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THE SECOND PILLAR OF THE SLOVAK PENSION SYSTEM – INTEREST RATE TARGETING

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This paper concentrates on the second pillar of the three-pillar pension system of Slovak Republic. The utility function is used to characterize the user's attitude to risk and to define the stochastic programming problem. A dynamic accumulation model for determining the optimal switching strategy between pension funds is presented. Numerical experiments are done to investigate the sensitivity of results on whether fund returns are constant over time or are dynamically changing.

K e y w o r d s: dynamic stochastic programming, utility function, Bellman equation

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1 INTRODUCTION

Before January 2005, the pension system in Slovakia was based on the unfunded pay-as-you-go system. Because of the unfavorable conditions (high unemployment, demography crisis, etc.) this system was replaced by a new one since 2005. The goals of the pension reform were to secure a stable flow of high pensions to the beneficiaries, and sustainability and overall stability of the system. The new system is based on three pillars: the mandatory non-funded first pillar (pay-as-you-go pillar); the mandatory fully funded second pillar; and the voluntary fully funded third pillar.

The contribution rates were set for the first pillar at 19.75 % (old age 9 %, disability and survival 6 % and reserve fund 4.75 %) and for the second pillar 9 %. A thorough description of the Slovak pension reform with calculations of the balance of the pension system and expected level of pensions in the new system could be found in [1] and [3].

Table 1. Limits for investment for the pension funds.

Fund type	Stocks	Bonds and money market instruments
Growth Fund	up to 80 %	at least 20 %
Balanced Fund	up to 50 %	at least 50 %
Conservative Fund	no stocks	100 %

The savings in the second pillar are managed by pension asset administrators. Each pension administrator manages three funds: Growth Fund, Balanced Fund and Conservative fund, each of them with different limits for investment (see Tab. 1). At the same time the savers may hold assets in one fund only. Up to 15 years before retirement, the saver may not hold assets in the Growth Fund and up to 7 years all assets must be in the Conservative Fund. Even with these restrictions the contributors have some space for individual decisions which fund is optimal in a specific situation (the age of the contributor, the saved amount, the past performance of the pension funds). The above restrictions for the funds were described by a mathematical model in [2] and the optimal strategies of switching between the pension funds (Growth, Balanced and Conservative) were calculated in the same work.

This paper is organized as follows: Section 2 contains the formulation of the dynamic stochastic programming accumulation model as it was done in [2]. In Section 3 we discuss the numerical scheme for finding a solution of this model. In Section 4 we present the form of the results and in Section 5 we do some numerical experiments regarding the fund returns. The last section contains final remarks and conclusions.

2 THE DYNAMIC STOCHASTIC PROGRAMMING ACCUMULATION MODEL

In this section we recall the basic steps of derivation of a dynamic stochastic accumulation model as it was done in [2]. Suppose that the future pensioner deposits once a year a τ-part of his/her yearly salary w_t to a pension fund $j \in \{1, 2, \dots, m\}$. Denote by s_t, $t = 1, 2, \dots T$ the accumulated sum at time t where T is the expected retirement time. Then the budget-constraint equations read as follows:

$$\begin{aligned} s_{t+1} &= s_t(1 + r_t^j) + w_{t+1}\tau\,,\ t = 1, 2, \dots, T-1\,, \\ s_1 &= w_1\tau \end{aligned} \tag{1}$$

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where r_t^j is the return of the fund j in the time period $[t, t+1)$. In terms of the ratio of the accumulative sum to the yearly salary $d_t = s_t/w_t$ (which is a more interesting variable for the pensioner), the budget constraint can be reformulated to

$$\begin{aligned} d_{t+1} &= F_t(d_t, j)\,,\ t = 1, 2, \ldots, T-1\,, \\ d_1 &= \tau \end{aligned} \tag{2}$$

where $F_t(d, j) = d\,\frac{1+r_t^j}{1+\varrho_t} + \tau\,,\ t = 1, 2, \ldots, T-1$ and ϱ_t denotes the wage growth defined by the equation

$$w_{t+1} = w_t(1 + \varrho_t)\,.$$

Suppose that each year the saver has the possibility to choose a fund $j(t, I_t) \in \{1, 2, \ldots, m\}$, where I_t denotes the information consisted of the history of returns $r_{t'}^j$, $t' = 1, 2, \ldots, t-1$, $j \in \{1, 2, \ldots, m\}$ and the wage growth $\varrho_{t'}$, $t' = 1, 2, \ldots, t-1$. Now suppose that the history of the wage growth ϱ_t, $t = 1, 2, \ldots, T-1$ is deterministic whereas the returns r_t^j are random and are assumed to be independent for different times $t = 1, 2, \ldots, T-1$. The relevant information is then the quantity d_t only. Hence $j(t, I_t) \equiv j(t, d_t)$. One can formulate a problem of dynamic stochastic programming

$$\max_J E(U(d_T)) \tag{3}$$

with the following recurrent budget constraint:

$$d_{t+1} = F_t(d_t, j(t, d_t))\,,\ t = 1, 2, \ldots, T-1\,, \quad d_1 = \tau \tag{4}$$

where the maximum is taken over all non-anticipative strategies $J = \{j(t, d_t) : t = 1, ..., T\}$. Here U stands for a given preferred utility function of wealth of the saver. Using the tower law for the conditional expectation

$$E(U(d_T)) = E(E(U(d_T)|I_t)) = E(E(U(d_T)|d_t))$$

we conclude that $E(U(d_T)|d_t)$ should be maximal. Let us denote $J_t = \{j(\tau, d_\tau) : \tau = t, ..., T\}$ and

$$V_t(d) = \max_{J_t} E(U(d_T)|d_t = d)\,.$$

Then by using the tower law

$$E(U(d_T)|d_t) = E(E(U(d_T)|d_{t+1})|d_t)$$

we obtain the Bellman equation

$$\begin{aligned} V_t(d) = &\max_{j\in\{1,2,\ldots,m\}} E[V_{t+1}(F_t(d, j))] \\ &= E[V_{t+1}(F_t(d, j(t, d)))]\,, \end{aligned} \tag{5}$$

for $t = 1, 2, \ldots, T-1$, where $V_T(d) = U(d)$. Thus, the optimal feedback strategy $j(t, d_t)$ can be constructed backwards. This strategy gives the saver the decision for the optimal fund for each time t and level of savings d_t.

Suppose that the stochastic returns r_t^j are represented by their densities f_t^j. Then equation (5) can be rewritten in the form

$$\begin{aligned} V_t(d) = &\max_{j\in\{1,2,\ldots,m\}} E[V_{t+1}(F_t(d, j))] \\ &= \max_j \int_{\mathbb{R}} V_{t+1}\left(d\,\frac{1+r}{1+\varrho_t} + \tau\right) f_t^j(r)\mathrm{d}r \\ &= \max_j \int_{\mathbb{R}} V_{t+1}(y) f_t^j\left((y-\tau)\frac{1+\varrho_t}{d} - 1\right)\frac{1+\varrho_t}{d}\mathrm{d}y \\ &= \int_{\mathbb{R}} V_{t+1}(y) f_t^{j(t,d)}\left((y-\tau)\frac{1+\varrho_t}{d} - 1\right)\frac{1+\varrho_t}{d}\mathrm{d}y \end{aligned} \tag{6}$$

where the substitution $y = d(1+r)(1+\varrho_t)^{-1} + \tau$ has been used.

An important part of the problem (3)-(4) is the choice of the utility function U. The utility function varies across the investors and represents their attitude to the risk. In our case we use the iso-elastic utility function

$$U(d) = \frac{1}{1-a}\left((\kappa d)^{1-a} - 1\right)$$

where $\kappa = 1/12$ scales the utility function to be "steeper" for reasonable values and the numerical procedure is more stable; $a > 0$ is the constant coefficient of proportional risk aversion. Problem (3)-(4) then maximizes the expected utility of savings (compared to the last yearly salary) appointed for $\kappa = 1/12$ of the yearly benefits (i.e. the benefits for 1 month). Clearly, maximizing monthly benefits or yearly benefits leads to the same strategy.

3 NUMERICAL APPROXIMATION SCHEME

The numerical scheme for solving the dynamic stochastic accumulation model has been proposed in [2]. We discuss some key ideas of it.

The main difficulty in computing the Bellman integral (6) resides in significant oscillations in the integrand function, which may attain both large values as well as low values of the order one. Therefore a scaling technique is needed when computing the integral (6).

Let $H_t(d)$ be any bounded positive function for $t = 1, 2, ..., T$. We scale the function V_t by H_t, i.e. we define a new auxiliary function

$$W_t(d) = H_t(d)V_t(d)\,.$$

Clearly, the original function $V_t(d)$ can be easily calculated from $W_t(d)$. Then, for each time step t from $t = T$

down to $t=2$ we have

$$W_T(d) = H_T(d)V_T(d) \qquad \text{and}$$

$$\begin{aligned} W_{t-1}(d) &= H_{t-1}(d)V_{t-1}(d) \\ &= \max_{j\in\{1,2,\dots,m\}} \int_{\mathbb{R}} H_{t-1}(d)V_t\Big(\frac{d}{1+\varrho_t}(1+r)+\tau\Big) f_t^j(r)\mathrm{d}r \\ &= \max_{j\in\{1,2,\dots,m\}} \int_{\mathbb{R}} \frac{H_{t-1}(d)W_t\Big(\frac{d}{1+\varrho_t}(1+r)+\tau\Big)}{H_t\Big(\frac{d}{1+\varrho_t}(1+r)+\tau\Big)} f_t^j(r)\mathrm{d}r \\ &= \max_{j} \int_{\mathbb{R}} \frac{H_{t-1}(d)W_t(y)}{H_t(y)} f_t^j((y-\tau)\frac{1+\varrho_t}{d}-1)\frac{1+\varrho_t}{d}\mathrm{d}y\,. \end{aligned}$$

It is worthwhile noting that any choice of the family $H_t, t=1,...,T$, of positive bounded scaling function preserves the result. It may however significantly improve the stability of numerical computation.

We recursively define the scaling functions $H_t, t = T, T-1, ..., 2, 1$, depending on the previously computed solution V_{t+1} as follows:

$$H_T = \frac{1}{\sqrt{1+V_T^2}}, \quad \text{and}$$

$$H_t = \frac{1}{\sqrt{1+V_{t+1}^2}} \quad \text{for} \quad t = T-1,\dots,1\,.$$

In our algorithm we compute values of the function $W_t = W_t(d)$ for discrete values of d from the time dependent interval $d \in (d_{min}, t/2)$, where we use $d_{min} = 0.09$. In each time level $t = T$ down to $t = 1$ we choose a uniform spatial discretization of the interval $(d_{min}, t/2)$ consisting of $k = 200$ mesh points. In order to compute the Bellman type integral with normal distribution densities f_i^j we use the Simpson rule with 11 grid points. We take into account the rapid decay of normal distribution densities f_i^j and we replace them by zero outside the interval of the range of the grid points.

The output of the numerical code is a matrix of size $(T = 40) \times (k = 200)$ allowing us to "browse" between different years (rows) t and different levels of d (columns). At a given cell of the table we can read the name of the fund $(j = 1, ..., m)$ which has to be chosen. Plots of computed output matrices adjusted to the domain $\{(d,t), t \in (0,T), d \in (d_{min}, t/2)\}$ are depicted in the next sections.

4 OPTIMAL CHOICE AND PENSION PORTFOLIO SIMULATIONS

In paper [2] we implemented the proposed scheme for parameters with values given in Tab.2 and Tab.3. The assumed period of saving was $T = 40$ years and the percentage of salary transferred each year to a pension fund was $\tau = 9\%$ according to Slovak legislature. The typical result is depicted in the upper graphics of Fig. 1. This graphical plot shows the three regions I, II, resp. III in the (d,t) plane, in which fund $j = 1,2,3$ is respectively the optimal choice $j = j(d,t)$. The solid curvilinear line represents the path of the averaged wealth $E(d_t)$ calculated from 10000 simulations of the wealth d_t. The dashed lines correspond to $E(d_t) \pm \sigma_t$ intervals where σ_t is the standard deviation of the random variable d_t. One can observe the points where the solid curvilinear line intersects the region borders, i.e. the moments of "switching" between funds. For details of computation see [2].

In this recent work we investigated the sensitivity of the resulting strategy to changes in some parameters, like e.g. the height of returns, or the utility function parameter a. Accepting a higher risk (lower a) in the strategy leads to a higher expected level of the future pension benefits. Higher stock returns turned out to imply a later switch to less risky funds accompanied with a higher risk. Similarly, higher bond returns cause an earlier switch to more conservative funds.

Table 2. Data used for computation.

Fund	Return	StdDev
F_1	$r^1 = 0.1166$	$\sigma_1 = 0.1247$
F_2	$r^2 = 0.0923$	$\sigma_2 = 0.0780$
F_3	$r^3 = 0.0516$	$\sigma_3 = 0.0082$

Table 3. Data used for computation.

Period	2006-08	2009-14	2015-21	2022-24	2025-...
wage gr.	1.075	1.070	1.065	1.060	1.050

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However, it can hardly be expected that the returns of funds will remain constant over the whole period of saving. Based on the calibration of Cox-Ingersoll-Ross interest rate model it was shown in [4] that it is reasonable to expect the bond return to be decreasing to the value of approximately 2% in the time horizon of "some" years.

Let us investigate how the resulting strategy changes when we replace constant returns by returns decreasing monotonically to some target level. Let us assume that the return of each fund j decreases exponentially from the starting value r_0^j and in the infinite time horizon it converges to its target value r_∞^j. Then the rates in years $i = 1,2,...,T$, are given by formula

$$r_i^j = r_\infty^j + (r_0^j - r_\infty^j)\exp^{-K_i/T} \tag{7}$$

for funds $j = 1,2,3$ and some coefficients K_i. Let r_0^j have the values given in Tab. 2, and let $r_\infty^j = r_0^j/2$ for

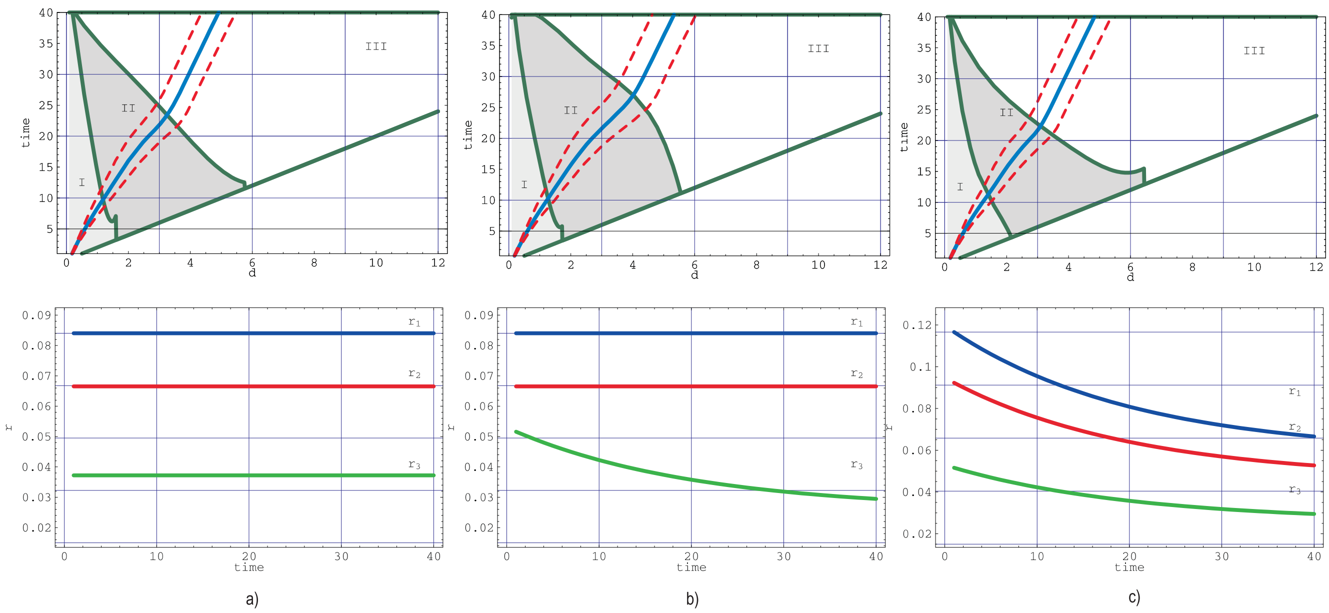

Fig. 1. Regions of optimal choice and the path of average saved return. a) r_1, r_2, r_3 constant, b) r_1, r_2 constant, r_3 monotonically decreasing, c) r_1, r_2, r_3 monotonically decreasing.

all j. Thus, $r_0^1 = 0.1166$, $r_0^2 = 0.0923$, $r_0^3 = 0.0516$, and $r_\infty^1 = 0.0583$, $r_\infty^2 = 0.0462$, $r_\infty^3 = 0.0258$.

First, we investigate what happens if the returns of the first two funds remain constant, and the bond fund return alone decreases exponentially to the target value. For the constant rates r_{aver}^j we take the geometric average return obtained from (7) for years $i = 1, \ldots, T (= 40)$ and $K_i = 2$. In the $a), b), c)$ part of Fig. 1 the rates of returns - all constant, one decreasing, and all decreasing, respectively (bottom) and the optimal choice regions with the simulated path (top) are depicted. Although the final wealth in the first two cases is not very different, we can observe the enlargement of the region II, i.e. a later switch to the conservative fund in case $b)$. The border between regions II and III is more curved than the one corresponding to the averaged value of r_{aver}^3. This is also in accordance to our intuition.

Second, we answer the question what happens if *all* fund returns are exponentially decreasing according to (7). We cannot expect anything by intuition because everything depends on the concrete starting and target returns and on the speed of decrease. It thus remains a question how the resulting optimal strategy will change. The part $c)$ of Fig. 1 gives the answer. The regions, the switching times, and the averaged final wealth do not change significantly compared to case $a)$. One can observe only a slight deformation of the region borders. This gives us the experience that decreasing returns lead to the same strategy as returns fixed on the level of *geometric mean of the returns in the first situation*. However, it can be expected that if the level of fixed returns is much higher or much lower than the mentioned geometric mean, the resulting strategies will be more different from each other.

6 CONCLUSIONS

We have presented a dynamic accumulation model for determining optimal switching strategies for choosing pension funds with different risk profiles. We tested the impact of non-constant fund returns on the optimal strategy in comparison to the optimal strategy obtained by considering fixed fund returns.

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