COMENIUS UNIVERSITY, BRATISLAVA

FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

RISK PREMIUMS IN SLOVAK GOVERNMENT BONDS

MASTER THESIS

2017 **Bc. Richard PRIESOL**

COMENIUS UNIVERSITY, BRATISLAVA

FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

RISK PREMIUMS IN SLOVAK GOVERNMENT BONDS

MASTER THESIS

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Supervisor: Pavol Povala PhD.

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Meno a priezvisko študenta: Bc. Richard Priesol
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Názov: Risk premiums in Slovak government bonds
Rizikové prirážky v slovenských štátnych dlhopisoch

Cieľ: Cieľom práce je porovnať rozdielne rizikové prirážky zohľadnené v slovenských štátnych dlhopisoch. Musíme preto špecifikovať a odhadnúť skupinu afinných dynamických modelov časovej štruktúry úrokových mier slovenských štátnych dlhopisov. Tieto modely umožňujú rozklad výnosov na rizikové prirážky a rizikovo neutrálne očakávania krátkodobých úrokových mier pre rôzne doby splatnosti. V ďalšom kroku sa pokúsime vysvetliť disperziu v odhadnutých rizikových prirážkach pomocou ekonomických a finančných premenných. Táto práca je zameraná na ekonometrické aspekty a implementáciu. Máme prístup k výnosom bezkupónových dlhopisov od Januára 2003 do Októbra 2016.

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THESIS ASSIGNMENT

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Field of Study:	Applied Mathematics
Type of Thesis:	Diploma Thesis
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Title: Risk premiums in Slovak government bonds

Aim: The aim of the thesis is to study different risk premiums reflected in Slovak government bond yields. To this end, the thesis should specify and estimate a set of affine dynamic term structure models for the Slovak government yield curve. These models allow for a decomposition of yields into risk premiums and risk-neutral short rate expectations at different maturities. In the next step, the thesis seeks to explain the variation in estimated risk premiums with economic and financial variables. This is an empirical thesis with a focus on econometric aspects and implementation. Zero-coupon yield data for the period January 2003 through October 2016 are available.

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Abstrakt

PRIESOL, Richard: Rizikové prirážky v slovenských štátnych dlhopisoch [Diplomová práca], Univerzita Komenského v Bratislave, Fakulta matematiky, fyziky a informatiky, Katedra aplikovanej matematiky a štatistiky, školiteľ: Pavol Povala PhD., Bratislava, 2017, 64 s.

Výnosy štátnych dlhopisov sú ovplyvnené rôznymi faktormi od očakávaní o bezrizikovej úrokovej miere po rizikové prirážky, ktoré sú špecifické pre danú krajinu. V tejto práci popíšeme najdôležitejšie faktory ovplyvňujúce výnosy slovenských štátnych dlhopisov pomocou dynamických modelov časovej štruktúry úrokových mier aplikovaných na slovenskú výnosovú krivku bezkupónových dlhopisov. Slovenskú výnosovú krivku pritom rozložíme na bezrizikovú krivku Eurozóny a rizikové prirážky špecifické pre Slovenskú republiku. Bezrizikovú krivku ďalej rozložíme na očakávania o bezrizikovej úrokovej miere a časové prirážky a podrobne popíšeme rizikové prirážky v slovenských štátnych dlhopisoch pomocou dynamického lineárneho modelu a Kalmanovho filtra. Ako výsledok dostaneme vývoj najdôležitejších faktorov ovplyvňujúcich výnosy slovenských štátnych dlhopisov, vysvetľujúci varianciu vo výnosoch a zachytávajúci najdôležitejšie udalosti v analyzovanom období od Januára 2009, kedy sme vstúpili do Eurozóny, do Októbra 2016.

Kľúčové slová: Štátne dlhopisy, Modely časovej štruktúry, Výnosová krivka, Rizikové prirážky, Modely skrytých faktorov

Abstract

PRIESOL, Richard: Risk premiums in Slovak government bonds [Master thesis], Comenius University in Bratislava, Faculty of Mathematics, Physics and Informatics, Department of Applied Mathematics and Statistics, Supervisor: Pavol Povala PhD., Bratislava, 2017, 64 p.

Government bond yields are driven by a number of different factors from the risk-free short rate expectations to the country-specific risk premiums. In this paper, we describe the most important drivers of yields on Slovak government bonds within a set of dynamic term structure models applied on the Slovak zero-coupon yield curve. To this end, we decompose the Slovak yield curve into the risk-free curve of Eurozone, represented by the term structure of Overnight Indexed Swaps, and additional risk premiums specific for Slovak Republic. Furthermore, we decompose the risk-free curve into the average short rate expectations and term premiums and make detailed specification of the country-related risk premiums through the methodology of dynamic linear models and application of the Kalman filter. As the result, we obtain the development of the yield-driving factors in the Slovak government bonds explaining the variance in the yields and capturing the main events in the sample period from January 2009, when we entered the Eurozone, to October 2016.

Keywords: Government bonds, Term structure models, Yield curve, Risk premiums, Latent factor models

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1 Introduction

Looking into standard macroeconomic literature, one can find many important indicators summarizing the economic situation of a particular country, from the gross domestic product (GDP), measuring total economic output of the country, through the inflation and unemployment rates, describing the development of the money and labour markets, to the interest rates on government bonds, representing the ability of the country to borrow in the financial markets. Yields on government bonds thus not only aggregate the investors views of future economic growth and inflation but also reflect the availability of capital that is essential for future existence of modern economies.

Development of the interest rates is usually captured by the zero-coupon yield curve, term structure of yields on zero-coupon bonds, that can be extracted from the coupon bonds by a variety of methods, the most popular are variants of Nelson and Siegel (1987) and even more widely used extension by Svensson (1994). Term structure of the government bond yields is of crucial importance not only because reflecting the market expectations of a particular economy in different horizons or because of the character as the price of capital but also for the application in many economical models and investment projects and the country-specific composition reflecting different aspects of the economy.

Composition of the government bond yields vary from country to country but we generally decompose the yield curve into the short rate expectations and risk premiums. Nominal short rate expectations can be then further decomposed into the real short rate expectations and inflation expectations and generally reflect a monetary policy impact on economy. Risk premiums can be decomposed into the term premium, the compensation that investors require for the uncertainty about future development, and country-specific risk premiums, while the most important one is the credit or default premium, reflecting the probability of potential default and the rating of a particular country, followed by the liquidity premium that can be represented as the compensation that investors require for illiquid assets and also as the price that investors are ready to pay for highly liquid assets.

We should mention also the redenomination premium, reflecting the risk arising from currency redenomination what is relatively specific for the Eurozone government bonds, and the safety premium, representing the price that investors are ready to pay for safe assets. However, there are also other significant channels affecting the government bond yields, for example the Quantitative Easing (QE), the expansive program of the European Central Bank (ECB) which was designed to lower the yields on Eurozone government bonds by inserting a large and relatively price-insensitive buyer to the market, or the market segmentation that can be summarized as the impact of government policies, amount and structure of regulation, possible investment constraints and other country-related specifications.

With regard to the country-specific differences in the composition of the zero-coupon yield curve, there is a number of methods for modelling the term structure of government yields. Majority of them are based on the latent factor models that are able to extract the driving factors of particular yields and thus model the risk-free curve and country-specific risk premiums together. However, since we share the monetary policy with ECB as a member country of the Eurozone, it is reasonable to first extract the risk-free curve that is determined by the monetary policy and is thus common for all Eurozone members, with the objective to model the risk-free curve and country-specific risk premiums separately.

We represent the risk-free curve of Eurozone by the term structure of Overnight Indexed Swaps (OIS), the interest rate swaps with floating payment determined by the best tradable approximation of the risk-free rate, so-called the overnight rate (EONIA). Working with this representation we can then decompose the risk-free curve of Eurozone into average short rate expectations and term premiums, since they are determined only by the monetary policy and respective uncertainties. We could perform this decomposition by a variety of models implementing the short rate problematics, so-called the short rate models, but a lot of them have an essential problem with negative interest rates or time-varying variance and they are thus not applicable in our dataset. Therefore, we use a robust and simple approach based on a three-step linear regression estimated via Ordinary Least Squares (OLS).

In the next section, we focus on the specification of the country-related risk premiums and thus extract the most important drivers of the yield spreads on Slovak government bonds, namely credit premium, liquidity premium and QE effect, through the methodology of dynamic linear models, also known as the state-space models, and application of the Kalman filter. At the beginning, we introduce a set of economic and financial variables representing the dynamics of risk premiums and explain the spreads between government bond yields and corresponding OIS through a couple of linear regressions to obtain a basic specification of risk premiums across maturities. This decomposition offers a brief look into development of analysed premiums but incorporate several inaccuracies and complications, like the presence of measurement errors in the explanatory variables that could eventuate into inaccurate specification of the risk premiums or the representation of regression residuals.

However, we can overcome these issues through the methodology of dynamic linear models and filter out potential inaccuracies by the Kalman filter. In fact, this is nothing else than the already mentioned latent factor analysis, since we are able to extract the filtered latent factors from the yield spreads on Slovak government bonds and other observable variables. We identify the model through the matrix of factor loadings, capturing the mutual relationships between latent factors and observables and thus defining the risk premiums, and estimate remaining model parameters via Maximum Likelihood Estimation (MLE).

In the last section, we discuss potential improvements of the dynamic linear model presented above and outline a general approach for the decomposition of government bond yields with an arbitrary maturity, due to the absence of underlying economic and financial variables. We also discuss further extensions of our work.

2 Related literature

When Nelson and Siegel (1987) published their pioneering work about parsimonious modelling of yield curves it started up a massive research of the term structure models eventuating into a lot of extensions of the original work, the most widely used by Svensson (1994), and also a lot of new discoveries on the field of interest rate models. The original model by Nelson and Siegel (1987) offers a simple procedure to extract the zero-coupon yields from the supplied coupon bonds through the decomposition of tradable financial products into cashflows and applying a functional relationship between yields and time to maturity based on a set of model parameters. Later extension by Svensson (1994) expands the original model based on four parameters into the six-parametrical model to improve the quality of the estimation.

The basic idea behind the term structure estimation is to write the discount factors as a function of model parameters, due to the functional relationship between yields and time to maturity, and compare the prices of coupon bonds with the sum of their discounted cash flows to obtain a set of model pricing errors. By minimalizing the sum of squared pricing errors, we then obtain the estimation of model parameters which we can use to extract the zero-coupon yields and thus the term structure of interest rates.

Availability of the parsimonious specification for interest rates eventuated into a lot of work on the field of interest rate models. However, before discussing the term structure models, one should not forget about the research on the related field of so-called short rate models. These models are based on a particular stochastic differential equation for instantaneous interest rate, also known as the short rate, and thus model the evolution of interest rates by modelling the evolution of short rate in the risk-neutral probability measure or by incorporating the market price of risk in the real-world probability measure. Since they predict future behaviour of interest rates in risk-neutral world, they are used for pricing of interest rate derivatives.

Short rate models can be generally divided into one-factor models, describing the evolution of short rate by a single stochastic differential equation, and multi-factor models based on a set of stochastic differential equations. We should mention the model by Vasicek (1977) and the model by Cox, Ingersoll and Ross (1985), also known as the CIR model. Both of them incorporate a mean-reversion process of short rate into the stochastic differential equation and are thus the modifications of so-called Ornstein-Uhlenbeck process. However, Vasicek model allows only for the constant variance and CIR model only for the non-negative values of short rate, they are thus based on too rigorous restrictions for practical implementation.

We should mention also the important generalization of the mean-reversion models proposed by Chan, Karolyi, Longstaff and Sanders (1992), also known as the CKLS model. This modification allows for more general definition of the variance in the stochastic differential equation and thus theoretically solve some practical problems of short rate modelling, like time-varying variance and negative values of interest rates. However, since we need to estimate the model parameters, CKLS model is practically still constrained to the non-negative interest rates. Another approach to the short rate modelling allowing for time-varying deterministic parameters was proposed by Ho and Lee (1986).

However, short rate models are not truly applicable for the decomposition of yield curve into the short rate expectations and risk premiums, since the decomposition is impossible in the risk-neutral probability measure and problematic in the real-world probability measure, due to the issues with estimation of the market price of risk. Different approach to the problematics of interest rate models was proposed by Heath, Jarrow and Morton (1987) by constructing so-called HJM framework. In contrast to classic short rate models, models developed within the HJM framework capture the dynamics of the entire term structure of interest rates by modelling the instantaneous forward rate curve.

Although the short rate models are not applicable for the yield curve decomposition, they can be viewed only as the modification of a substantial group of interest rate models, so-called the affine term structure models. Generally, these models propose a relationship between the term structure of particular yields and underlying pricing factors based on the no-arbitrage restrictions. Pricing factors are then often determined by a set of stochastic differential equations for instantaneous interest rate and thus by a particular short rate model but can be also determined otherwise. Classic literature incorporating the affine term structure models was proposed by Duffie and Kan (1996), Dai and Singleton (2000) or Duffee (2002).

Pricing factors can be handled in numerous ways, for example like the latent factors, see Kim and Wright (2005), macroeconomic variables, see Rudebusch, Swanson and Wu (2006), or like the principal components of yields, see Adrian, Crump and Moench (2013, 2014). Estimation of the model parameters is usually performed by maximizing the likelihood or log-likelihood function of the underlying model and thus via standard Maximum Likelihood Estimation. Modern literature applying the affine term structure models, like Joslin, Singleton and Zhu (2011) or Hamilton and Wu (2012), is also working with the MLE.

Different approach to the problematics of term structure models was proposed by Adrian, Crump and Moench (2013, 2014) by developing the regression-based term structure model estimated via Ordinary Least Squares. Specifically, the model is based on the observable pricing factors represented by the principal components of yields and a three-step regression estimation. In the first step, a vector autoregressive process (VAR) is applied on the principal components to obtain the feedback matrix and residuals. In the second step, a standard linear regression is applied on the excess returns and in the third step, a market price of risk is obtained through the cross-sectional regression between estimated parameters. Finally, the no-arbitrage restrictions are used to extract the model-implied yields.

Decomposition of the risk-free curve into average short rate expectations and term premiums can be then performed through the adjustment of the risk-related parameters. Specifically, by setting up the market price of risk to zero, we obtain the estimation for average short rate expectations, and by keeping it unrestricted, we obtain the model-implied yields. Furthermore, since the only risk factor emerging in the risk-free curve is driven by the time uncertainty, we can obtain the estimation for term premium as the difference between model-implied yields and average short rate expectations. Adrian, Crump and Moench (2013) applied this methodology on the Treasury yields of United States but the model is fully applicable also for the Overnight Indexed Swap yields and thus for the risk-free curve of Eurozone.

This methodology can be viewed as the modification of traditional estimation approaches to the asset pricing models, for example the static cross-sectional approach developed by Fama and MacBeth (1973) or the later extension by Ferson and Harvey (1991). Another estimation method, based on the principal components of yields and additional pricing factor designed to predict the excess returns through a linear combination of forward rates, so-called the CP factor, was proposed by Cochrane and Piazzesi (2008).

Specification of particular risk premiums in the term structure of yields is usually performed through the latent factor models, since they are able to extract the driving factors of yields. Probably the most popular group of models are the dynamic linear models, also known as the state-space models, determined by a transition equation, describing the dynamics of latent factors, and a measurement equation, describing the relationship between the latent factors and corresponding observables. Identification of the model is usually performed through the identification of the matrix of factor loadings, due to the structure of observables and risk premiums, and estimation of remaining parameters via MLE. Evaluation of the latent factors for a particular model definition is meanwhile performed by the Kalman filter.

Decomposition of the yield spreads on Slovak government bond into particular risk premiums is primary based on the work of Krishnamurthy, Nagel and Vissing-Jorgensen (2015) and Ejsing, Grothe and Grothe (2015). Both papers apply the methodology of dynamic linear model and Kalman filter but differ in the approach to the model identification.

Krishnamurthy, Nagel and Vissing-Jorgensen (2015) analysed the impact of different expansionary monetary policies on the government bond yields of peripheral Eurozone countries, mostly Italy, Spain and Portugal, during the Eurozone debt crisis. Due to this objective, they decomposed the government bond yields into the risk-free yields represented by the OIS and country-specific risk premiums, namely credit, liquidity and segmentation, and analysed how these risk premiums react on the monetary policies. Model identification was performed through the fully-supplied matrix of factor loadings and estimation of the dynamic system via MLE. This kind of identification was possible due to the availability of underlying financial variables, different government and corporate yields, that defined the latent factors and thus the risk premiums through the mutual relationships.

Authors also incorporated the relationship between the observables and lagged factors, since the yields demonstrably reacted to the monetary policies in delayed fashion, and implemented this modification through the weighted Kalman filter. They also allowed for the time-varying covariance matrices captured by the observation that the higher values of government bond yields eventuate into the higher variance of factors and observables.

Ejsing, Grothe and Grothe (2015) analyzed different risk premiums in the German and French government bonds by the mutual relationships with the government-guaranteed agency bonds. More specifically, they analyzed the dynamics of credit and liquidity premiums by setting up the credit factor in sovereign and agency bonds one-to-one and capturing the differences through the liquidity factor. Therefore, they allowed for the unrestricted loadings for the sovereign and agency bonds with respect to the liquidity factor and thus estimated the dynamic system with not fully-supplied matrix of factor loadings.

Furthermore, authors restricted the off-diagonal elements of feedback matrix and both covariance matrices to zero and thus assumed the uncorrelated factors and uncorrelated innovations. They also discussed these assumptions for highly rated and highly liquid assets. By the estimation of liquidity factor in the government bonds, authors in fact captured the demand of investors for the safe destinations and thus the safe-haven flows into German and French government bonds during the Eurozone debt crisis.

Finally, we should mention the work of Ódor and Povala (2016) studying the most important drivers of the yields on Slovak government bonds. After adjusting the yield curve by the term structure of OIS and thus obtaining the term structure of risk premiums, authors performed two different regression-based approaches to the analysis. The first one study a decomposition of the credit spread on the international government bonds by a set of domestic macroeconomic variables, like Debt-to-GDP ratio or Terms-of-Trade index, together with a set of global financial variables, like TED spread or VIX index.

The second approach is based on the entire term structure of domestic government bond yields but this analysis was possible only after the adoption of Euro, due to the currency-related issues in the times of Slovak koruna. Term structure of risk premiums was decomposed into credit, liquidity and safety premiums, represented by the particular financial variables, effect of the Quantitative Easing, represented by a dummy variable, and effect of the market segmentation, represented by the regression residuals. Our paper is based on the work by Ódor and Povala (2016) but presents important improvements of the original analysis, mostly the estimation of risk premiums through the Kalman filtering.

3 Data and methodology

Our work is based on the zero-coupon yield curve published by the Ministry of Finance of Slovak Republic. This curve is continuously estimated by the Institute for Financial Policy and represents the term structure of yields on Slovak government bonds. Estimation of the Slovak zero-coupon curve is based on the methodology of Svensson (1994) with the detailed manual proposed by Ódor and Povala (2015). The manual describes the original algorithm together with the specifications for the Slovak government bonds.

Generally, to obtain a good estimation of the yield curve we need to have sufficient amount of issued coupon bonds. This is a problematic precondition for Slovak Republic, since we do not issue so many bonds like more tradable countries, but we work with the best approximation of the yield curve that is available. Only government bonds were applied for the estimation, since the corporate bonds can carry more specific risk factors. International bonds and foreign currency bonds were also excluded as they have different structure. The estimation is thus based on the government bonds issued in local currency on domestic market and the three-month Euribor rate applied to improve the short-end of the yield curve.

3.1 Zero-coupon curve

Due to the data availability and structure, we work with weekly data from January 2009 until October 2016. Application of daily data is not necessary, since the yields are relatively consistent over time. Although the estimation has been performed since January 2003, there are theoretical and practical issues with data consistency before and after the Euro adoption in January 2009. Due to the different currencies of issued bonds, we have to adjust the currency-related risk factors in the time period of Slovak koruna. If we ignore this adjustment, the zero-coupon curve cannot be consistently decomposed into the short rate expectations and risk premiums. Data consistency issues are also discussed in Ódor and Povala (2016).

We look at two different approaches to address this issue. The first one is based on the adjustment of the exchange rate risk factor through the difference between Slovak koruna and Euro swap rates. However, it seems to overcompensate the impact of exchange rate risk factor, since the estimated risk premiums became negative in corresponding time period without valid economic interpretation. There is also a data-related issue with this approach. Since the currency swap differentials are only available for the 5-years maturity, we would not be able to analyse the entire term structure of estimated yields.

The second approach estimates the yields from the international bonds issued in Euro and thus avoids the currency-related issues. However, if we apply the technique discussed in Ódor and Povala (2016), we obtain only the estimation for 10-years maturity and thus would not be able to analyse the entire term structure of estimated yields, similarly to the currency swap differentials. We can also estimate the entire zero-coupon curve from the international bonds but due to the lack of available bonds, we would not obtain correct estimation. Since we did not find appropriate solution to the currency-related issues, we work only with the consistent data after the adoption of Euro in January 2009.

Estimation of the zero-coupon curve is performed from the 1-year to the 10-years maturity until May 2006 and from the 1-year to the 15-years maturity afterwards. Due to the lack of applicable short-term bonds, we are not able to accurately estimate the yields with maturity under one year. On the other hand, maximal maturity is determined by the availability of long-term bonds but we work only with the maturities up to ten years, due to the issues at the long-end of the yield curve. Evolution of the estimated yields on zero-coupon bonds with 1-year, 5-years and 10-years maturity is displayed in Figure 1.

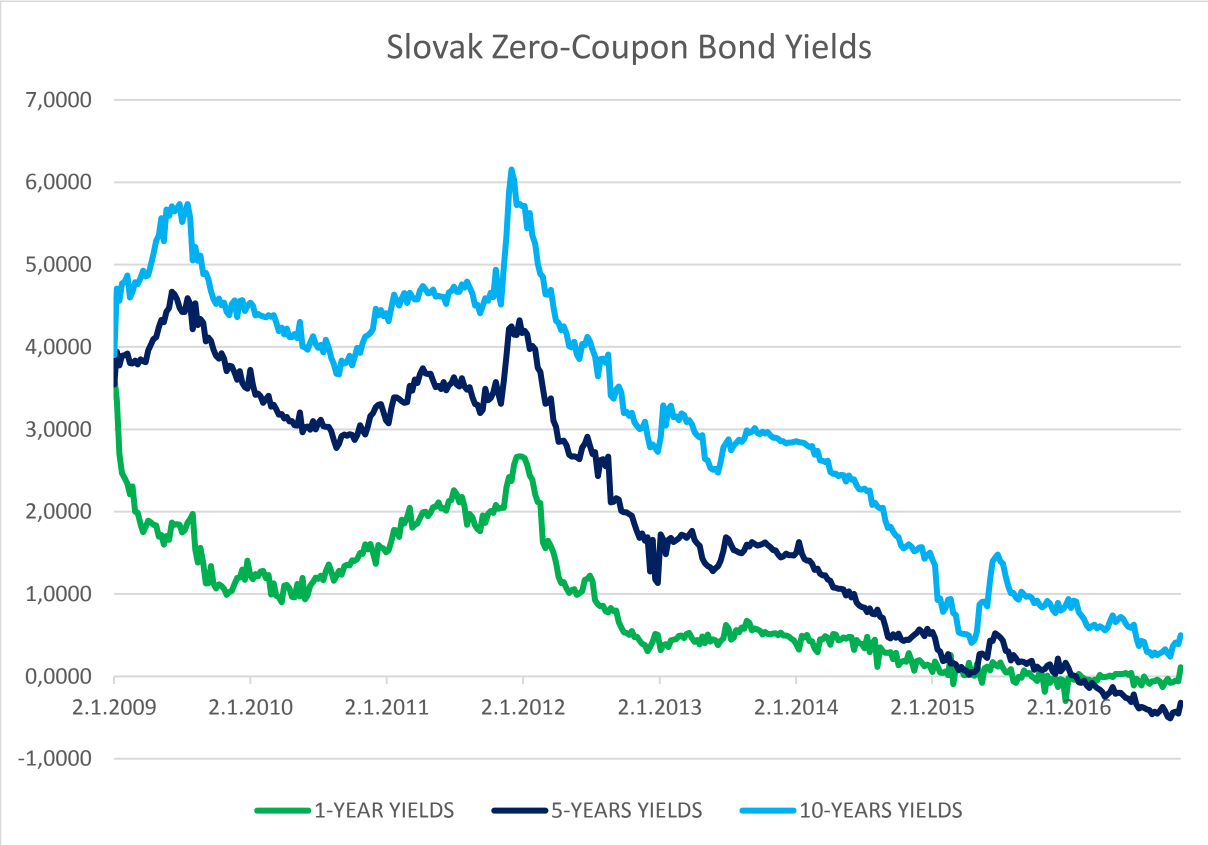

Figure 1: Evolution of the yields on Slovak zero-coupon bonds with 1-year maturity pictured by green colour, with 5-years maturity pictured by dark-blue colour and with 10-years maturity pictured by light-blue colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

3.2 Model overview

Our decomposition of the Slovak zero-coupon yield curve is principally based on the model introduced in Ódor and Povala (2016). This model assumes a decomposition of the yields on zero-coupon bonds with n-years maturity in particular time t into average short rate expectations, term premium and country-specific risk premiums. However, our model is based on a reduced number of the risk premiums, due to their significance, structure and availability of underlying financial variables. We can write the model as follows:

$$y_t^n = {}^1/_n \sum_{j=1}^{n} E_t(i_{t+j}) + tp_t^n + def_t^n + liq_t^n + QE_t^n \tag{1}$$

The first component denotes a sum of expectations in particular time t about future short rates over next n years divided by the number of years. We can represent this component as the average short rate expectation over maturity of a particular zero-coupon bond. Although this component represents only nominal short rate expectations, we do not make further decomposition into the real short rate expectations and inflation expectations, like Cieslak and Povala (2015), since they are not the key drivers of the yields on Slovak government bonds. The second component then denotes the term premium in the corresponding bonds, the compensation that investors require for holding the bonds with particular maturity. Specifically, the term premium reflects the uncertainty about the future path of policy rate. These two components thus represent the risk-free curve of Eurozone and are common for all Eurozone countries, due to the common monetary policy determined by the ECB.

Other components denote the risk premiums in Slovak government bonds that are specific for each Eurozone country. The first one is credit or default premium that represents the compensation for holding out the risk of potential country default. This is usually the most significant risk premium reflected in the government bonds. The second one is liquidity premium that can be positive in the case of highly liquid bonds and negative in the case of illiquid bonds. Specifically, in the case of government bonds with high liquidity, investors are ready to pay additional charge for these assets. On the other hand, in the case of government bonds with low liquidity, investors require additional compensation for holding out the illiquidity risk. The last component is not standard risk premium but denotes the effect of QE, nonstandard monetary policy applied by the ECB to stimulate loans, investments and economic growth through lowering the long-term yields on government bonds, due to the low level of short-term interest rates and thus the absence of standard monetary policy.

As discussed above, there are also other factors affecting the government bond yields. Safety premium represents the demand for safe assets and thus the safety level of particular government bonds. Redenomination risk represents the problems related to the currency redenomination, for example the change of currency value in the case of high inflation or the change of the entire currency, like the adoption of Euro. Finally, the segmentation effect represents the market segmentation arising from the differences in regulatory rules or possible investment constraints. However, we do not include these factors in our decomposition.

We have two arguments to exclude the direct representation of safety premium. The first one is based on the work of Ódor and Povala (2016) where authors represented the safety premium by the spread between the three-month German T-Bills and corresponding OIS yields. This spread should capture the demand for liquidity and safety, since both of these assets carry no additional risk premiums and the difference between them is thus fully determined by the liquidity and safety premiums. We discuss the arguments for this assumption later.

The difference between the three-month German T-Bills and corresponding OIS yields was then applied as the driver for the safety factor in a linear regression what eventuated into positive correlation with the yields on Slovak government bonds. However, positive impact of the safety premium on the government bond yields misses valid economic interpretation, since the safety premium is defined as the price that investors are ready to pay for safe assets and not as additional compensation for investors. This positive impact can be thus interpreted only as the absence of the safety premium and further creates the spurious relationship in the regression that could be allocated between other risk premiums. Due to these empirical issues, we do not represent the safety factor by the difference between German T-Bills and OIS yields and since we did not find any alternative representation, we exclude the direct safety premium from our decomposition of the yields on Slovak government bonds.

The second argument is based on the work of Ejsing, Grothe and Grothe (2015). Authors analyzed credit and liquidity premiums in the German and French government bonds with the objective to capture the safe-haven flows into these assets through the dynamics of the liquidity premium. Negative values of the liquidity premium can be thus represented not only as the price that investors are ready to pay for highly liquid assets but also as the price they are ready to pay for safe assets, especially during the uncertain times like the Eurozone debt crisis, when we can observe significant decrease in the level of liquidity premiums of particular government bonds. Therefore, there is not strong motivation to capture the direct safety premium, since we can represent this factor through the dynamics of liquidity premium.

We exclude the redenomination premium, since it should not be the significant driver of the yields on Slovak government bonds. Again, we have two arguments for this assumption. The first one is based on the low debt-to-GDP ratio and thus the little incentive to redenominate the government debt and the high GDP growth which supports the convergence to the Eurozone level. The basic problem for the countries like Italy is the high debt-to-GDP ratio together with the small economic growth and low level of the inflation, further increasing the real debt, what eventuates into the increased redenomination risk.

The second argument is based on the low probability of the currency redenomination, since after the adoption of Euro there is not strong motivation for the reversing redenomination of the currency. Specifically, we do not have any rational motivation for the return to Slovak koruna, since the adoption of Euro significantly increased the real economic growth, as discussed in Žúdel and Melioris (2016), and moreover eliminated a number of currency-related issues. Therefore, the redenomination risk could be the significant driver of the Slovak sovereign bond yields only immediately after the adoption of Euro, due to the uncertainty about future development, but not afterwards and we thus exclude this risk factor from our decomposition.

Finally, we exclude the segmentation effect, since we do not have enough information to correctly estimate this factor. Generally, there are two alternatives how to estimate the segmentation effect using the methodology of dynamic linear models. The first one is based on its direct representation by a combination of particular financial assets and variables but we did not find any representation for this factor. The second one is to treat the segmentation effect like the residual factor identified by the other components of yields. This approach was applied by Krishnamurthy, Nagel and Vissing-Jorgensen (2015) but requires additional assumptions about the model development. Specifically, to ensure correct model identification, the model should be identified through the fully-supplied matrix of factor loadings. This assumption prevents from potential ambiguities of the estimation that can arise from the combination of residual factor and not fully-determined matrix of factor loadings.

Since we cannot supply the entire matrix of factor loadings, as will be discussed later, we should not incorporate any residual factor into the model development and thus have to exclude the segmentation effect from our decomposition. Although Krishnamurthy, Nagel and Vising-Jorgensen (2015) discussed the presence of segmentation effect in a variety of asset markets, Ódor and Povala (2016) treated this factor only as the regression residuals. Therefore, if we can represent most of the variation in the yields with the other factors, we can exclude the segmentation effect without too rigid assumptions about the model development.

3.3 Overnight indexed swaps

As discussed above, we prefer to first decompose the yield curve into the risk-free curve and the country-specific risk premiums and then model them separately over modelling the entire curve together. We have two important arguments for this alternative. On the one hand, the dynamics of the risk-free curve and the dynamics of the risk premiums are relatively independent, since the risk-free curve is driven by the monetary policy of ECB and the risk premiums by the specifications of particular country. On the other hand, the risk-free curve of Eurozone is very similar to the one modelled by Adrian, Crump and Moench (2013).

Due to this objective, we have to find a good approximation for the risk-free curve of Eurozone. We can either apply the German zero-coupon curve, since the German bunds are considered to be the safest and most liquid bonds in Eurozone, or the term structure of Overnight indexed swaps, the interest rate swaps tied to the almost risk-free overnight rate (EONIA), the best tradable approximation for the risk-free rate. We obtain the German zero-coupon curve from the Deutsche Bundesbank website and information about OIS from the Bloomberg. Data are obtained from January 2009 until October 2016 on weekly basis to be consistent with the Slovak zero-coupon curve. We can see the evolution of the yields on 1-year and 10-years German bunds together with corresponding OIS yields in Figure 2.

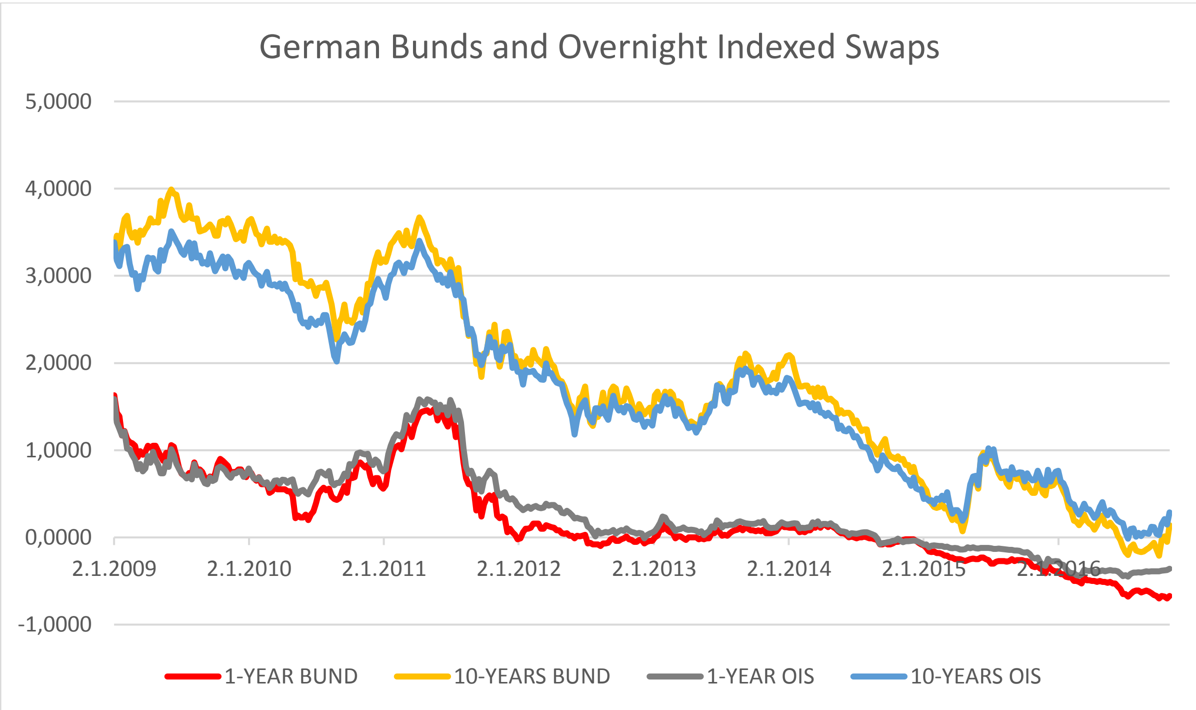

Figure 2: Evolution of the yields on German bunds with 1-year maturity pictured by red colour and with 10-years maturity by orange colour. OIS yields with 1-year maturity are pictured by grey colour and with 10-years maturity by blue colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

Although both of these assets approximate the risk-free curve of Eurozone, there are some significant differences between them, as discussed in Ódor and Povala (2016). OIS carry minimal credit premium, since they do not reflect the banking system credit risk, in contrast to the other interest rate swaps, and due to their non-cash nature, they also do not carry any liquidity or safety premiums. However, liquidity and safety premiums in German bunds take mostly negative values, since they provide high-liquidity and safe-storage services to investors. The short-end of the German yield curve carry no additional credit premium but there is a significant credit premium at the long-end, due to the higher probability of default in time.

We can see these inconsistencies also in Figure 2. Generally, German bunds offer lower yields than the OIS for the 1-year maturity, due to the negative values of liquidity and safety premiums. On the other hand, credit premium in the 10-years German bunds generally overcomes liquidity and safety premiums and thus the long-term German bunds generally offer higher yields than the corresponding OIS. We should also mention that over the last year we observe the opposite effect. Due to the significant risk premiums in the German bunds, we approximate the Eurozone risk-free curve by the term structure of OIS.

3.4 Credit default swaps

We perform the specification of the country-related risk premiums, or rather the driving factors of the government bond spreads, through the methodology of dynamic linear models and Kalman filtering. Specifically, we need to estimate the credit and liquidity premiums and the effect of QE through the Kalman filter after the estimation of corresponding dynamic linear model. Furthermore, this procedure is based on a set of financial variables, or the observables, that identify the latent factors through the mutual relationships captured by the matrix of factor loadings. In our work, we identify each factor by corresponding financial variable and then specify their relationships with the Slovak government bonds.

We represent the credit premiums by the Credit Default Swaps (CDS), the most basic type of the credit derivative. Generally, CDS work as the insurance against a potential default when the buyer agrees to pay periodical payments to the seller over the maturity of the particular CDS and in the case of country or company default obtains an arranged amount of compensation. CDS thus capture the credit risk of a particular country or company and represent a good approximation for the credit premiums in government bonds. However, there are several issues arising from this implementation and assumption about one-to-one effect of the credit factor on the government bonds and corresponding CDS.

To be more specific, Krishnamurthy, Nagel and Vissing-Jorgensen (2015) have two arguments against the application of CDS as the identification for credit premium. The first one is based on the observation that not all default-related events, for example a debt restructuring, have to trigger corresponding CDS. Due to this assumption, empirically supported by the uncertainty around the restructuring of the Greek debt during the Eurozone debt crisis, CDS in fact underestimate the underlying default risk because the market participants believe that it would not be triggered in all default-related events.

The second argument is based on the assumption that the market segmentation affects not only the government bond yields but also corresponding CDS. Therefore, the segmentation effect should be incorporated into CDS in the same way as in the government bonds but probably with different magnitude. The main issue arises from the fact that we do not have additional information about the relative magnitudes of segmentation effect and thus cannot effectively incorporate these differences into the model.

Moreover, the market with Slovak sovereign CDS is very illiquid what produces additional issues for this implementation. On the one hand, low liquidity of the sovereign CDS could eventuate into the significant liquidity premium as in the case of government bonds. This is very similar to the segmentation effect problem, since we do not have additional information about the relationship between the liquidity factor in the government bond yields and the liquidity factor in corresponding CDS. On the other hand, the low-liquid market with sovereign CDS eventuates into the application of the Dollar denominated swaps, despite the Euro denominated ones, due to their higher liquidity. However, this representation produces additional complications with the currency-related risk factors.

We can theoretically overcome some of these issues if we estimate the relationship between the government bonds and corresponding CDS unrestricted but we would not obtain the correct estimation of the model parameters, due to the lack of additional information and the ambiguity of the estimation. Another alternative is to forget on the application of CDS and represent the credit premium by the Dollar denominated government bonds, as discussed by Krishnamurthy, Nagel and Vissing-Jorgensen (2015). However, we do not have a sufficient amount of bonds denominated in foreign currencies to apply this procedure. Moreover, additional complications would arise from the estimation of liquidity factor, since the magnitude of liquidity factor in the Euro and Dollar denominated government bonds could be different.

Due to the absence of alternatives, we represent the credit risk by the sovereign CDS denominated in US Dollar and assume one-to-one effect of the credit factor on the government bonds and corresponding CDS. Specifically, we apply the CDS with 5-years and 10-years maturity, due to their solid liquidity, and thus capture the middle and the long-end of the credit factor. Furthermore, we apply the CDS with 1-year maturity to pin down also the short-end of the credit factor. Data are obtained from the Bloomberg and are consistent with the yields on Slovak government bonds in the terms of time period and frequency.

Returning to the main issues with CDS, the problem with segmentation effect was solved itself, since we do not incorporate segmentation in our decomposition and the problem with currency-related risk factors is secondary, due to the nature of credit contracts that carry almost zero currency risk. Of course, there are also exceptions, partially captured by the quanto spreads. The main issue thus remains the relationship between the credit and liquidity factors in the government bonds and corresponding CDS.

3.5 Illiquidity measure

We represent the liquidity premium by the aggregate level of illiquidity in Slovak government bonds. Estimation of the illiquidity is based on the assumption that arbitrageurs exploit small pricing errors in the assets with similar maturities and thus tie the yields on these assets together, as the effect of the arbitrage exploiting. Therefore, large price deviations are eliminated by arbitrageurs in the assets with normal liquidity and observed differences between related assets can be in fact viewed as the estimation of illiquidity.

Specifically, we measure the illiquidity in Slovak government bonds by averaging the pricing errors between the observed coupon bond yields and their modelled counterparts obtained by the methodology of Svensson (1994) for each trading day. This illiquidity measure based on the noise in observed yields was proposed by Hu, Pan and Wang (2013) and we can write the aggregate measure of illiquidity and thus the approximation for the liquidity premium in the Slovak government bonds in particular time t as follows:

$$Liq_t = \sqrt{{}^1/_{N_t} \sum_{j=1}^{N_t} (y_t^j - \hat{y}_t^j)^2} \tag{2}$$

where element y_t^j denotes the observed coupon bond yield and element $\hat{y}_t^j$ its modelled counterpart. N_t then denotes the number of observable coupon bonds in particular time t. This measurement of aggregate illiquidity is thus the root-mean-squared error (RMSE) of the applied yield curve model in a particular trading day. The illiquidity measure by Hu, Pan and Wang (2013) is published together with the Slovak zero-coupon curve and thus we obtain these values from the Institute for Financial Policy. Estimation of the illiquidity is performed on the issued bonds with maturity from one to ten years and is thus consistent with our original dataset. We can read further information in the manual by Ódor and Povala (2015).

Since we apply the aggregate measure of illiquidity and not the maturity-specific approximation for the liquidity premiums and do not have additional information about the relative magnitudes of the liquidity factor in the government bonds and the illiquidity measure, we cannot work with the fully-supplied matrix of factor loadings and have to estimate these relationships unrestricted, together with the other parameters of the dynamic linear model.

3.6 Synthetic control method

There are numerous ways to capture the QE effect on the government bonds and a number of works analysing the problematics, for example the one by Andrade, Breckenfelder, De Fiore, Karadi and Tristani (2016). Our representation of the QE effect is based on the results of the Synthetic Control Method (SCM) applied on the Slovak government bond yields. This methodology is similar to the one implemented by Žúdel and Melioris (2016) but instead of focusing on the economic growth analyses the yields on government bonds. We obtain the SCM results capturing the QE effect from the Institute for Financial Policy. These results are thus of the independent analysis and were not estimated within this paper.

Synthetic Control Method was developed by Abadie and Gardeazabal (2003) and later extended by Abadie, Diamond and Hainmueller (2010, 2015). This method compares the countries related in the development of observed variables, for example the yields on government bonds, and constructs the synthetic counterparts of the particular country variables through the weighted average of these variables in the related countries. SCM is thus a suitable method to capture the effect of a new intervention, for example the QE, since we can estimate the weights of the related countries in the times before the intervention and construct the synthetic counterparts of particular variables and then interpret the differences between the observed and synthetic variables after the intervention as the impact of this intervention.

Žúdel and Melioris (2016) implemented the SCM to estimate the impact of the Euro adoption on the Slovak economic growth. They further discussed the potential advantages of the SCM with regard to standard regression techniques, like elimination of the potential biases or restricting the weights to lie between zero and one. Estimation of the QE effect is based on a similar SCM model assuming that the QE is limited only to the countries within the Eurozone and thus estimating the yields on Slovak government bonds through the government bond yields of European countries outside the Eurozone and also through some non-European countries. The difference between the observed and synthetic yields after the QE announcement can be then represented as the impact of the QE. The best results were obtained at the long-end of the yield curve, especially for the bonds with 10-years maturity, while the short-end of the yield curve was imprecisely estimated, probably due to the liquidity and measurement issues.

We represent the QE by the difference between the observed and synthetic yields on the Slovak government bonds with 10-years maturity, due to the quality of the estimation, and thus capture the long-end of the yield curve. Since we are not able to capture the short-end, due to the inaccuracy of the SCM estimation and also the questionable impact of QE on the short-end, we pin down at least the middle of the yield curve. We apply the SCM results for the 4-years maturity, since the QE effect for the 5-years maturity was estimated positive in a significant time period what misses a valid economic interpretation. We display the SCM estimation of the QE effect on the Slovak government bonds in Figure 3.

Again, there are several issues arising from this implementation. The first one is related to the methodology of the estimation, since the SCM model was applied on the government bond yields issued in local currencies. Therefore, the estimated QE effect can carry additional currency risk and as discussed in Du and Schreger (2016), local currency risk factors can be strongly significant eventuating into the problems with currency differences. We can theoretically overcome this issue by swapping the yields from local currencies to Euro but the currency swap market is not working properly after the economic and financial crisis, as discussed by Du, Tepper and Verdelhan (2017). Another alternative is to apply the SCM methodology on the Euro denominated bonds if they are available.

Other issues arise from the data inconsistency, since the SCM model was not applied on the zero-coupon curves of particular countries but rather on their tracked yields, due to the limited availability of the zero-coupon curves. Therefore, the estimated QE effect is not fully consistent with the yields on zero-coupon government bonds. Another inconsistency is driven by the fact that the SCM estimation was applied on the data before the QE announcement in September 2014 and not the QE settlement in January 2015 and thus did not incorporate additional four months of data development. Due to these inconsistencies, we estimate the impact of QE on the government bond yields unrestricted.

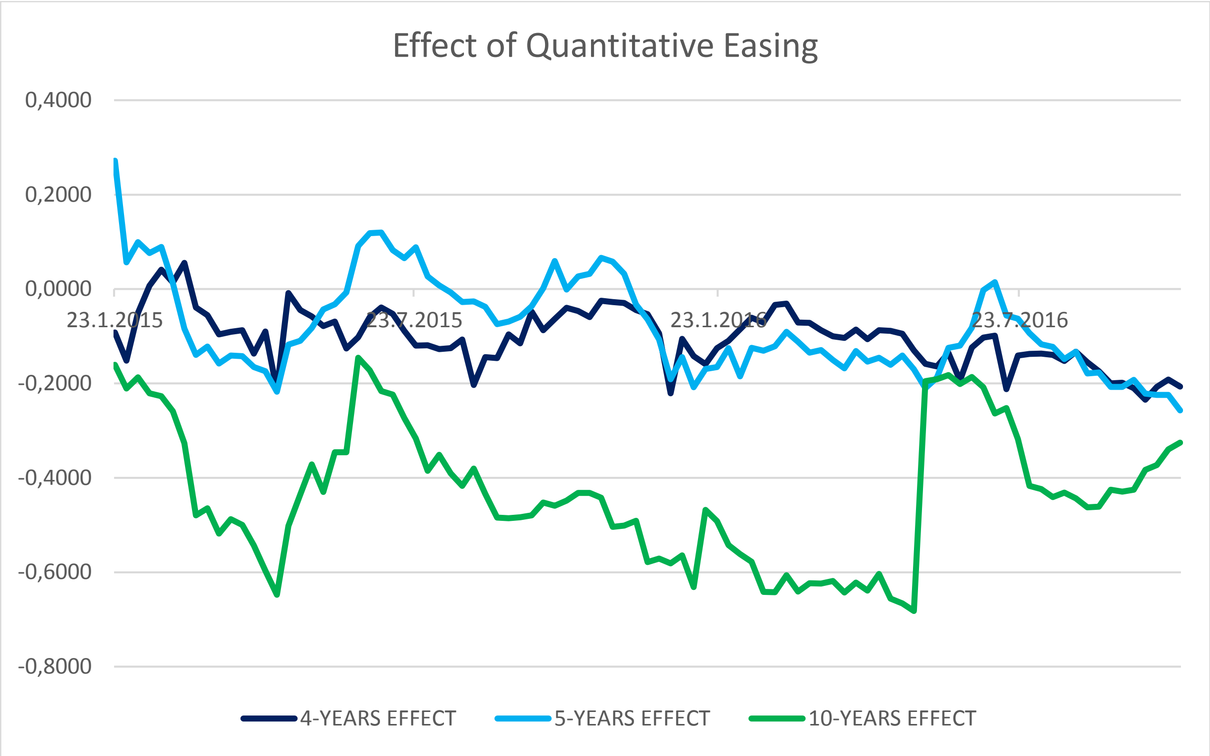

Figure 3: Evolution of the QE effect on the Slovak government bonds with 4-years maturity pictured by dark-blue colour, with 5-years maturity pictured by light-blue colour and with 10-years maturity pictured by green colour. Time period is from 23.1.2015, when the QE was launched, to 28.10.2016. Data are weekly.

4 OIS decomposition

Our objective in this chapter is make the decomposition of the Overnight Indexed Swap yields into average short rate expectations and term premiums to better understand the drivers of the Eurozone risk-free curve. As discussed above, we use the model developed by Adrian, Crump and Moench (2013, 2014), further denoted as the ACM model. In contrast to standard latent factor models, the ACM model makes unobservable factors observable and thus simplifies the estimation of a particular term structure. Model parameters are then estimated by a three-step regression approach and application of the OLS estimators, in contrast to standard MLE estimators applied in latent factor models.

We work with weekly yields from January 2006 until December 2015 with maturities from one to ten years and also with 1-week maturity to approximate the risk-free rate. There is a practical problem with the estimation starting in January 2009, since we cannot obtain a good quality of approximation for the yields. This issue is probably caused by the small data sample, impact of the economic and financial crisis and mostly by the low level of the yields with small volatility, also known as the zero lower bound. We therefore add additional three years to the data sample to improve the quality of the estimation.

4.1 Principal components

We represent the pricing factors by the principal components of the yields, like Adrian, Crump and Moench (2013, 2014), but apply only first three principal components, since more factors would not significantly improve the estimation, as discussed in Ódor and Povala (2016). This observation is in contrast to the original work by Adrian, Crump and Moench (2013), since they applied first five principal components in the decomposition of US Treasury yields. We extract the principal components from the term structure of OIS through the Principal Components Analysis (PCA) applied on the data adjusted by the mean and volatility. Correlations between particular OIS yields and first three principal components are displayed in Table 1.

	1-Week OIS	1-Year OIS	5-Years OIS	10-Years OIS
First PC	-0.28945	-0.29615	-0.30554	-0.30045
Second PC	-0.56169	-0.45769	0.08721	0.34187
Third PC	-0.72484	0.18025	0.12776	-0.23882

Table 1: Correlation between the yields on OIS with 1-week, 1-year, 5-years and 10-years maturity and the first three principal components extracted from the term structure of OIS via PCA.

The first principal component thus represents the level of the yields and explains around 97% of the variance in data. The second principal component represents the slope and explains more than 2.6% of the variance and the third principal component represents the curvature and explains more than 0.2% of the variance. Cumulatively they explain around 99.96% of the variance in the term structure of OIS. First three principal components are therefore a good representation for the pricing factors.

4.2 ACM model

As discussed above, affine term structure models assume a relationship between the term structure of particular yields and underlying pricing factors. ACM model further makes the pricing factors observable through the principal components of the yields. We represent the yields in the same way as Adrian, Crump and Moench (2013) and thus through the prices of corresponding zero-coupon bonds, captured by the Overnight Indexed Swaps, and a set of equations where the Equation (3) describes the relationship between the pricing factors and the prices on zero-coupon OIS and the Equation (4) mutual relationship between the zero-coupon bond prices and corresponding yields:

$$lnP_t^n = A_n + B_n^T X_t + u_t^n \tag{3}$$

$$y_t^n = -{}^{m}/_{n}\, lnP_t^n \tag{4}$$

Specifically, element P_t^n denotes the price of zero-coupon bond with n-period maturity in particular time t and element y_t^n corresponding yield. Since we apply weekly data, the model is based on the weekly periods and therefore we denote the number of periods per year by the term $m = 52$. Furthermore, element X_t denotes a vector of pricing factors in particular time t and elements A_n and B_n pricing parameters for n-period maturity. Element u_t^n then represents the residual for corresponding time and maturity. We can see that pricing factors are driven only by particular time and pricing parameters only by maturity. We further suppose continuous compounding of interest rates and thus can rewrite a price of zero-coupon bond with unit face value through the exponential discount factor, as we can see in Equation (4).

By the estimation of pricing parameters A_n and B_n, we can therefore estimate the OIS yields and by setting up the risk-free pricing parameters also average short rate expectations over next n periods. We can then capture a term premium for particular bond as the difference between model-implied yields and model-implied short rate expectations. Estimation of the pricing parameters in ACM model is made through a three-step regression approach and application of the no-arbitrage restrictions. We begin with assumption that the dynamics of pricing factors follows a vector autoregressive process of first order with unrestricted feedback matrix and thus write the dynamics of pricing factors as follows:

$$X_{t+1} = \mu + \phi X_t + v_{t+1} \tag{5}$$

As discussed in Adrian, Crump and Moench (2013), we can assume zero means of the pricing factors and thus set $\mu = 0$, since the principal components are extracted from the standardised yields. Particular values of pricing factors therefore depend only on their lagged values and corresponding residuals. However, we are not able to explicitly estimate parameters A_n and B_n only through the yields and corresponding dynamics of the pricing factors but instead we have to construct so-called excess returns that are further analysed through the ACM model. Logarithmic excess returns are defined as follows:

$$rx_{t+1}^{n-1} = lnP_{t+1}^{n-1} - lnP_t^n + lnP_t^1 \tag{6}$$

Element P_t^1 represents the price of zero-coupon bond with 1-period maturity and the yield of this bond is thus the approximation for the risk-free rate. More specifically, the risk-free rate is approximated by the weekly OIS yield. Due to the continuous compounding of interest rates, we can further rewrite the term $-lnP_t^n$ through the corresponding OIS yield multiplied by the number of years to maturity. Only remaining issue is thus the approximation of yields with n-1 weeks to maturity, since we apply only annual OIS yields. However, we can overcome this issue through the linear interpolation by the closest available yields.

As discussed by Adrian, Crump and Moench (2013), affine term structure models of interest rates usually start with three assumptions: exponentially affine pricing kernel in the shocks driving the economy, affine prices of risk in the state variables, conditionally normal innovations to state variables and log yield observation errors. Due to these assumptions, Adrian, Crump and Moench (2013) constructed the return generating process for log excess returns that can be written as follows:

$$rx_{t+1}^{n-1} = \beta_{n-1}^T(\lambda_0 + \lambda_1 X_t) - {}^1/_2\,(\beta_{n-1}^T \Sigma \beta_{n-1} + \sigma^2) + \beta_{n-1}^T v_{t+1} + e_{t+1}^{n-1} \tag{7}$$

where elements λ_0 and λ_1 represent the market price of risk and matrix Σ with value σ^2 variance and covariance components. Element β_{n-1} further captures the relationship between excess returns and pricing factors and element v_{t+1} residuals from the vector autoregression process of the pricing factors. Estimation of these parameters is performed through a three-step regression approach developed by Adrian, Crump and Moench (2013, 2014).

Representation of particular components of the return generating process is meanwhile the following: the first component of the sum denotes expected excess returns based on the pricing factors adjusted by the market price of risk, the next component represents additional convexity adjustment to improve the quality of the estimation, the next to last one captures the excess returns innovations based on the innovations of the pricing factors and the last component denotes corresponding excess returns residuals.

4.3 Model estimation

In the first step, we estimate the dynamics of the pricing factors through the vector autoregression defined in Equation (5). Estimation is performed via the OLS and results into the approximation of the feedback matrix ϕ and the matrix of innovations $\hat{V}$ constructed from the innovations v_{t+1}. If we further denote the number of periods minus one by T, due to the logical absence of the residuals for the first period, we can estimate the variance-covariance matrix of the pricing factors as $\hat{\sum} = \hat{V}\hat{V}^T/T$. We continue with the second step and model the excess returns through the return generating process defined in Equation (7). We can simplify this relationship by stacking the components corresponding to the pricing factors, components corresponding to the innovations and remaining components to three independent vectors and substitute the relationship into Equation (8). If we further rewrite the relationship in a matrix form and collect all explanatory variables in a single matrix Z, we can estimate the model parameters $\hat{a}, \hat{\beta}, \hat{c}$ by a simple OLS estimator described in Equation (9):

$$rx_{t+1}^{n-1} = a_{n-1} + \beta_{n-1}^T v_{t+1} + c_{n-1}^T X_t + e_{t+1}^{n-1} \tag{8}$$

$$\left[\hat{a}, \hat{\beta}^T, \hat{c}\right] = rxZ^T(ZZ^T)^{-1} \tag{9}$$

We stack the residuals from the excess returns regression into the matrix $\hat{E}$ with the number of rows corresponding to the number of annual maturities N and the number of columns corresponding to the number of model-related periods T. We further compute the variance σ^2 as the trace of the matrix $\hat{E}\hat{E}^T/NT$ and continue with the last step where we estimate the market price of risk λ_0 and λ_1 through the relationship between Equation (7) and Equation (8). Since we denoted particular model components by the parameters a and c, we can backwards apply this substitution to estimate the market price of risk as follows:

$$\widehat{\lambda_0} = \left(\hat{\beta}\hat{\beta}^T\right)^{-1}\hat{\beta}\left(\hat{a} + {}^1\!/_2\left(\hat{R} + \hat{\sigma}^2\right)\right) \tag{10}$$

$$\widehat{\lambda_1} = \left(\hat{\beta}\hat{\beta}^T\right)^{-1}\hat{\beta}\hat{c} \tag{11}$$

where the components of vector $\hat{R}$ are equal to the values of $\hat{\beta}_{n-1}^T\hat{\Sigma}\hat{\beta}_{n-1}$ computed for each time to maturity. After estimating the market price of risk, we obtain all components of the return generating process described in Equation (7) and are thus able to model the log excess returns. Furthermore, by applying the no-arbitrage principle we are able to extract the pricing parameters A_n and B_n and thus model the OIS yields. We should also mention that in the first two steps we estimated time series data whereas in the third step we estimated cross-sectional data and thus applied so-called cross-sectional regression.

4.4 No-arbitrage restrictions

One of the basic premises in financial models is the no-arbitrage principle. Specifically, there should not be any free-money opportunity in the financial markets or the particular model is essentially wrong. Due to this premise, we can create the no-arbitrage restrictions for the pricing parameters A_n and B_n based on the excess returns. If we rewrite the definition of logarithmic excess returns from Equation (6) through the decomposition of bond prices in Equation (3) and also rewrite the pricing factors through their dynamics described in Equation (5), we obtain the following decomposition for the log excess returns:

$$rx_{t+1}^{n-1} = A_{n-1} + B_{n-1}^T(\mu + \phi X_t + v_{t+1}) + u_{t+1}^{n-1} - A_n - B_n^T X_t - u_t^n + A_1 + B_1^T X_t + u_t^1 \tag{12}$$

Due to the no-arbitrage principle, this formulation has to be equal to the return generating process defined in Equation (7) for all times and maturities. We can thus extract the relationship between pricing errors into separate equation and also assume the equality between B_n and β_n. Finally, we can decompose the relationship between Equation (7) and Equation (12) into the terms corresponding to the pricing factors and remaining terms. The no-arbitrage restrictions for the pricing parameters A_n and B_n can be then written as follows:

$$A_n = A_{n-1} + B_{n-1}^T(\mu - \lambda_0) + {}^1\!/_2\, B_{n-1}^T \Sigma B_{n-1} + {}^1\!/_2\, \sigma^2 + A_1 \tag{13}$$

$$B_n^T = B_{n-1}^T(\phi - \lambda_1) + B_1^T \tag{14}$$

We thus obtain the system of recursive equations for the pricing parameters A_n and B_n. Furthermore, we denote $\delta_0 = A_1$ and $\delta_1 = B_1$ and estimate these parameters through the OLS applied on Equation (3) for the time series of yields with 1-month maturity. Due to the no-arbitrage principle, we do not obtain any yield for the bond with zero periods to maturity and therefore can set the starting values for the pricing parameters to $A_0 = 0$ and $B_0 = 0$. Since we already estimated the model parameters and set the starting values, we can recursively estimate the pricing parameters A_n and B_n for all maturities through Equation (12) and Equation (13). When we obtain the approximation of the pricing parameters, we can substitute them into Equation (3) and estimate the term structure of the OIS yields through the sample period by the first three principal components.

We did not estimate all the pricing parameters through Equation (3), since we would ignore the relationships between maturities, dynamics of the pricing factors and also the no-arbitrage restrictions. We highlight one more curiosity. Although the estimation of the parameter B_1 through the no-arbitrage restrictions is equal to the estimation through the OLS methodology, recursive estimator of the parameter A_1 is not equal to the original OLS estimator. More specifically, the difference consists in the appearance of the term ${}^1/_2\,\sigma^2$ in the no-arbitrage restrictions. As discussed by Adrian, Crump and Moench (2013), this difference arises due to the fact that we incorporate pricing errors directly into the no-arbitrage restrictions and thus adjust the original estimators of the pricing parameters by the pricing errors.

4.5 Empirical results

Decomposition of the yields into average short-rate expectations and term premiums is performed through the definition of the return generating process in Equation (7). Due to the fact, that we can represent λ_0 and λ_1 as the market price of risk incorporated in the pricing parameters A_n and B_n and thus capture the risk factors, we can generate the risk-free yields carrying no additional risk premiums. Specifically, setting the market price of risk to zero in Equation (13) and Equation (14) generates a new set of pricing parameters A_n^{RF} and B_n^{RF} adjusted by the additional risk factors. These risk-free pricing parameters can be then substituted into Equation (3) to obtain the risk-free yields and thus the average short rate expectations over next n periods in particular time t.

Term premiums in the OIS yields are then estimated as the difference between model-implied yields and model-implied short rate expectations and thus as the risk factor in the OIS yields. We can make this assumption, since we work with the risk-free curve of Eurozone with the only source of risk related to the uncertainties about monetary policy and future development and therefore captured by the term premium. We can see the decomposition of the OIS yields into average short rate expectations and term premiums for the 1-year maturity in Figure 4 and for the 10-years maturity in Figure 5.

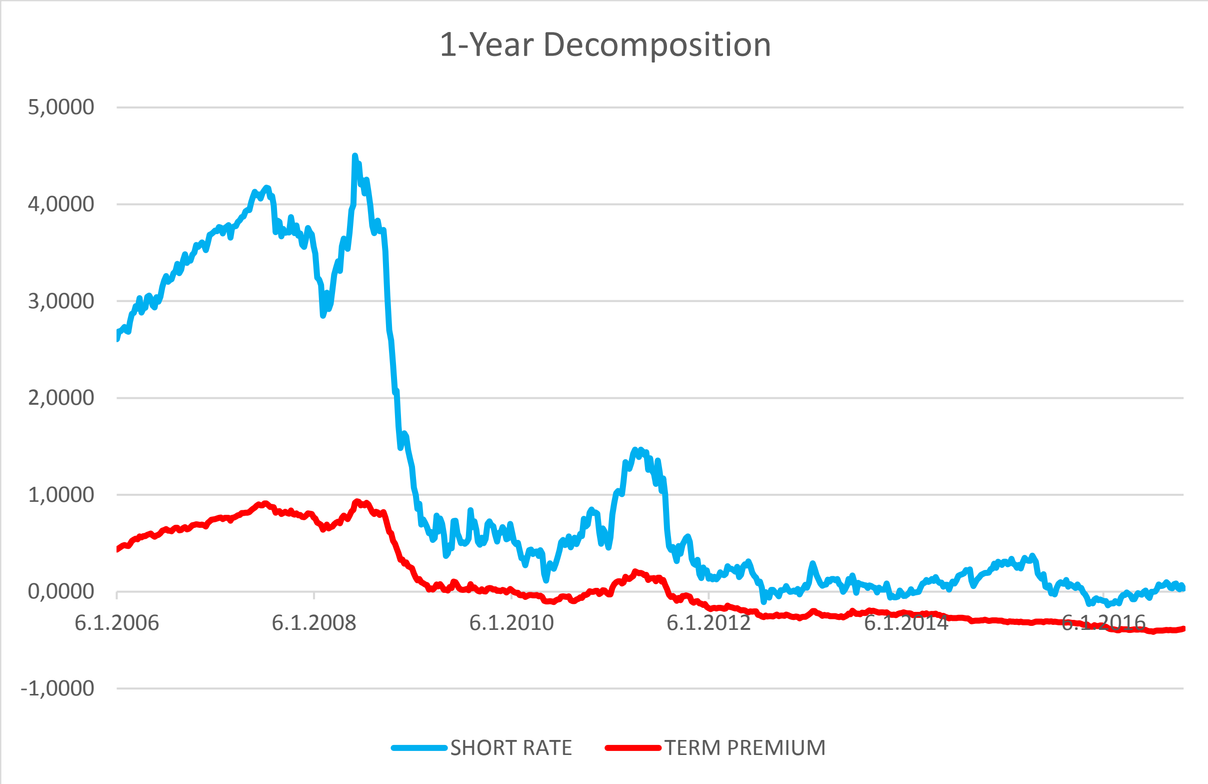

Figure 4: Decomposition of the yields on 1-year OIS into average short rate expectations pictured by blue colour and maturity specific term premium pictured by red colour. Time period is from January 2006 to October 2016. Data are weekly.

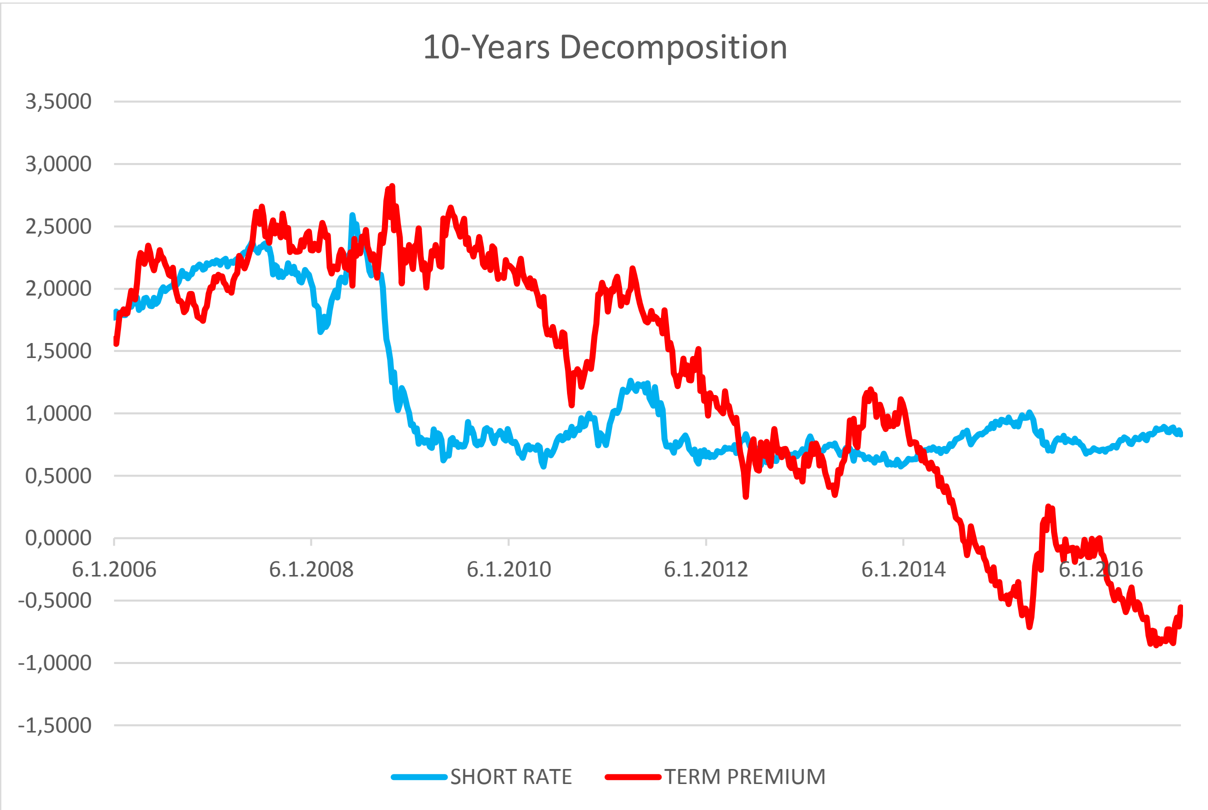

Figure 5: Decomposition of the yields on 10-years OIS into average short rate expectations pictured by blue colour and maturity specific term premium pictured by red colour. Time period is from January 2006 to October 2016. Data are weekly.

We can see in Figure 4 that the yields on 1-year Overnight Indexed Swaps are driven mostly by the short rate expectations what is consistent with our assumptions, since the short-term assets that are considered to be safe should carry minimal risk premiums. Average short rate expectations over the next one year were moving on a relatively high level before the impact of the economic and financial crisis and fell down rapidly afterwards thus following the global evolution of the short-term interest rates. Term premium was also following this evolution albeit on a lower level what can be explained through the variation in the interest rates. Specifically, in the times of high interest rates there is more uncertainty about the future development of the yields than in the times of interest rates close to zero.

On the other hand, we can see in Figure 5 that the average short rate expectations for the Overnight Indexed Swaps with 10-years maturity are not so volatile and were not affected so widely by the economic and financial crisis as the short-term ones. This observation is also consistent with our assumptions, since the expectations over a longer period of time are usually more resistant to the economic shocks. Term premium is non-trivial, what is again consistent with our assumptions, and has mostly decreasing tendency through the sample period what eventuates into the negative values over the last two years. This phenomenon was probably boosted by the launch of the Quantitative Easing in January 2015.

4.6 Model performance

In this subsection, we take a closer look at the results of the ACM model applied on the weekly OIS yields. Specifically, we compare the model applied on weekly data with the model applied on monthly data, the approach implemented by Ódor and Povala (2016). As was discussed before, the quality of the OIS yields approximation is poorly when applied on the time period after January 2009, due to the great impact of the zero-lower bound, and is thus necessary to add additional three years of data development to improve the quality of the estimation. It is therefore reasonable to expect that the model applied on weekly data would give more biased approximation of the yields than the model applied on monthly data, due to the greater impact of the yields with the zero lower bound.

We compare the performance of both models on the OIS yields for the 1-year and 10-years maturities to capture the differences on the short-end as well as on the long-end of the zero-coupon curve. We can see the approximation of the OIS yields by the model applied on weekly data in Figure 6 and by the model applied on monthly data in Figure 7.

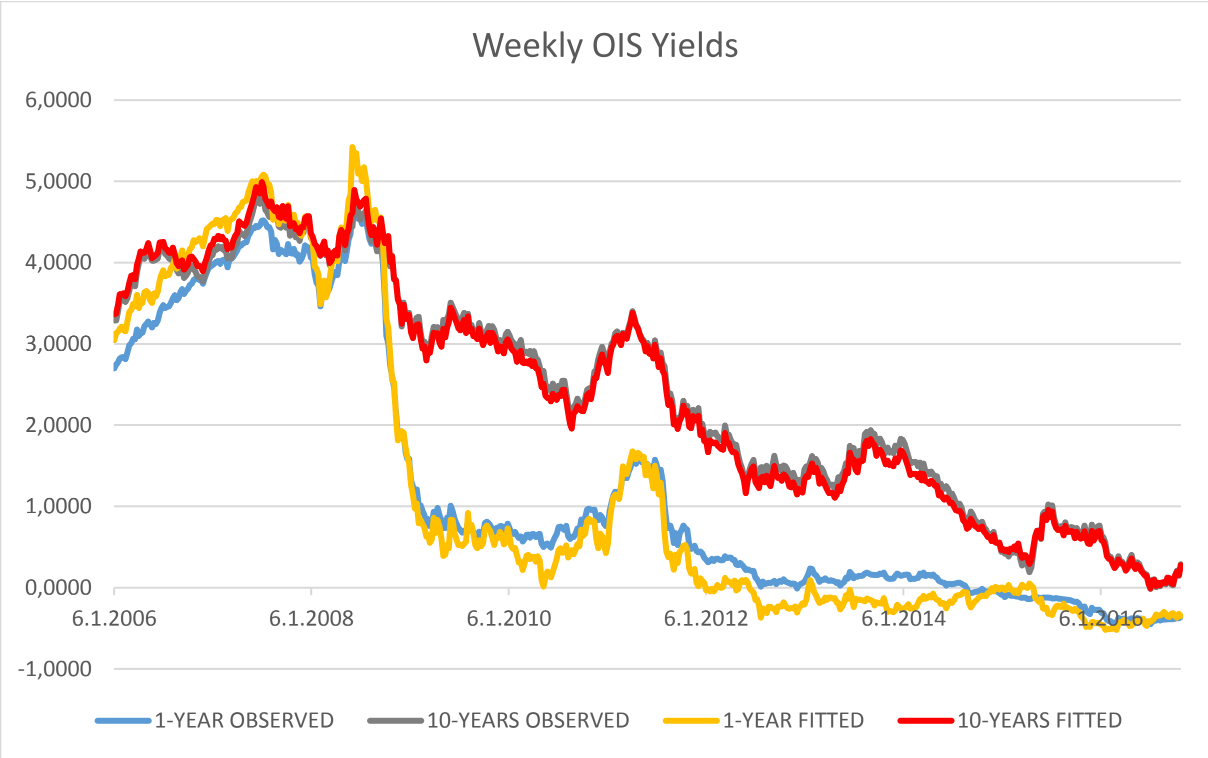

Figure 6: Weekly observed yields on the OIS with 1-year maturity pictured by blue colour and with 10-years maturity by grey colour. OIS yields fitted by the ACM model are pictured by orange colour for 1-year maturity and by red colour for 10-years maturity. Time period is from January 2006 to October 2016. Data are weekly.

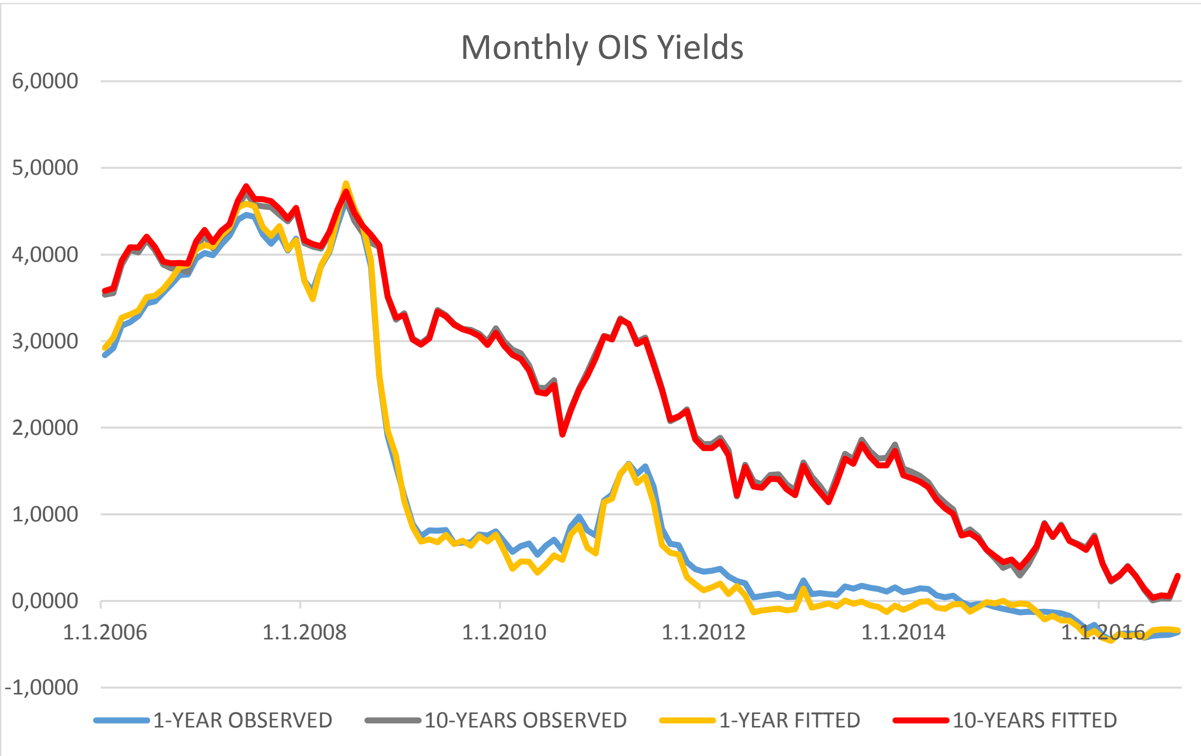

Figure 7: Monthly observed yields on the OIS with 1-year maturity pictured by blue colour and with 10-years maturity by grey colour. OIS yields fitted by the ACM model are pictured by orange colour for 1-year maturity and by red colour for 10-years maturity. Time period is from January 2006 to October 2016. Data are monthly.

Performance of both models is almost the same at the long-end of the yield curve with a high quality of approximation for the OIS yields. However, estimation at the short-end of the yield curve is more biased and through the first three years of the sample period we can further observe significant differences between the models. Specifically, ACM model applied on weekly data is estimating significantly higher yields than the observed ones while the model applied on monthly data keeps a solid quality of the approximation. ACM model applied on monthly data is thus performing slightly better, especially at the beginning of the sample period, and therefore makes sense to work with the monthly OIS yields like Ódor and Povala (2016). However, we apply the weekly yields to be consistent with the Slovak zero-coupon curve.

We should discuss also decomposition of the yields, since our estimation of term premiums is based on the difference between corresponding model-implied yields and model-implied short rate expectations. Another way is to estimate the difference between observed yields and short rate expectations and thus capture the entire dynamics of the yields. We decided against this approach, since the yield pricing errors are probably incorporated also in the estimated short rate expectations and thus if we estimate the term premiums as the difference over observed yields, we in fact transfer the pricing errors from the short rate expectations into the term premiums. On the other hand, if we apply the model-implied yields, we divide the pricing errors between both of these components.

5 Risk premiums

Country-specific risk premiums vary not only from country to country, as captured by the work of Krishnamurthy, Nagel and Vissing-Jorgensen (2015) and Ejsing, Grothe and Grothe (2015), but also over time. For example, the redenomination risk in Slovak government bonds was probably strongly significant before and immediately after the Euro adoption but not afterwards. Another example is the safety premium in German bunds that have arisen especially during and after the Eurozone debt crisis. It is thus important to specify the most important drivers of the government bond yields to fully understand their dynamics. As discussed above, we focus on three different yield-driving factors, namely credit premium, liquidity premium and effect of the Quantitative Easing, and make their further specification through the methodology of dynamic linear models and Kalman filtering.

Specifically, we decompose the Slovak yield spread, obtained as the difference between zero-coupon yields and corresponding Overnight Index Swaps, on 1-year, 5-years and 10-years government bonds and thus capture the major points of the yield term structure. Decomposition of the 10-years spread is meanwhile performed by the following variables: 10-years Credit Default Swap as the approximation for the credit factor, aggregate measure of illiquidity in Slovak government bonds as the approximation for the liquidity factor and difference between 10-years observed and synthetic yields as the approximation for the QE. 5-years spread is decomposed in the similar way, specifically by the 5-years CDS, aggregate illiquidity measure and 4-years QE approximation. Finally, 1-year spread is decomposed solely by the 1-year CDS and aggregate illiquidity measure, since the Quantitative Easing affected only bonds over two years to maturity and thus we do not expect any QE effect here.

5.1 Dynamic linear model

Development of the country-specific risk premiums is captured by the dynamic linear model. This group of models, also known as the state space models, are defined by a set of equations identifying the latent factors through the observable variables, specifically a state equation describing the dynamics of state variables, or the latent factors, and a measurement equation describing the relationships between the state and observed variables. General dynamic linear model with deterministic parameters can be thus written as follows:

$$x_t = c + Fx_{t-1} + v_t \tag{15}$$

$$y_t = Hx_t + w_t \tag{16}$$

where x_t denotes the vector of state variables in particular time t and y_t the vector of observables. Vector c and feedback matrix F further capture the dynamics of the state variables through the vector autoregressive process of first order and matrix of factor loadings H identify the state variables through the relationships with the observables. Residuals v_t and w_t are normally distributed with zero expected values and covariance matrices Q_t and R_t respectively. All model parameters can be generally deterministic or time-varying, in our representation we allow only for deterministic parameters c, F and H but for time-varying covariance matrices Q_t and R_t to better capture the variance in the state as well as in the observed variables.

We allow for constant term in the state equation, since we expect the risk factors with non-trivial mean and thus should capture the mean of the latent variables through the constant term. On the other hand, we do not allow for constant term in the measurement equation, since we want to capture the entire dynamics of the risk factors through the observables. Allowing for the constant term in measurement equation is reasonable only for the purposes of bias modelling. Furthermore, we create the vector of observables in particular time t by stacking the corresponding government bond yield spread, Credit Default Swap, illiquidity measure and Quantitative Easing approximation into one vector. We thus obtain 4xT matrix of observables where T denotes the number of observed weekly periods.

Identification of the covariance matrices Q_t and R_t is then performed through the lagged values of observed yield spread. Specifically, we assume higher variance in the state variables as well as in the observables in the times of higher yields, since the higher values of government bond spread imply also its higher variance and all remaining variables are in fact specification for particular components of the spread and thus their variance should be linked with the variance of the spread. This definition of covariance matrices should therefore reflect the real development of model innovations. We further apply the lagged yields to ensure the model predictability and thus assume the state covariance matrix in the form $Q_t = Q|y_{1,t-1}|$ and the measurement covariance matrix in the form $R_t = R|y_{1,t-1}|$, where Q and R are deterministic covariance matrices. The first-step covariance matrices are further estimated through the maximal value of the corresponding yield spread.

This representation is almost identical to the one implemented by Krishnamurthy, Nagel and Vissing-Jorgensen (2015) but we apply the absolute values of the lagged yields, since the Slovak government bond spreads are negative in the non-trivial time period, in contrast to strictly positive spreads of Spain, Italy or Portugal. Related approach was implemented by

Feldhutter and Lando (2008) but while they applied the lagged values of the latent variables to model the covariance matrix of state equation, we use the lagged values of the observable variable. Moreover, we allow for negative values of the latent factors, especially for negative values of the liquidity premium, and thus would have to apply the absolute values of the lagged factors, since the components of covariance matrix are strictly non-negative. These arguments were discussed also by Krishnamurthy, Nagel and Vissing-Jorgensen (2015).

5.2 Model identification

Dynamic linear models are usually estimated via the Maximum Likelihood Estimation based on the underlying Kalman filter. However, we should at first specify the relationships between the latent factors and the observables to ensure the correct interpretation of the risk factors and to avoid the ambiguity of the estimation. Model identification is thus performed through the matrix of factor loadings. Specifically, we apply the following matrix H where the rows represent particular observables, the columns particular state variables and components of the matrix their mutual relationships:

$$H_t = \begin{bmatrix} 1 & \alpha & \beta \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

Order of the observables was defined before and order of the state variables is the following: credit factor, liquidity factor and the QE. As was already discussed, we assume one-to-one effect of the credit factor on the government bond spread and corresponding CDS and unrestricted effect of the liquidity factor and the QE effect. We further standardise all of the relationships for convenience. This matrix of factor loadings thus identifies the model for the 5-years and the 10-years government bond spread while in the case of the 1-year spread we simply adjust the model by the QE effect and therefore forget the last row and the last column of the identification matrix. Our model identification is similar to the one applied by Ejsing, Grothe and Grothe (2015), since they also implemented not fully-supplied matrix of factor loadings for the purpose of liquidity factor modelling.

Although we made the model identification through the matrix of factor loadings, we still do not have enough information to estimate the other parameters of the dynamic linear model unrestricted and avoid the ambiguity of the estimation. Therefore, we have to make additional restrictions to the model. To ensure the explicit estimation we simplify the model in the maximal way and thus to the diagonal model when we restrict the off-diagonal elements of the matrices F, Q_t and R_t to zero. Diagonal model in fact assumes the uncorrelated factors as well as the uncorrelated factor innovations and the uncorrelated measurement residuals. This modification was also implemented by Ejsing, Grothe and Grothe (2015).

It is the empirical evidence that particular risk factors are mutually correlated. For example, Christensen and Gillan (2016) or Schlepper, Hofer, Riordan and Schrimpf (2017) discussed potential impact of the Quantitative Easing on the bonds liquidity and Beber, Brandt and Kavajecz (2009) relationship between credit and liquidity premiums in the government bonds. Another non-trivial channel is probably the effect of liquidity factor on the Quantitative Easing, since the high-liquid bonds should be more available for the QE than the bonds with low liquidity. Of course, it would be therefore convenient to estimate all of these relationships unrestricted but due to the practical issues arising from their implementation, we decided against this. Furthermore, although the mutual correlations between the factors are probably non-trivial, they should not cause essential inconsistencies in the estimation.

As was discussed by Ejsing, Grothe and Grothe (2015), innovations of the credit and liquidity factors could be uncorrelated for the highly rated and highly liquid assets, since there are historically several instances when the rating downgrades for large liquid markets had no significant impact on the liquidity, like the downgrade of US sovereign debt in August 2011. Assumption about the orthogonality of credit and liquidity factors is also used in the related literature, for example in Beber, Brandt and Kavajecz (2009). Of course, even if these assumptions are correct for highly rated and highly liquid markets, like the German bund market, they could not be convenient for the small markets like the one with Slovak government bonds. Other issues arise from the potential correlations with the innovations of the QE effect. However, we do not have additional information about any of these relationships.

Furthermore, since we apply unrestricted components of the matrix of factor loadings H and diagonal feedback matrix F, we have to ensure the model identifiability, as discussed by Ejsing, Grothe and Grothe (2015). Specifically, the dynamic linear model is identifiable, if different values of components in identification matrix generate different dynamics of the pricing factors. Model identifiability is thus always ensured in the case of fully-supplied matrix of factor loadings but not otherwise. However, we can ensure this property in the case of diagonal feedback matrix by setting the off-diagonal components of the state covariance matrix to zero and thus applying the diagonal covariance matrix Q_t. For detailed explanation and proof of this statement see Ejsing, Grothe and Grothe (2015).

Correlation of the measurement residuals is the general problem in the term structure modelling and it is difficult to overcome this issue without additional information about the measurement noise or other potential inaccuracies. We should at least mention that the most of the estimated measurement residuals were significantly correlated with the strongest negative correlation between the yield spreads and corresponding CDS.

5.3 Kalman filter

Estimation of the latent factors for a given set of parameters of the dynamic linear model is performed by the Kalman filter. This pioneering algorithm developed by Kalman (1960) recursively estimates the latent factors from supplied observables and thus filter out measurement noise and other potential inaccuracies. Specifically, the algorithm estimates the optimal value of the latent factors in a particular time as the weighted average between factors dynamics and linear combination of the observables while the optimal weights are captured by so-called Kalman gain. Dynamics of the latent factors as well as their relationships with the observables is meanwhile defined by the underlying state space model. Although the later extensions like the Extended Kalman filter or the Unscented Kalman filter allow also for non-linear state space models, the original implementation is based on the linearity of the state equation as well as the measurement equation.

Generally, standard Kalman filter consists of two set of equations, the prediction step and the update step. While the prediction step predicts the state estimate as well as its covariance matrix through the state equation capturing the dynamics of the latent factors, the update step incorporates the values of the observables and update the state estimate and covariance matrix through the measurement equation and optimal Kalman gain. Prediction step therefore estimates the predicted values of the latent factors and update step their filtered values. We can write the prediction step for the applied dynamic linear model as follows:

$$x_{t|t-1} = c + Fx_{t-1|t-1} \tag{18}$$

$$P_{t|t-1} = FP_{t-1|t-1}F^T + Q_t \tag{19}$$

where $x_{t|t-1}$ denotes the predicted state estimation in particular time t and $x_{t-1|t-1}$ the filtered state estimation in previous time t-1. On the other hand, $P_{t|t-1}$ denotes the predicted state covariance matrix and $P_{t-1|t-1}$ the filtered state covariance matrix. Prediction step is thus fully defined through the Equation (15). After predicting the state estimate and variance, Kalman filter continues with the update step that can be written as follows:

$$x_{t|t} = x_{t|t-1} + K_t y_t - K_t H x_{t|t-1} \tag{20}$$

$$P_{t|t} = P_{t|t-1} - K_t H P_{t|t-1} \tag{21}$$

where $x_{t|t}$ denotes the filtered state estimation and $P_{t|t}$ the filtered state covariance matrix in the corresponding time t. Element K_t then denotes the optimal Kalman gain. Estimation of the state value as well as its covariance matrix thus depends only on the value of the Kalman gain that defines the weights for the predicted state and corresponding measurements. Optimal value of the Kalman gain is then estimated through the methodology developed by Kalman (1960) and can be described by the following equation:

$$K_t = P_{t|t-1} H^T (H P_{t|t-1} H^T + R_t)^{-1} \tag{22}$$

If we simplify the underlying state space model to the 1x1 alternative with the unit matrix of factor loadings H, Kalman filter becomes the filter in the true meaning, since it would serve only to filter out measurement noise and other inaccuracies in the underlying observable. Furthermore, it is then obvious from Equation (22) that the optimal Kalman gain always lies between zero and one, since we divide two non-negative numbers where the numerator is always smaller or equal to the denominator, and the filtered state always lies between the predicted state and the observable, as we can see from Equation (20). Specifically, if we set the Kalman gain to zero, we would obtain the predicted state, and if we set the Kalman gain to one, we would obtain corresponding observable. This property then ensures the correct interpretation of the Kalman gain as well as the appropriateness of the entire algorithm. More complicated models can be then viewed as the modifications of this alternative.

However, due to the well-known issues with the numerical instability, direct implementation of the Kalman filter is not recommended for more complicated models. Specifically, computational rounding errors may cause that the state covariance matrix lose the property of positive semi-definiteness what results not only to theoretical problems, since the covariance matrix is positive semi-definite from the definition, but also to potential divergence of the Kalman filter. There is a number of potential extensions improving the numerical stability of the Kalman filter, for example so-called Joseph form of the state covariance matrix that keeps the matrix in the form more resistant to numerical inaccuracies, but the true solution to this issue was proposed by the development of the square root covariance filters. In contrast to classic covariance filters, these algorithms keep the state covariance matrix in the square root form and thus ensure its positive semi-definiteness.

Different forms of the square root filter were developed by Potter and Stern (1963) and Thornton and Bierman (1975). In general, these algorithms keep the covariance matrix in the form of Cholesky decomposition $P = SS^T$, where S is the square root matrix, or in the triangular form $P = UDU^T$, where U is the upper triangular matrix with unit diagonal and D is the diagonal matrix. However, we obtain the square root form of corresponding matrices through the singular value decomposition (SVD). In general, SVD makes the factorization of an arbitrary rectangular matrix A in the form A = $V\sum W^T$, where $\sum$ is the matrix of singular values and V with W are the corresponding square matrices. Moreover, in the case of normal and positive semi-definite matrix N, we can write the SVD in the form $N = V\sum V^T$. This kind of factorization can be applied also for the covariance matrices, since they are symmetric, and thus normal, and positive semi-definite from the definition.

Another potential issue arises from the computation of the inverse covariance matrix during the estimation of the Kalman gain in Equation (22), since we have to ensure the invertibility and thus the regularity of the underlying covariance matrix. However, since the singular matrices have at least one zero eigenvalue and eigenvalues are equal to singular values for normal and positive semi-definite matrices, we can check for the numerical singularity of the underlying covariance matrix through the extracted singular values.

To practically apply the Kalman filter, we need to at first set the initial values for the state estimate and covariance matrix. We can estimate these values together with the parameters of the dynamic linear model via the MLE or set an arbitrary value for the state estimate together with high values for the state covariance matrix to ensure the small impact of the initial state. However, probably the best solution is to apply the unconditional mean and unconditional variance of the latent factors that can be obtained if we forget the time indices in Equation (15) and thus obtain the following estimators:

$$E(x) = c + FE(x) \tag{23}$$

$$D(x) = FD(x)F^T + Qy_1 \tag{24}$$

Estimation of the unconditional mean through Equation (23) is thus simple and we need to only ensure the invertibility of the matrix $I - F$, where I represents the corresponding identity matrix. Since we apply the diagonal feedback matrix F, we need to in fact ensure that none of the autoregressive terms is equal to one. Estimation of the unconditional covariance is more complicated and in general we need to apply the Kronecker product of matrices. However, since we implement the diagonal model, we can change the order of the matrices in Equation (24) and thus obtain the simple estimator for the unconditional variance. Now we need to ensure the invertibility of the matrix $I - F^2$ and therefore again set the autoregressive terms different from one. Finally, we estimate the unconditional value of the government bond spread y_1 as the sample mean of the observed spread.

However, implementation of the standard one-step Kalman filter is not appropriate for our decomposition, since the Quantitative Easing was launched only in January 2015 and we should therefore not expect any effect on the government bond yields beforehand. We need to in fact ensure that the Kalman filter understands the zero effect of the QE before its settlement as the true zero values and not as the regular measurements, since this implementation would eventuate into additional inaccuracies in the latent factors and also into potential distortion of the parameters estimation. We overcome this issue by implementing the alternative two-step Kalman filter. In the first step, we apply the Kalman filter on the model with two latent factors, credit and liquidity, and estimate their development until the QE settlement in January 2015. Then we add additional third factor and run the second step of the Kalman filtering.

The only remaining issue is the appropriate setting of the initial values for the state estimates and covariance matrices but since we apply the diagonal model, we can solve this easily. Specifically, we firstly estimate the unconditional mean and variance of the credit and liquidity factor and apply them as the initial values for the first step of Kalman filtering. After expanding the 2x2 model into the 3x3 alternative in January 2015, we simply apply the last values of the state estimate and covariance matrix as the initial values of the credit and liquidity factor for the second of Kalman filtering and set the initial values of the QE effect as the unconditional mean and variance of this latent factor. This implementation should thus capture the true development of the latent factors as well as the model quality.

5.4 Maximum likelihood estimation

Estimation of the dynamic linear model is meanwhile performed through the maximum likelihood. We thus handle the Kalman filter as the underlying function that not only estimates the latent factors for a given set of parameters of the dynamic linear model but also determines the quality of the estimation and thus the quality of the model parameters. We can then optimize the model parameters by maximizing the quality of the estimation through the underlying optimization routine. Specifically, we measure the quality of the predicted states with regard to corresponding observables and thus the development of the measurement residuals, since the quality of the model can be viewed as the quality of its predictability.

Furthermore, since the measurement residuals are normally distributed, the predictability and thus the quality of the model can be captured by the likelihood function of the multivariate normal distribution. For convenience, we apply the logarithm of the likelihood function, also known as the log-likelihood function, that can be written as follows:

$$\ln(L) = -{}^{1}/_{2}\, np\ln(2\pi) - {}^{1}/_{2}\sum_{t=1}^{n} \ln|S_t| + e_t^T S_t^{-1} e_t \tag{25}$$

where e_t denotes the vector of measurement residuals and S_t their covariance matrix in particular time t. Since the measurement residuals have zero expected value, we do not incorporate this parameter in the log-likelihood function. Element n then denotes the number of observations and p the dimension of the residual vector. We further write the definition for the measurement residuals and their covariance matrix as follows:

$$e_t = y_t - Hx_{t|t-1} \tag{26}$$

$$S_t = HP_{t|t-1}H^T + R_t \tag{27}$$

Measurement residuals as well as their covariance matrix were thus already incorporated in the Kalman filter, specifically in the update of the state estimate in Equation (20) and in the estimation of the optimal Kalman gain in Equation (22). We can further exclude the constant term from the definition of the log-likelihood in Equation (25), since it does not have any impact on the parameters estimation, and the log-likelihood function is therefore based solely on the value of the measurement residuals and their variance in each step of the Kalman filter. Optimization of the model parameters is then performed by maximizing the log-likelihood function and thus by the Maximum Likelihood Estimation.

There is a variety of different optimization methods. Probably the most popular group of the optimization algorithms are so-called gradient methods that can find the optimum of particular function through the function gradient. We should mention the standard Newton method or the group of so-called Quasi-Newton methods that do not require the direct evaluation of the Hessian matrix and are thus based on the less rigid assumptions about the underlying function than the standard Newton method. The most widely used modification of the Quasi-Newton methods is the well-known BFGS algorithm. However, the group of gradient based methods are able to find only a local optimum of the underlying function that is at least one time differentiable (Quasi-Newton methods) or two times differentiable (Newton method) and are thus not applicable for more complex problems.

Another approach to the optimization was proposed by Nelder and Mead (1965) in their pioneering work. They developed the simplex based algorithm that minimize the underlying function by a set of different simplex transformations. Although this algorithm is not a true global optimization routine, it has practically good performance even for the functions with more than one local minima. Moreover, since this algorithm is the direct and not the gradient based method, the underlying function need not to be differentiable.

However, we may encounter the optimization problem with many local optima and thus should apply a true global optimization algorithm that can handle with the problematics. There is a number of stochastic metaheuristic algorithms for the global optimization, for example the Simulated Annealing, based on the progressive cooling of materials, the Particle Swarm Optimization, based on the behavior of swarms, or the Genetic algorithms, large group of population-based optimization procedures. We decided to implement the Differential Evolution algorithm developed by Storn and Price (1997), due to the combination of simplicity and good performance on the field of continuous optimization. This method can be in fact viewed as the modification to the Genetic algorithms, although some differences exist. We can represent the basic Differential Evolution by the following equation:

$$X_{t+1}^{i} = CR_t^i\left(A_t^i + F\left(B_t^i - C_t^i\right)\right) + \left(1 - CR_t^i\right)X_t^i \tag{28}$$

where X_t^i denotes the vector of the parameter values for the i-th element of the population in particular time t and A_t^i with B_t^i and C_t^i random candidates from the population different from each other and also from X_t^i. Each element of the population is therefore the vector with the actual parameter values. Term CR_t^i then denotes the corresponding vector consisting of zeros and ones, each one of them equal to one with the given crossover probability, and F the given value of the mutation weight. Furthermore, all of the terms in Equation (28) are multiplied element-by-element. Each parameter value in the new i-th element of the population is thus equal either to the original value of this parameter or to the linear combination of the corresponding parameter values obtained from the selected candidates. The probability of the latter is meanwhile called the crossover probability.

Setting of the parameters for the Differential Evolution, namely number of the elements in the population, crossover probability and mutation weight, is a general issue in the practical implementation of this algorithm, since these values are usually related to the particular problem and its characteristics. Generally, crossover probability lies between zero and one and mutation weight in the interval [0,2]. However, it is strongly recommended to set the mutation weight from the interval [0.5,1], since different values of the mutation weight rarely give a better solution. We decided to apply the crossover probability equal to 0.9, the mutation weight to 0.8 and the number of the elements in the population to five times number of the parameters to optimize, as the compromise between fast convergence and robustness of the algorithm. Finally, we set the number of iterations to 1000 and since we apply only limited number of iterations, we run also the additional interior-point algorithm to fine-tune the convergence to the global optima determined by the Differential Evolution.

Due to the natural constraints for particular model parameters, we have to incorporate them into the optimization bounds. Specifically, the covariance matrices have to be symmetric and positive semi-definite from the definition what can be in general controlled through the Cholesky decomposition, since the matrices that are kept in the form of the square root matrix in Cholesky decomposition are always symmetric and positive semi-definite. However, since we apply the diagonal covariance matrices, we need to only ensure that their components are not negative. Other constraints are imposed on the autoregressive terms that should be greater than zero and smaller than one to ensure the stationarity as well as the continuity of the latent factors. Moreover, they should be different from one, to ensure the invertibility of the corresponding matrices in Equation (23) and Equation (24). Other constraints for the model parameters were set empirically to improve the quality and speed of the estimation.

5.5 Regression analysis

Before studying the specification of the risk premiums through the implemented dynamic linear model, we perform a simple regression analysis based on the underlying financial variables representing particular yield-driving factors and corresponding government bond spreads. This analysis is based on the similar assumptions as the general one but instead of applying the methodology of the dynamic linear models works with a set of simple linear regressions that capture the relationships between particular risk premiums and Slovak yield spreads and thus supply the basic information about the development of the risk premiums.

We can see the decomposition of the spread on Slovak government bonds with 1-year, 5-years and 10-years maturity in Table 2. Specifically, we supply the regression coefficients for each of the yield-driving factors, namely credit premium, liquidity premium and QE effect, and also the standard deviations of the estimated coefficients supplied in the parenthesis. Risk premiums, or the yield-driving factors, are meanwhile represented by the same financial variables as in the implemented dynamic linear model.

	Credit	Liquidity	QE Effect
1-Year Spread	0.6304 (0.0170)	2.7634 (0.1199)	0.0000 (0.0000)
5-Years Spread	0.7091 (0.0252)	4.4036 (0.2936)	3.6564 (0.2976)
10-Years Spread	0.7740 (0.0219)	5.3927 (0.3012)	1.4137 (0.0793)

Table 2: Regression coefficients with corresponding standard deviations in parenthesis from the linear regression of the Credit Default Swap, aggregate measure of illiquidity in the Slovak government bonds and approximation of the Quantitative Easing on the corresponding spreads on Slovak government bonds with 1-year, 5-years and 10-years maturity.

All of the risk premiums were identified as strongly significant for every yield spread. Regression coefficient for the credit factor represented by corresponding Credit Default Swap seems to have increasing tendency across the maturities and in general is significantly lower than one. However, this is in contrast to the original observation of Krishnamurthy, Nagel and Vissing-Jorgensen (2015), since the authors expect higher amount of the credit risk in the government bonds than in corresponding CDS, due to the fact that CDS can underestimate the underlying credit risk. Dynamics of the credit premium in the government bonds and its relationship with corresponding CDS is thus not well explained. In contrast to the linear regression approach, applied also by Ódor and Povala (2016), we assume one-to-one effect of the credit factor on government bond yields and corresponding CDS.

Regression coefficient for the liquidity premium has also the increasing tendency across the maturities, as was already captured by Ódor and Povala (2016). However, since the underlying illiquidity measure is strictly non-negative from the definition in Equation (2), we obtain only positive liquidity premium in Slovak government bonds, measuring the additional premium that investors require for holding the bonds with low liquidity. Although the Slovak government bonds are in general illiquid, and thus carrying the positive liquidity premium for the investors, we would like to capture also potential negative premium in particular time periods when these bonds could be attractive for the investors, for example in the times after the Euro adoption or during the Eurozone debt crisis.

Moreover, due to the character of the illiquidity measure that captures only the illiquidity in the government bonds, we would not be able to obtain negative liquidity premium even for the highly liquid assets. Specifically, even if the illiquidity measure would be equal to zero and thus expecting the asset with no illiquidity, the liquidity premium in corresponding asset would be also equal to zero. However, it is reasonable to expect that the absolutely liquid asset would carry significantly negative liquidity premium, since it offers additional service for the investors, for example over the cash that can be viewed as the asset with zero liquidity premium. Combination of the illiquidity measure and regression analysis is therefore not the best solution to capture the liquidity premium in government bonds but since we apply the methodology of dynamic linear models and Kalman filtering, we allow for positive as well as the negative liquidity premium in Slovak government bonds.

Finally, the regression coefficients capturing the QE effect are significantly greater than one what leads to the assumption that the QE approximation through the Synthetic Control Method can be underestimating the true QE effect on Slovak government bonds, probably due to the already discussed inconsistencies and inaccuracies in the SCM estimation, and is thus reasonable to estimate this component unrestricted. Furthermore, this phenomenon is much more significant in the middle than on the long-end of the yield curve, where we obtain the best results of the SCM estimation, what is deepening our assumption. We display also the evolution of the extracted risk premiums in the government bonds with 10-years maturity together with the regression residuals in Figure 8.

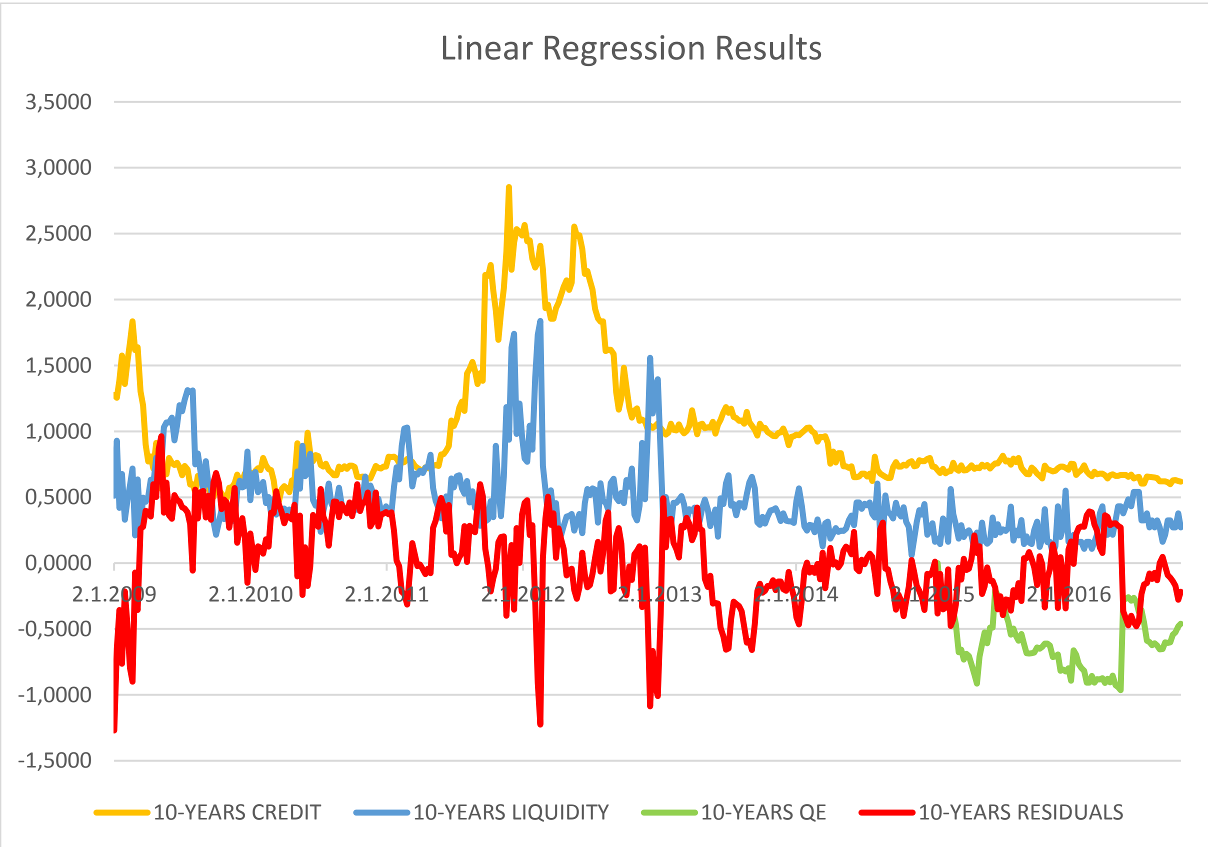

Figure 8: Decomposition of the spread on Slovak government bonds with 10-years maturity into risk premiums obtained by the linear regression. Credit premium is displayed by orange colour, liquidity premium by blue colour, QE effect by green colour and regression residuals by red colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

It is obvious that the residuals are strongly significant over the entire sample period what makes their representation problematic. However, since the most significant and generally negative impact of the residuals can be observed in the beginning of the sample period and in the times of the Eurozone debt crisis, we can understand this unexplained part of the yields as the potential liquidity premium that was not captured by the illiquidity measure. Another explanation for the regression residuals was proposed by Ódor and Povala (2016), since they treat them as the segmentation effect that was not captured by applied financial variables.

5.6 Empirical results

Estimation of the parameters of the dynamic linear model defined by Equation (15) and Equation (16) is performed on the full data sample for the spreads on 5-years and 10-years government bonds but only on the limited data sample for the spread on 1-year government bonds, due to the practical issues with the implementation. Specifically, we were not able to obtain correct estimation of the model parameters for the full data sample, due to the evolution of the short-term spread at the end of the sample period, with or without incorporating the QE. Evolution of the short-term yields on Slovak government bonds is generally problematic over the last years, due to a lack of issued short-term government bonds and their low liquidity in combination with the zero-lower bound. Therefore, we apply the estimation of the model parameters only on the sample period until January 2015.

When we obtain the optimal set of parameters, we run the additional Kalman filter to estimate the development of the latent factors and thus the specification of particular risk premiums in the Slovak government bonds. We can see the decomposition of the yield spread on 1-year government bonds into credit and liquidity premiums in Figure 9, and decomposition of the yield spreads on 5-years and 10-years government bonds into credit premium, liquidity premium and QE effect in Figure 10 and Figure 11 respectively.

Evolution of the credit premium is consistent across the maturities and is generally captured by the underlying Credit Default Swaps. We observe relatively high values of the credit premium at the beginning of the sample period, specifically at the start of the year 2009, due to the economic and financial crisis that was reaching its peak, and also over the years 2011 and 2012, due to the Eurozone debt crisis. We can further see that the Eurozone debt crisis affected the credit factor in Slovak government bonds more than the worldwide economic crisis. Through the rest of the sample period we observe consistent and relatively small values of the credit premium representing the stability in the Slovak government bonds.

However, we can see also some inconsistences across the maturities. For example, magnitude of the credit factor in 5-years and 10-years government bonds was almost the same until the Eurozone debt crisis, in contrast to more convenient development of the credit factor with different magnitudes for different maturities afterwards. Another inconsistency can be observed after the Eurozone debt crisis. Although the credit premium in 1-year government bonds fell down almost immediately after the crisis, credit premiums in 5-years and 10-years government bonds were descending more gradually.

Evolution of the liquidity premium offers more interesting interpretation. We observe significantly negative values of the liquidity premium at the beginning of the sample period immediately followed by the high illiquidity in the Slovak government bonds. This phenomenon can be explained as the impact of the speculations with the currency exchange rates during the adoption of the Euro, eventuating into increased demand for the Slovak government bonds reflected in the liquidity premium and followed by the fall in the demand for these assets afterwards, since they were not more attractive for the currency-related investments. We should mention that the increased demand for the Slovak government bonds could have been reflected in the liquidity premium even shortly after the Euro adoption, since the investors did not have to sell these assets immediately.

Liquidity premium in the middle and the long-end of the yield curve was significantly positive until the Eurozone debt crisis, representing the illiquidity in the Slovak government bonds. However, liquidity factor again reached negative values during the Eurozone debt crisis what partially captures the emerging safe-haven flows. Specifically, investors were ready to pay additional cost to place their money into safe assets in the times of high uncertainty. This phenomenon is typical for the highly-rated assets like German government bonds, as discussed by Ejsing, Grothe and Grothe (2015), but although the Slovak government bonds do not have so high rating as the German ones, they could have been also affected by the safe-haven flows, since they offer a compromise between the safety and profit for investors. We thus capture the safety factor in the Slovak government bonds that was not obtainable by the regression approach, as discussed by Ódor and Povala (2016).

Negative values of the liquidity premium are observed even after the Eurozone debt crisis, especially in the middle and the long-end of the yield curve, but in the recent years we observe increasing tendency of the liquidity factor eventuating into significantly positive values at the end of the sample period. Slovak government bonds in recent days are thus significantly illiquid what is consistent with the empirical observations of the financial markets.

Evolution of the QE effect is different in the middle and in the long-end of the yield curve. Specifically, we observe almost immediate and strong impact of the Quantitative Easing on the government bonds with 10-years maturity, while in the rest of the sample period we can see relatively continuous evolution of this premium. On the other hand, impact of the Quantitative Easing on the government bonds with 5-years maturity has increasing tendency, reaching its peak at the end of the sample period. However, the evolution in the long-end of the yield curve seems more reliable, due to the quality of the underlying financial variables.

Finally, we compare the decomposition of the spread on 10-years government bonds obtained by the Kalman filtering with the regression based decomposition. Evolution of the credit premium is very similar for both approaches, although the credit factor estimated through the linear regression is generally on a lower level. Evolution of the QE effect obtained by the Kalman filter is much smoother than the one from the regression decomposition, since we observe relatively continuous evolution of this factor after the initial shock.

However, the most significant and most important differences are observed for the liquidity premium. Since the liquidity factor from the regression decomposition can take only positive values, all additional effects were captured by the regression residuals. On the other hand, the liquidity premium obtained by the Kalman filter take both positive and negative values, due to the current economic situation and development of the financial markets. We were thus able to put together the liquidity premium and the residuals from the regression approach eventuating into more complex evolution of the liquidity premium that captures the main events through the sample period with a valid economic interpretation. Specification of the yield-driving factors performed by the methodology of dynamic linear models and Kalman filtering therefore offers significant improvement over the regression approach.

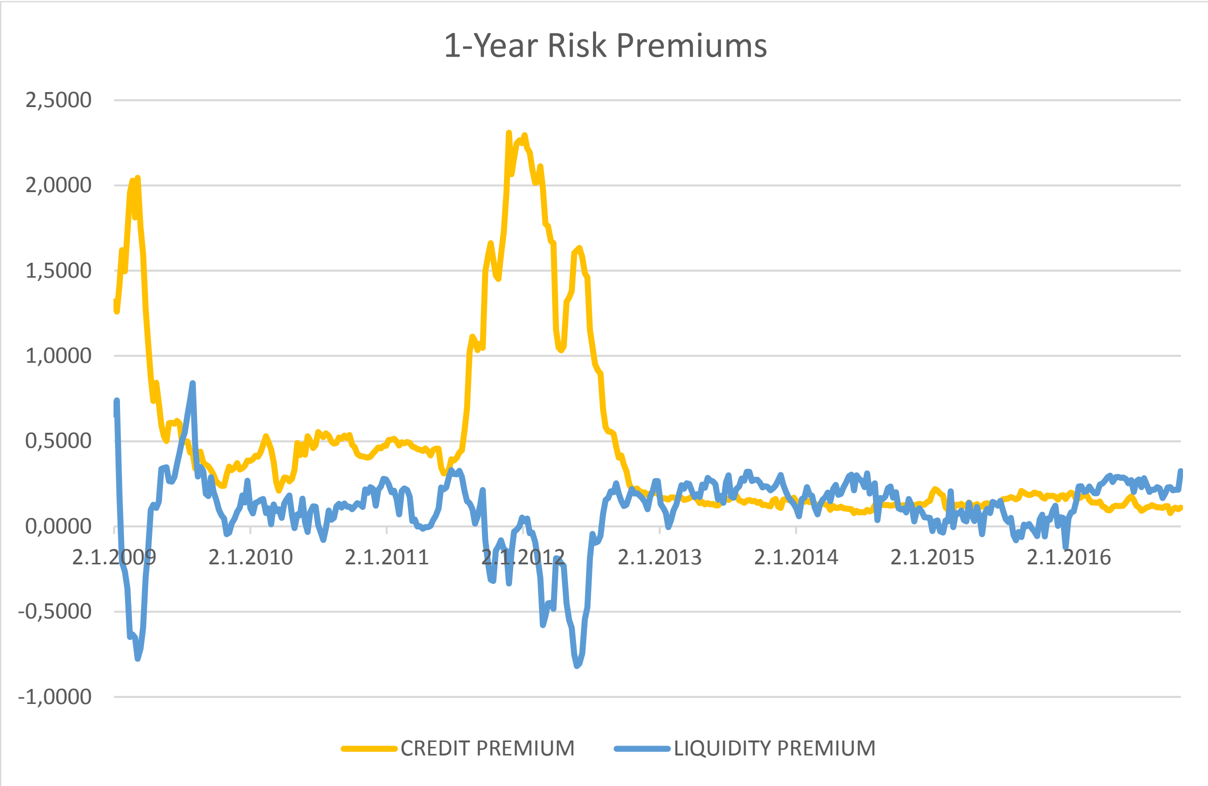

Figure 9: Decomposition of the spread on Slovak government bonds with 1-year maturity into risk premiums obtained by the Kalman filter. Credit premium is displayed by orange colour and liquidity premium by blue colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

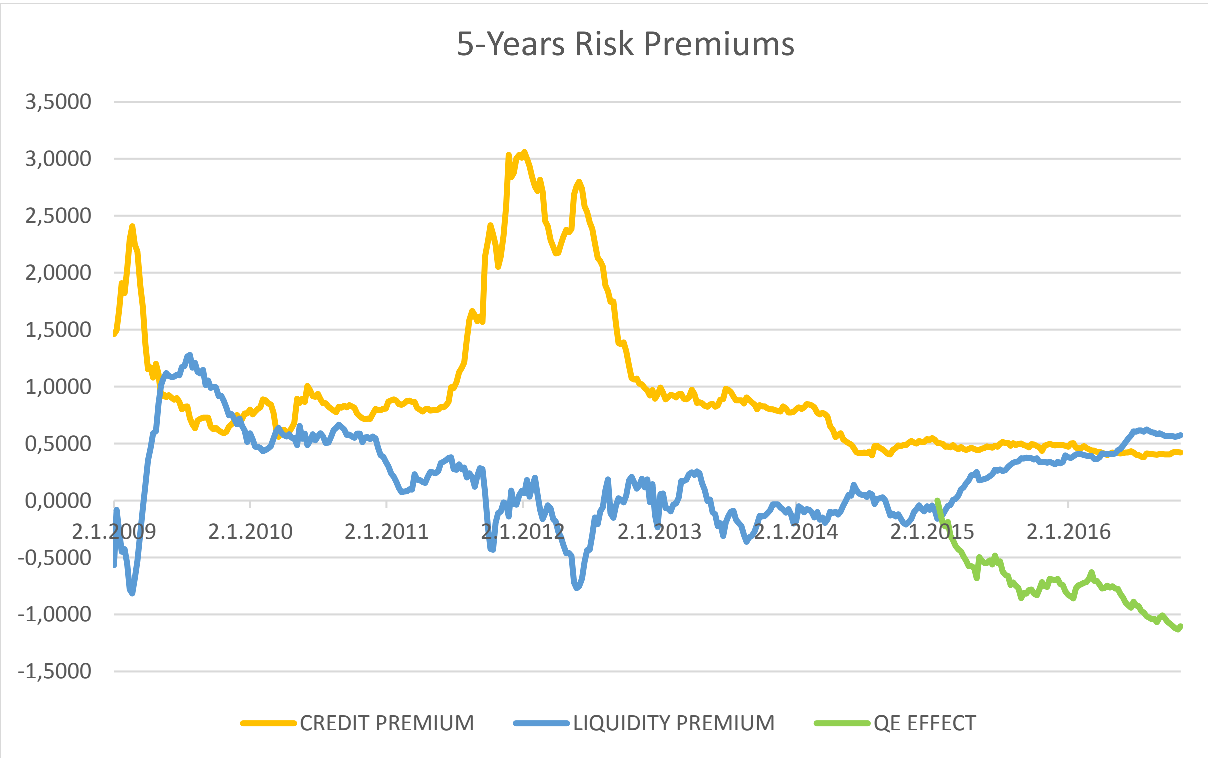

Figure 10: Decomposition of the spread on Slovak government bonds with 5-years maturity into risk premiums obtained by the Kalman filter. Credit premium is displayed by orange colour, liquidity premium by blue colour and QE effect by green colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

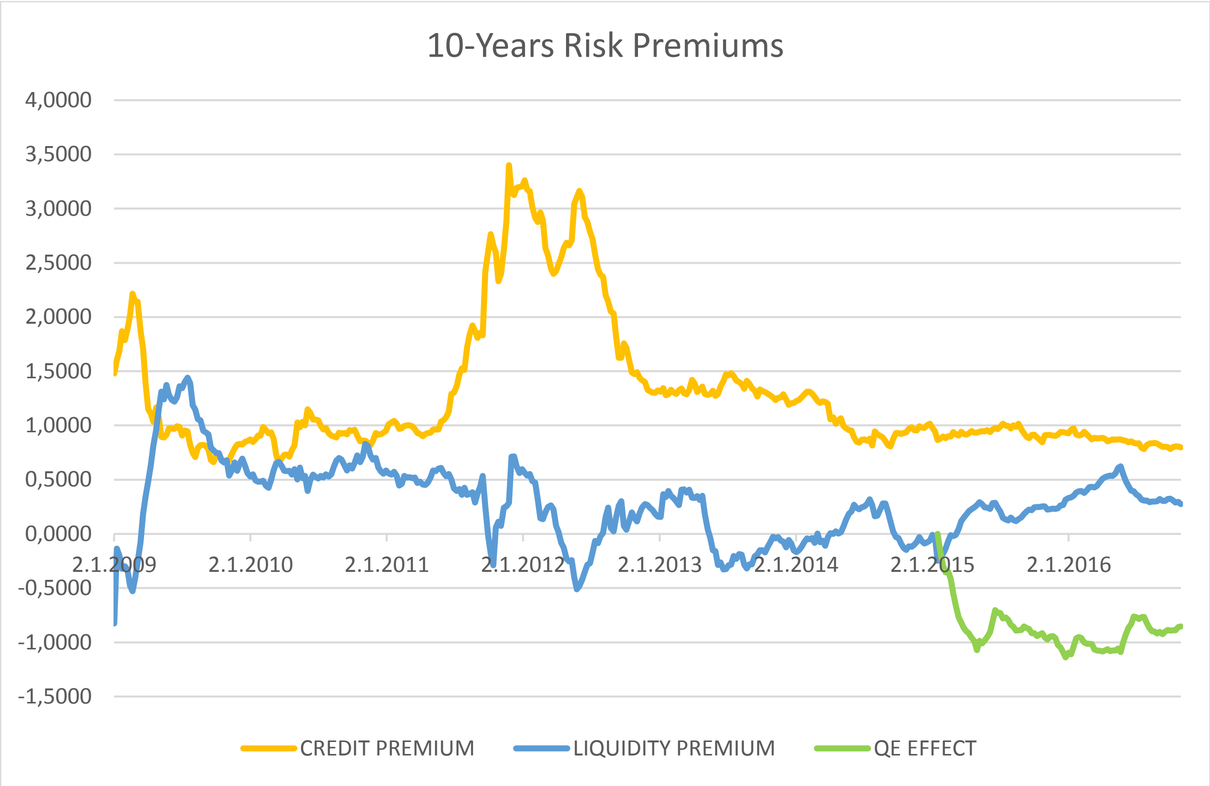

Figure 11: Decomposition of the spread on Slovak government bonds with 10-years maturity into risk premiums obtained by the Kalman filter. Credit premium is displayed by orange colour, liquidity premium by blue colour and QE effect by green colour. Time period is from 2.1.2009 to 28.10.2016. Data are weekly.

6 Further extensions

There is a number of potential improvements of the underlying dynamic linear model. Firstly, we would like to incorporate the diagonal elements to the feedback matrix and thus allow for the correlations between particular risk premiums, since they are probably non-trivial, due to the empirical observations of the financial markets. Specifically, we would like to incorporate the impact of the illiquidity in Slovak government bonds on the Quantitative Easing and also the relationships between the credit and liquidity factors. This implementation is theoretically possible even without additional information from the financial markets but we have to solve the issues with the optimal set of initial values for the two-step Kalman filter. If incorporating the off-diagonal elements of the feedback matrix, we advise to add these parameters stepwise one-by-one for the most important correlations and observe potential improvements over the original model, rather than instantly apply the full-blown feedback matrix.

Secondly, we could estimate also the correlations between factors innovations but this further extension is probably not applicable without additional information about the development of the risk premiums. Specifically, if we want to incorporate correlations between factors as well as between their innovations, we need more information from the financial markets to obtain correct estimation of the dynamic linear model.

Another potential improvement is to incorporate the correlations between the measurement residuals. However, to estimate the entire measurement covariance matrix unrestricted we would need additional information about the measurement noise or other inaccuracies. It is thus reasonable to incorporate at least the most significant correlations between the measurement residuals, empirically the one between the yield spreads and corresponding CDS followed by the mutual correlations of these observables with the illiquidity measure. Correlations of the QE approximation with other financial variables seem to be the least significant.

Finally, we can assume different dynamics of the state covariance matrix. We can for example apply the approach implemented by Feldhutter and Lando (2008) and model the state covariance matrix through the lagged values of the latent factors. However, we have to transform the liquidity factor to non-negative numbers to ensure the convenient definition of the state covariance matrix. We can either apply the absolute values of the latent factors, similarly to the implementation of the lagged yields, or shift the liquidity factor from the negative values. We can then compare the results obtained by this approach and the original one based on the lagged values of the yield spreads.

Furthermore, we discuss the implementation of the dynamic linear model for the yields with different maturities to obtain the specification of the risk premiums for the entire term structure of the government bond yields. However, decomposition of the remaining yield spreads is more problematic, due to the lack of underlying financial variables. We have to therefore make the model identification based on the available maturities for the Credit Default Swaps and the Quantitative Easing approximation.

Since we have already captured the short end as well as the middle and the long end of the CDS curve, we can estimate the CDS dynamics also for the remaining maturities. Specifically, if the information about particular CDS are not available, we can either interpolate them through the closest available maturities or implement the identification similar to the one of Ódor and Povala (2016), where authors represented the level of the CDS curve by the 5-years CDS and its slope by the difference between 10-years and 5-years ones. They later applied both of these values in the linear regression to capture the dynamics of the credit factor across maturities. Analogy for the dynamic linear models would be to apply two different drivers of the credit factor, represented by the CDS with the closest available maturities and difference between them. Identification is then performed through the matrix of factor loadings.

Implementation of the QE effect is more problematic, since we can capture only the middle and the long-end of this factor. However, since we estimate this parameter unrestricted, we allow for higher inaccuracies in the underlying variables. One of the possible alternatives is to approximate the QE effects on the short-end of the yield curve by the shortest available maturity and thus by the 4-years QE estimation, since the QE effects on the short-end are probably similar to each other, and interpolate other maturities through the available ones.

Other extensions are mostly related to the improvement of the underlying financial variables. As was discussed above, alternative implementation for the credit factor based on the government bonds issued in foreign currencies, similar to the one of Krishnamurthy, Nagel and Vissing-Jorgensen (2015), is problematic not only due to the lack of these bonds but also for the estimation of the liquidity premium and effect of the QE. Finding the more appropriate estimation of the liquidity factor is also problematic, since the Slovak Republic do not issue the agency bonds and we thus cannot apply the same model identification as Ejsing, Grothe and Grothe (2015). However, since the main issue with the aggregate illiquidity measure by Hu, Pan and Wang (2013) is the limitation to the non-negative numbers and thus the inability to capture the negative liquidity premiums, we can theoretically overcome this issue if we could somehow shift the illiquidity measure also to the negative numbers.

Finally, we discuss the potential improvements of the Quantitative Easing approximation but rather than finding the alternative estimation to the applied Synthetic Control Method, we prefer to perform the SCM estimation on the data adjusted by the currency related risk factors. As was already discussed, there are generally two alternatives how to perform this adjustment. The first one is to swap the local currencies to Euro through the available currency swaps and the second one is to apply the Euro denominated bonds despite the bonds issued in local currencies. However, both of these approaches can be problematic due to the availability and the quality of the underlying financial variables.

The last discussed extension is the implementation of the redenomination risk and the segmentation effect. Although the redenomination is probably not significant driver of the yields on Slovak government bonds through the majority of the sample period, it could be significant at the beginning of the period, due to the adoption of the Euro. One of the alternatives how to identify the redenomination risk is to implement the corporate bonds with the corresponding corporate CDS like Krishnamurthy, Nagel and Vissing-Jorgensen (2015). Effect of the market segmentation can be also an important driver of the government bond yields but since we apply the unrestricted components in the matrix of factor loadings, we cannot implement it like the residual factor. Therefore, if we want to implement the segmentation effect, we need to find some direct approximation of this factor.

7 Conclusions

Yields on government bonds are affected by a number of different factors, from the risk-free short rate expectations, capturing the expectations about future development of the risk-free interest rates, to the country-specific risk premiums, capturing the most important additional premiums in the government bonds specific for particular country. Since the dynamics of the government bond yields are related to a particular country, time period and also maturity of the underlying bonds, it is important to explain the most significant drivers of the government bond yields to capture their entire dynamics.

Due to this objective, we described a set of term structure models analyzing the term structure of interest rates and making their further decomposition into the most important components. We were thus able to decompose the term structure of the yields on Slovak government bonds, represented by the zero-coupon yield curve, and specify the basic interest rate components as well as the most significant risk premiums in the Slovak government bonds. This is only the second paper making a detailed analysis of the Slovak zero-coupon yield curve, after the work of Ódor and Povala (2016), that is further proposing the significant improvements of the original yield curve decomposition.

Since the Slovak Republic as the member country of the Eurozone shares the monetary policy with the European Central Bank, we can in general decompose the Slovak yield curve into the components determined by the monetary policy and thus common for all Eurozone countries and components specific for the Slovak Republic. Therefore, we extracted the risk-free curve of Eurozone represented by the term structure of Overnight Indexed Swaps and further modelled the risk-free curve and the country-specific risk premiums separately.

Decomposition of the risk-free curve into average short rate expectations and term premiums was performed through the model developed by Adrian, Crump and Moench (2013, 2014). This model applies the principal components of yields as the pricing factors and makes a simple estimation of the pricing parameters through a set of linear regressions and application of the no-arbitrage restrictions. Furthermore, when we adjusted the pricing parameters by the market price of risk, we obtained the estimation of the risk-free short rate expectations over the maturities and thus the decomposition of the risk-free curve.

Specification of the country-related risk premiums was performed through the methodology of dynamic linear models and Kalman filtering. We treated the risk factors as the latent variables and extracted them applying a set of observable variables containing the spread between the

Slovak yield curve and the term structure of OIS, the variable to be decomposed, and a set of financial variables representing particular risk factors. Identification of the dynamic linear model was then performed through the identification of the matrix of factor loadings and Maximum Likelihood Estimation.

This work makes a number of important contributions to the analysis of the Slovak government bonds. Firstly, we describe the most important drivers of the yields on Slovak government bonds, namely credit premium, liquidity premium and effect of the Quantitative Easing, and make their further specification. Secondly, we discuss the implementation of particular financial variables as the approximation for the analyzed risk factors, reminding their weaknesses and also potential alternatives.

Specifically, we discuss the absence of alternatives to the Credit Default Swaps in the representation of the credit factor and implementation of the aggregate illiquidity measure in the Slovak government bonds as the approximation for the liquidity factor with issues arising from the non-negative character of the illiquidity measure and potentially negative values of the liquidity premium. Finally, we approximate the QE effect on the Slovak government bonds as the difference between the observed and synthetic government bond yields obtained through the Synthetic Control Method, what can be considered as the significant improvement over the approach applied by Ódor and Povala (2016).

However, the most important contribution of this work and also the most significant improvement over the regression-based approach applied by Ódor and Povala (2016) is the identification of the liquidity premium in the Slovak government bonds. Specifically, since we apply the methodology of dynamic linear models and filter the underlying financial variables through the Kalman filter, we are able to obtain more complex estimation of the liquidity premium taking the positive as well as the negative values, in contrast to the regression-based approach limited only to the positive liquidity premium, and capturing the most important economic and financial events in the recent years.

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