	-0.0001	0.0013	0.0829	0.0870	A	0.0009	0.0601
DF	0.0010	0.0017	0.1905	0.1254	D	-0.0061	0.0929
CD	0.0010	0.0017	0.1124	0.1208	B	0.0330	0.0958
AD	0.0000	-0.0020	0.1083	0.1067	C	-0.0371	0.0774
AF	0.0021	0.0008	0.0999	0.1789	E	-0.0405	0.0560
BD	0.0053	0.0051	0.0729	0.0818	F	0.0573	0.1076
EF	0.0062	0.0067	0.0917	0.1269			
AE	-0.0103	-0.0115	0.0893	0.1137			
CF	0.0125	0.0128	0.0797	0.0921			
BE	-0.0208	0.0240	0.1390	0.3844			
BC	0.0238	0.0270	0.1137	0.1453			
AC	-0.0336	-0.0343	0.0897	0.0659			

Tab. 7. Comparison of parameters of models

The table shows comparison between theta coefficient for individuals and pairs. In the left part there are parameters for pairs and parameters for individuals are in the right part. If the theta coefficient of a pair is close to the coefficient of one of the individuals, these numbers are shaded by the same color. Pairs in table are sorted by maximum absolute value of two theta coefficients.

Higher values of parameter theta and sigma produces higher angle deviations that indicates weaker ability to keep a straight direction of walking. The aim of our work was to examine if the mutual cooperation in pair can improve such ability of individuals. The most significant improve is visible on the pair EF. Individuals E and F have the most significant bias to one side as individuals. While cooperating as the pair their bias is much lower. Their result could have been even better unless F would start leading and deviate the pair strongly to the left. The participant F improved his score in a cooperation with D. Their interesting strategy is described in section 1.3.2. A significant improvement of theta was detected also at the pair CE. Following the participant with a lower bias has showed as a good strategy for pairs AF, BD and CD. All volunteers knew that D had an exceptionally good navigational ability because they knew the results from our work in 2013 [3] (we are measuring the same group of volunteers in both works). Thus we expected that volunteer D would be dominant in most of the pairs or the other participants would try to cooperate with her. Although the obvious dominancy haven't been observed it is clear from the parameters theta that volunteers tend to adapt to D. By this behavior volunteers B, C and F were able to decrease their bias to the much lower values of D. Dominant and also faster member in pair the AC was C. So the result of the pair is very close to the result of C as an individual and so the member A increased his bias. In this pair the strong effect of walking speed on the trajectory of the pair is obvious. Pairs have in general slightly higher standard deviation than individuals (except for pairs AC and CF).

If one individual starts to walk faster, the slower one is pulled by the rope. When this occurs the distance between volunteers rises. For this reason we are studing distance between volunteers. On the other hand when participants get too close to each other they feel unconfident about their partner's position which resulted in tendency to slow down and increase the distance between volunteers. This tendency haven't been observed every time when volunteers get close to each other. Since the rope had fixed length (in relaxed state) we can predict that optimal distance between volunteers in pair exist. For this reason we modelled a distance between volunteers in pairs by linear mean return model.

$$d_{n+1} = d_n + B(D - d_n) + \sigma\xi_n \tag{12}$$

We assume a strong correlation between successive distance changes because rope was stretched or pulled down continuously as the pairs walked. This process is visible on the index plot of distances.

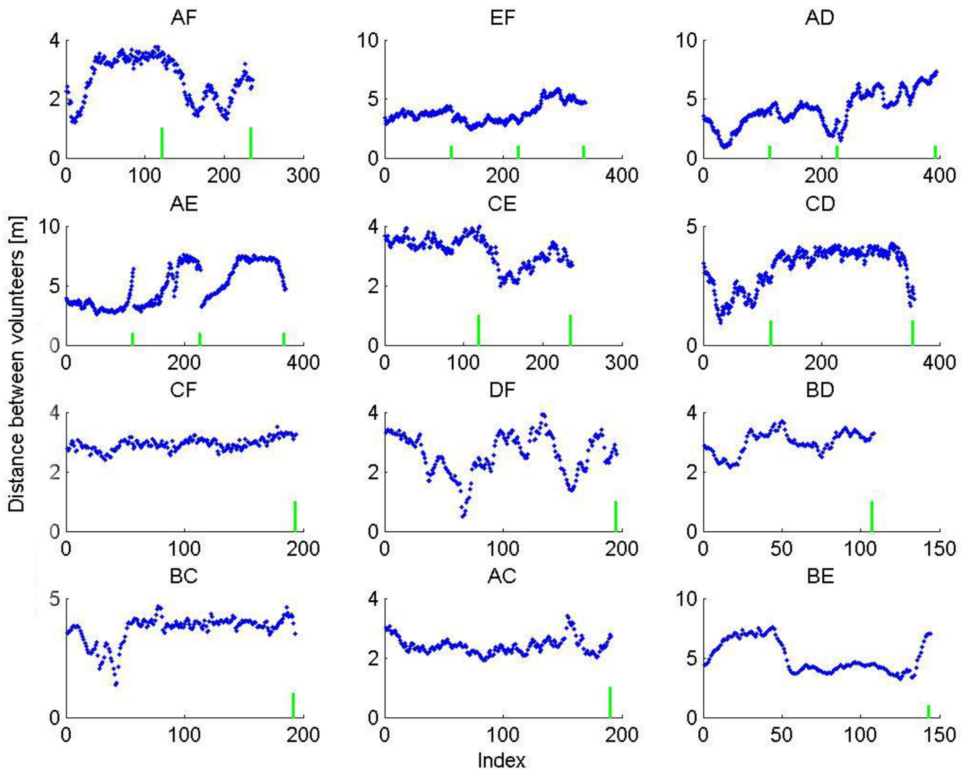

Fig. 15. Index plot of relative distance in pairs

The plot displays the evolution in time of individual's distance in pair. Green vertical line indicates start of the next experiment.

We estimate parameters by linear regression and sigma as standard deviation of residuals. The estimated parameters are on the table 8:

	D	B	σ	θ_1	θ_2		θ
CE	3.1059	0.0309	0.0130	-0.0001	0.0013	A	0.0009
DF	2.3854	0.0761	0.0260	0.0010	0.0017	B	0.0330
CD	2.4617	0.0309	0.0190	0.0010	0.0017	C	-0.0371
AD	4.1808	0.0111	0.0260	0.0000	-0.0020	D	-0.0061
AF	5.0976	0.0141	0.0230	0.0021	0.0008	E	-0.0405
BD	2.8097	0.0100	0.0090	0.0053	0.0051	F	0.0573
EF*	4.0000	0.0109	0.0160	0.0062	0.0067		
AE*	4.0000	0.0026	0.0790	-0.0103	-0.0115		
CF	3.6955	0.0430	0.0110	0.0125	0.0128		
BE	2.9403	0.1489	0.0410	-0.0208	0.0240		
BC	3.0161	0.0292	0.0280	0.0238	0.0270		
AC	2.7390	0.0214	0.0090	-0.0336	-0.0343		

Tab. 8. Parameters of model for relative distances

In the first three columns table shows parameters of model for relative distances. If an estimate of *B* coefficient was close to zero the D coefficient was set to 4 and *B* was recalculated. Such pairs are denoted by *. Two columns in the middle of table are displaying theta parameters for pairs. Two columns on the right side are summarizing theta parameters of individuals. Pairs in table are sorted by maximum absolute value of two theta coefficients.

The parameter D which stands for optimal length of the rope have expected values for all the pairs (except BE and AD) since the maximum stretch of the rope was approximately 5.5m. The too high values of D for BE and AD are caused by too low parameter B. Because when we estimate parameters by regression $d_{n+1} = A\, d_n + C + \sigma\xi_n$ then

$$B = 1 - A \quad \text{and} \quad D = \frac{C}{B} \tag{13}$$

So for the low values of B the D rises to unexpected values. For these pairs we replaced their estimate of D by fixed value of 4 and recalculated B. The parameter D can be a measure of cooperation. Values greater than 2.5 (length of rope in relaxed state) and lower than 4 (the maximum stretch of the rope was 5.5) indicates mutual cooperation in pair.

Same way that in the models of individuals we study the residuals of the models. We need to examine our assumption that random part of the model $\sigma\xi$ is independent normally distributed. The histogram of residuals is on the figure 16:

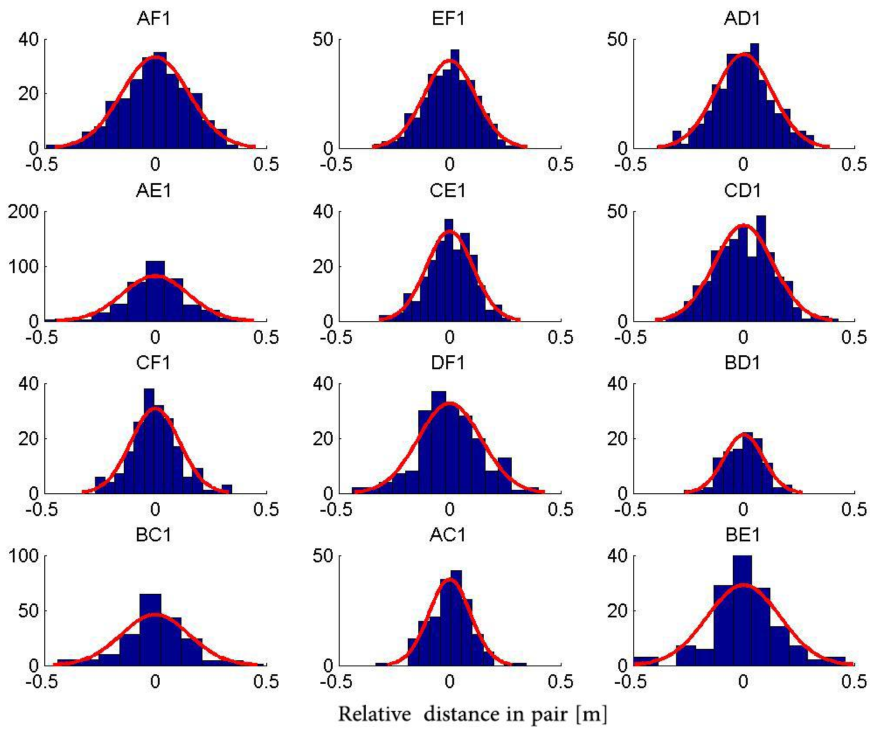

Fig. 16. Histograms of residuals relative distance in pairs

Histograms of residuals for all 12 pairs. There are two plots for each pair. Red line is a fit of normal distribution to the histogram.

Normal distribution fits histogram very well on the figure 16 in most cases so our assumption of normal distribution is correct.

2.3 Second and third approach

This chapter is devoted to two additional methods that process data's noise. The second method is using simple mathematical filter and the third is comparing angle deviations *k* seconds lagged from each other. It is not a goal of this chapter to conduct the analysis and comparison of trajectories of individuals and pairs as it was in the previous chapter. We just want to introduce alternative methods to analyze noised data and compare their performance. For this reason we analyze only the trajectories of individuals.

2.3.1 Mathematical filter

Data smoothing is often used in image processing where data contain the high amount of noise [18]. The filter can decrease the noise and help to find searched patterns. Using a filter to data has also unwanted drawbacks that differ among the filters. Therefore it is crucial to choose a correct filter. To find the most proper one we should understand where the noise in data comes from.

In our case the noise could come from several sources. First source of noise could be that sampling interval of 1s is so small that we are unable to observe any patterns as a correlation or other functional dependences. This effect is mentioned in work of Codling and Hill [15] who studied effect of the sampling rate on the parameter estimation. Also roughness of the terrain could produce some noise, even though a significant change of participant's direction has been observed only few times. The errors in measurements and thus the noise could be naturally produced by measuring devices. Another source of noise could be that a walking person is bending regularly from side to side. This regular swing is made with the same frequency as steps frequency. However during our measurements the data were obtained by frequency of 1s and volunteers walked with different frequencies. On the sampling rate of 1s this bends could have produced an unwanted randomness in angle deviations. We consider this as the main source of the noise in our data.

For this reason we are using moving average filter to the X-Y coordinates obtained during the experiments. We recalculated each position of a participant as an average of three successive positions. In this process X-coordinates and Y-coordinates were calculated separately. This procedure reduced volatility (and thus the noise) to one third.

The drawback of this filter is that absolute values all turns decreased so we may lost also part of a real variability of the trajectories.

The plot of $\varphi_n$against $\varphi_{n+1}$of filtered data:

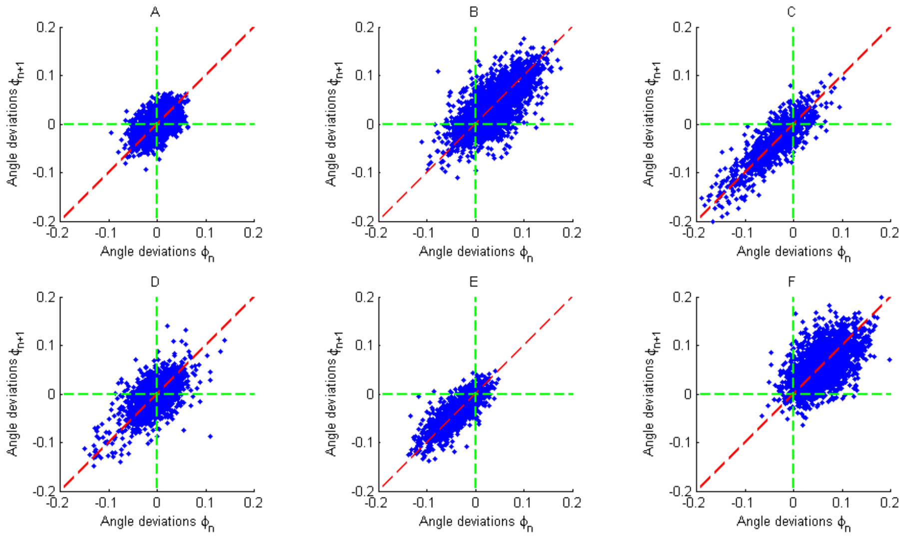

Fig. 17. Scatter plots step size 1s filtered data

Scatter plots of angle deviations for all six measured volunteers using filtered data. Figure displays φ_n against φ_{n+1}. The scale is in rad for both axes. The red line indicates direct proportion. The correlation coefficient is displayed on the top of each graph.

The previous picture confirms our assumption about linear model. Coefficients are estimated by regression:

	θ	β	σ	θ first
A	0.0009	0.4880	0.0003	0.0009
B	0.0335	0.2905	0.0008	0.033
C	-0.0374	0.1534	0.0006	-0.0371
D	-0.0064	0.3716	0.0007	-0.0061
E	-0.0409	0.1888	0.0003	-0.0405
F	0.0580	0.4727	0.0010	0.0573

Tab. 9. Parameters of model for individuals filtered data

The table shows parameters of the model for angle deviations. The last column is theta estimated by the first approach.

When we compare theta estimated from filtered data with our estimates from 2.2. we can see that the estimates are almost the same. So the filter haven't changed equilibrium angle deviation. So the systematic bias of the volunteer has not been violated. Although the estimates from this method are similar to the first approach, we has not proved that use of this method is legitimate.

The residuals of the model fits the normal distribution very well.

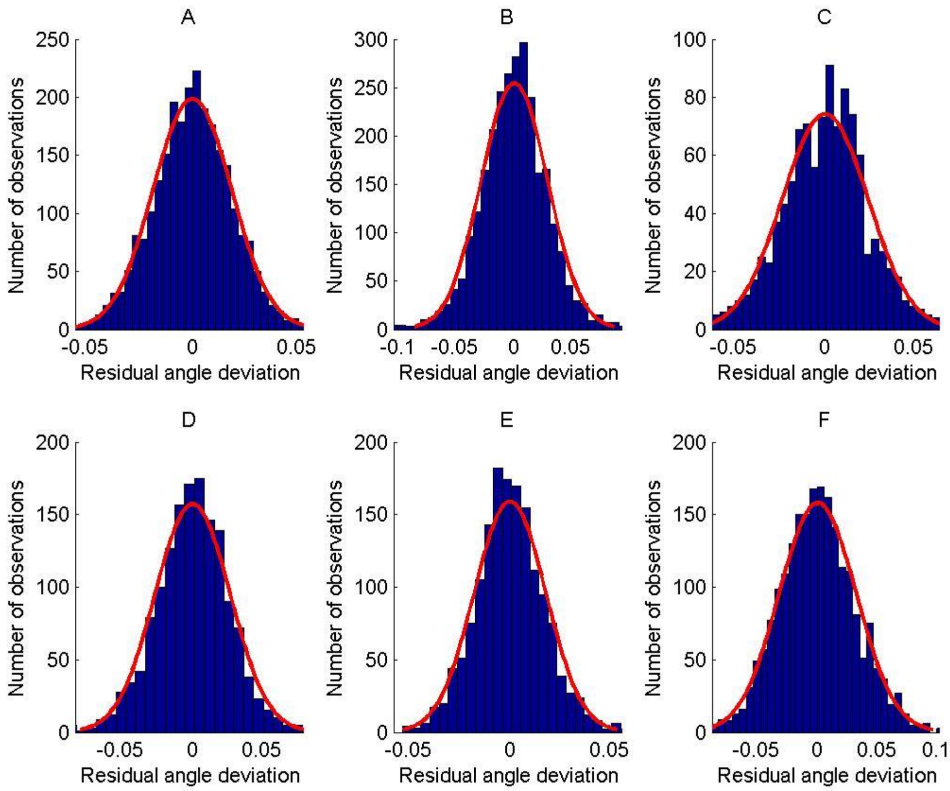

Fig. 18. Histogram of residuals

Histogram of residuals of angle deviation calculated from filtered data. The red curve is a normal distribution fit. We can conclude that residuals seems that they comes from normal distribution.

2.3.2 Lagged angles

In this chapter we estimate the original linear model for angle deviations of the form (5) from data that are dominated by noise. The original model is:

$$\varphi_{n+1} = \varphi_n + \beta(\theta - \varphi_n) + \sigma\xi_n \tag{5}$$

In the figure 10 we drew angles φ_n against φ_{n+1}. In this part we estimate parameters of linear model of the form (5) using angle deviation φ_{n+k} instead the φ_{n+1} in the original formula in order to decrease the noise. Let k be an integer greater than one and we call it lag. At first we derive formulas to calculate the parameters of original model based on estimates from lagged angles.

In our calculations unknown variables β and θ are estimated by linear regression (least squares method). For every lag k least squares method use the model:

$$\varphi_{n+k} = A_k\varphi_n + B_k + S\xi_n \tag{14}$$

We interpret A_k as the influence of the angle deviation k seconds before (persistence coefficient) and B_k as an equilibrium state of model with lag k. From these parameters we want to estimate persistence coefficient *a* and the bias *b* per one second.

$$\varphi_{n+1} = a\varphi_n + b + \sigma\xi_n \tag{15}$$

The aim is to estimate *a* and *b* by A_k and B_k and then β and θ by *a* and b. Starting from:

$$\varphi_{n+1} = a\varphi_n + b + \sigma\xi_n \tag{16}$$

$$\begin{aligned} \varphi_{n+k} = a\varphi_{n+k-1} + b = a(a\varphi_{n+k-1} + b + \sigma\xi_n) + b + \sigma\xi_n = \\ = a(a(a\varphi_{n+k-2} + b + \sigma\xi_n) + b + \sigma\xi_n) + b + \sigma\xi_n \\ = a^k\varphi_n + b\sum_{i=0}^{k-1}(a^i) + \xi_n\sigma\sum_{i=0}^{k-1}(a^i) \end{aligned} \tag{17}$$

Then we can compare members next to the φ_n and intercepts and standard deviation in equations (14) and (17). It follows that:

$$a \approx \sqrt[k]{A_k} \tag{18}$$

The root can produce complex numbers so we can improve our estimate by adding an absolute value to the approximation. There is no need to multiply the root by the sign function because we assume the positive correlation between the results. If negative correlation occurs for one particular lag it is caused by superposition of noised angles so we don't take it into account. The estimate of β is:

$$\beta \approx 1 - a = 1 - \sqrt[k]{|A_k|} \tag{19}$$

To estimate the standard deviation we should compare the volatility of respective normal distributions.

$$S^2 = \left(\sum_{i=0}^{k-1}(a^i)\right)^2 \sigma^2 = \sigma^2\left(\frac{(1-a^k)}{(1-a)}\right)^2 \tag{20}$$

So the estimate of $\sigma$is

$$\sigma = \frac{S\,(1-a)}{1-a^k} = \frac{S\,(1-a)}{1-A_k} \tag{21}$$

We can estimate *b* similar way as *a* to get:

$$b = \frac{B_k}{\sum_{i=0}^{k-1}(a^i)} = \frac{B_k\,(1-a)}{(1-a^k)} = \frac{B_k\,\beta}{1-A_k} \tag{22}$$

Then estimate of θ is:

$$\theta = \frac{b}{\beta} = \frac{B_k\,\beta}{(1 - A_k)\beta} = \frac{B_k}{1 - A_k} \tag{23}$$

But this estimate is actually equal to the estimate of θ in model with lag k. So there is no time correction. Therefore θ cannot be calculated this way and we should use different approach. We compare the equilibrium state of the models. The model that we can estimate by regression is:

$$1.\ \ \varphi_{n+k} = A_k\varphi_n + B_k + \sigma\xi_n \tag{24}$$

The model that we want to estimate is:

$$2.\ \ \varphi_{n+1} = \varphi_n + \beta(\theta - \varphi_n) + \sigma\xi_n \tag{5}$$

Equilibrium of the model is state where angle deviation in time *t+1* is the same as in time *t*. Let $\overline{\varphi_1^k}$ be the equilibrium of the model 1 with lag k and let $\overline{\varphi_2^1}$ be the equilibrium of the desired model (time step 1s).

We can calculate equilibrium of model 1 in time k and 2 in time 1.

$$\overline{\varphi_1^k} = \frac{B_k}{1 - A_k} \tag{25}$$

$$\overline{\varphi_2^1} = \theta \tag{26}$$

Since we assume the equilibrium state of model angle $\overline{\varphi_1^k}$is angle deviation made during k seconds and it will not change during the time. Therefore we can divide it and write.

$$\varphi_1^1 = \frac{\varphi_k^1}{k} \tag{27}$$

We assume that both models describes the same motion so they should have same equilibrium angle deviation. So the estimate of θ (the equilibrium angle of model 2) can be obtained as follows:

$$\theta = \overline{\varphi_2^1} = \overline{\varphi_1^1} \approx \frac{\overline{\varphi_1^k}}{k} = \frac{B_k}{(1 - A_k)k} \tag{28}$$

So the theta per second could be estimated as

$$\theta \approx \frac{B_k}{(1 - A_k)k} \tag{29}$$

To summarize:

Parameters with lag k:	Parameters per second:
$\beta_k \approx 1 - A_k$ $\quad \theta_k \approx \frac{B_k}{1-A_k}$ $\quad S$	$\beta \approx 1 - \sqrt[k]{\lvert A_k \rvert}$ $\theta \approx \frac{B_k}{(1-A_k)k}$ $\sigma = \frac{S\,(1-a)}{1-A_k}$

We estimated all parameters for each volunteer separately using a lag of 1 to 15. All parameters are estimated by regression. We assume that after some lags the noise decrease and the estimated parameters will have similar values. The graphs of β, σ and θ are on the figures 19, 20 and 21:

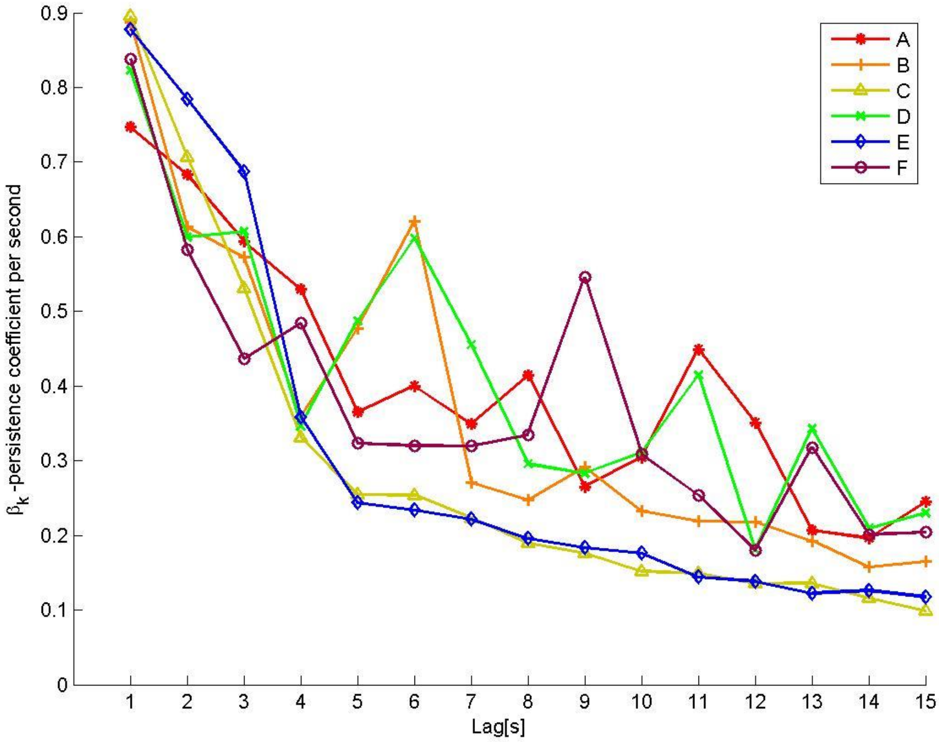

Fig. 19. Persistence coefficient β per second

The figure shows persistence coefficient per second. The lag k is selected from 1 to 15.

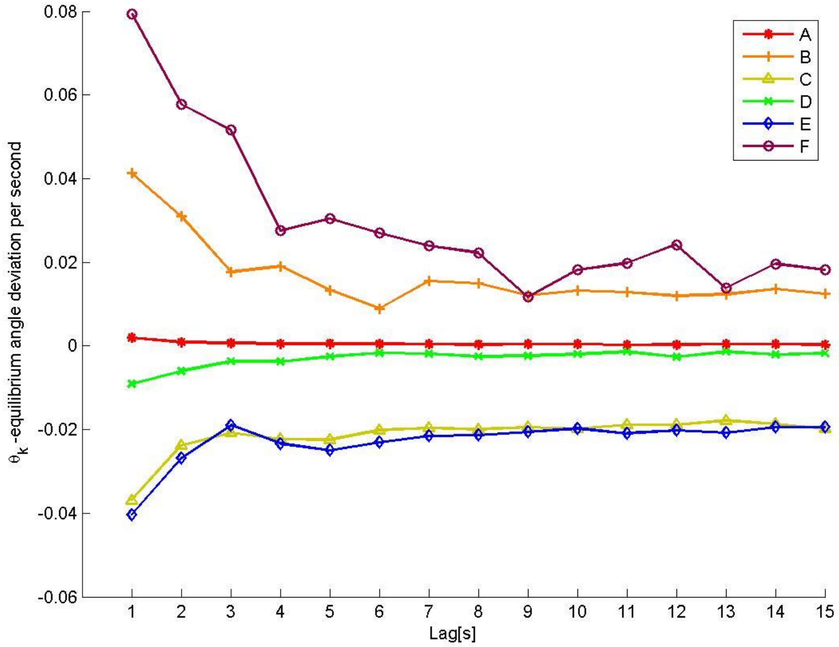

Fig. 20. Equilibrium angle deviations θ per second

The figure shows θ the equilibrium angle deviation per seconds. The lag k is selected from 1 to 15. The variable θ is in rad and it is calculated for each volunteer separately.

We calculated the standard deviation of residuals for each lag. On the figure 21 we can see gradual decrease in standard deviation.

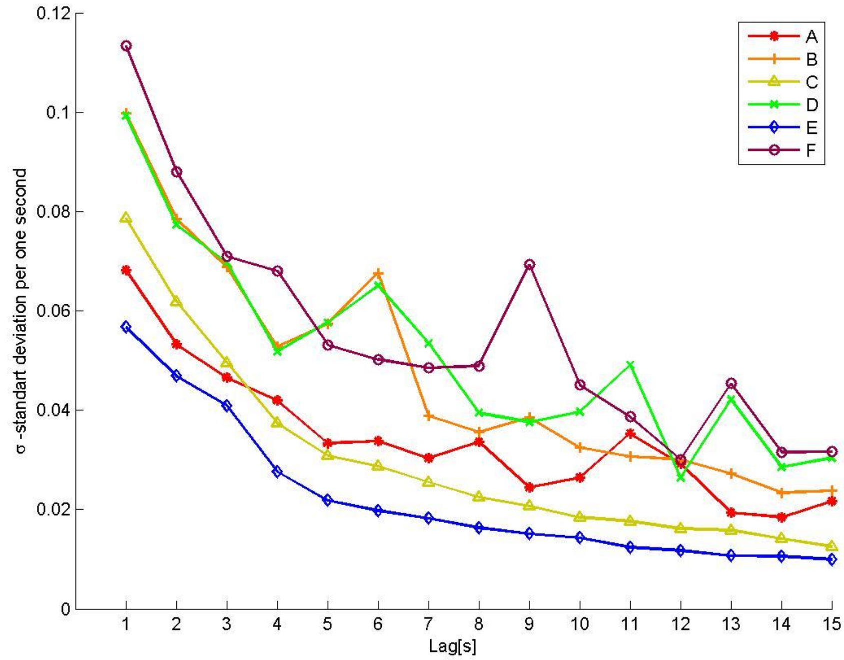

Fig. 21. Standard deviation σ per second

The figure shows gradual decrease in standard deviation per seconds. The lag k is selected from 1 to 15.

On the figure 21 the dependence between sigma and beta is visible. We can divide all three graphs of parameters into three areas. During the lags 1 to 5 the standard deviation (and thus the noise) rapidly declines. The estimates from lag 6 are stabilized. For too high lags all conclusions should be made with care since for such large lags the original patterns in data could be lost although it is not visible in our graphs. We should choose the lag higher than 6 but not too high. It is necessary to choose same lag for all volunteers since we want to compare the estimates. Peaks for volunteer B and D (lags 5 to 7) on the figure of beta are caused by negative correlation. The negative correlation is caused by superposition of noised angles so we don't choose these lags. The same situation for beta occurs for volunteer F lag 9. All other original correlations are positive. For this reasons we choose lag 8 to estimate our parameters. The table 10 shows comparison of all three approaches:

	First approach		Second approach (filter)			Third approach (Lagged)		
	σ_{res}	θ	θ	β	σ	θ	β	σ
A	0.0009	0.0625	0.0009	0.4880	0.0003	0.0003	0.4145	0.0283
B	0.0330	0.0977	0.0335	0.2905	0.0008	0.0150	0.2470	0.0456
C	-0.0371	0.0792	-0.0374	0.1534	0.0006	-0.0199	0.1894	0.0341
D	-0.0061	0.0946	-0.0064	0.3716	0.0007	-0.0025	0.2956	0.0443
E	-0.0405	0.0571	-0.0409	0.1888	0.0003	-0.0214	0.1955	0.0243
F	0.0573	0.1094	0.0580	0.4727	0.0010	0.0223	0.3350	0.0497

Tab. 10. Comparison of parameters for individuals

The table shows parameters of the model for angle deviations of individuals. Three approaches are used to estimate the parameters. The last column is theta estimated by the first approach.

When we order the individuals according to any parameter the resulting order would be the same for all three approaches. The third approach produced smaller values of theta but relative results are the same. The estimates of theta by the second approach are surprisingly almost the same estimates of theta as the first approach. As expected the standard deviation of the second approach is far lower than in the other two approaches. The estimates beta coefficients are close to each other in both the second and third approaches. On the other side both second and third approaches contains a problematic assumption. For the second it is an effect of the averages to the real randomness of trajectories and for the third approach it is choice of lag k.

Conclusion

In this thesis we have studied the human ability of orientation in the environment which lacks the possibility of orientation based on sensory cues. Our aim was to find out whether the mutual cooperation in a group can improve such an ability.

We conducted two experiments with individuals and pairs as a smallest possible group. The same six volunteers as in our previous work [3] agreed to participate. They were measured as individuals and as pairs. The experiments were designed to avoid external factors that affect the walking as much as possible. The participants were not allowed to use any navigational equipment and the visual and auditory orientation was also excluded. Participants walked on a large field and their positions were measured each second by geodetic GPS devices. The interaction in pairs was established by the elastic rope they were holding. This type of interaction showed to be chosen correctly because we saw all decisions and interactions instantly as a tension of the rope. We measured only 12 (out of 15) pairs because we underestimated time of pair's experiment setup. Too time consuming part was choosing members of pair before measurements without giving them information who is their partner (this process is described in chapter 1.2.1. page 15).

The experiments were recorded by a digital camera which showed to be very useful in analyzing strategies of pairs. Our previous research [3] shows that walking pairs are strongly affected by the presence of a dominant members who are trying to enforce their direction. The videos from experiments were very useful to found later on whether such members in our pairs are and how their presence affects them. We constructed a graph of dominances that is on the figure 3. Despite the dominance being an important factor which affects the trajectory of pairs it is not included in our model for trajectories. Development of such a model thus remains for the further research.

During data processing we calculated angle deviations and traveled distances of volunteers in each second. We were developing models for angle deviations that describe their evolution in time. We assumed a linear model with a constant magnitude of noise. During the model development we found that the data contain a large amount of noise so we are unable to discover dependences from them without extensive processing.

In the first approach we tried to remove the noise by calculating angle deviations per more seconds but we were unable to estimate the parameters per one second from such angles. Based on the observation of angle deviations per several seconds we found that the data can be modeled from original angle deviations by a simpler constant model which demonstrated not to be influenced by noise in such extent. In the last part of the work we presented also second and third alternative approaches to elimination of the noise. Their advantage was that we were able to estimate the original linear model.

The second is a simple mathematical filter. Each filtered position is calculated as a mean of three successive positions. In this approach we significantly reduce the noise as well as the real variability of the trajectory. Although the estimates from this method are similar to the first approach, we haven't proved that use of this method is legitimate.

In the second approach we modelled a dependence between angle deviations φ_{n+k} and φ_n instead of successive angles. From these "shifted" models we were able to estimate the parameters of the original linear model. The problem in this approach was the choice of the shift k. We assumed that for a higher k, values of the estimates would be similar and we estimated parameter by this value. For some coefficients of the model this was not the case.

We have chosen first approach and estimated constant model with constant magnitude of noise and compared the results for individuals and pairs. This comparison showed that by a reciprocal cooperation in pairs it is possible to decrease bias of pairs to values which are lower than the values of its members as individuals. This has happened to DF and EF pairs. The strategy, when one of the pair let the one with a better ability of orientation lead, may strongly reduce the bias of an individual with worse orientation ability.

We have chosen to measure the same volunteers as in our previous work in 2013 [3] to have the opportunity to compare the results. In 2013 six individuals and four pairs (DE, CD, AC and BE) were measured. The experiments were different in [3] but we can compare the relative results when we compare the order of the volunteers sorted by the precision in keeping the straight direction. We sort results from this work by theta coefficient. We obtained the same results as in 2013 for the pairs AC and BE who showed low precision. Also pair CD showed relatively high precision in both works. The pair DE was not measured in our work. When comparing individuals for participant D relatively high precision and for E relatively low precision was observed in both works. For other individuals different results were observed. It may be caused by the fact that the experiments were conducted indoors and there was another type of communication in pairs. The differences between this two experiments are described in the section 1.1.1.

The results from pairs can be extended to the larger groups. If people in a larger group can estimate their abilities correctly then they can choose the two most efficient members to navigate the group.

Although this thesis does not provide a clear answer to the question whether group or individual is better at maintaining a straight direction of walk it is one of the first studies that measured detailed trajectories of pairs in the environment which lacks the possibility of orientation based on sensory cues. It provides data that can bring more results after an extensive analysis. It also gives suggestions for further research. The more complex models for trajectories that includes the effect of dominant member, interaction or changes in speeds can be introduced. An interesting question is whether blind participants are better in maintaining the straight direction. We assume that such an ability can be improved by using it so we predict that blind people would achieve better results than our participants.

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