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The Effects of the Euro Area Entrance on the Monetary Transmission Mechanism in Slovakia in Light of the Global Economic Recession*

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Abstract
In this paper we estimate the monetary policy reaction function of the National Bank of Slovakia and the possible impact of an independent monetary policy on the Slovak economy in 2009 and 2010, when the global economic recession had the strongest impact on Slovakia. We estimate a small macroeconomic VEC model using a modified version of a Bayesian estimation technique developed for models using data observed with different frequencies with core inflation, the exchange rate, the real growth rate of GDP, the balance of trade and the interbank interest rate as endogenous domestic variables. Based on counterfactual simulations, we show that while an independent monetary policy would not be able to mitigate the drop in GDP in the first half of 2009, the recovery phase would have been positively affected.

1. Introduction

After Slovakia became a member of the euro area in 2009, monetary policy decisions were delegated from the National Bank of Slovakia (NBS) to the European Central Bank (ECB). A natural question is therefore whether monetary policy would have helped, had id been independent, to dampen the impact of the global financial crisis and the economic recession that led, *inter alia*, to a decline of gross domestic product in the course of 2008 and 2009, increasing unemployment and raising inflation after an initial lowering in 2009.

To answer this question, it is necessary to first investigate the monetary policy of the NBS and the transmission of monetary policy decisions to the real economy. A widely used method that is useful for studying monetary policy transmission and the effects of monetary policy shocks is the VAR/VEC framework. However, despite a relatively rich literature studying monetary policy-related topics using the above-mentioned modeling framework, research has still remained constrained in the case of Slovakia.

In this paper, we study the monetary policy of the NBS and the effects of monetary policy shocks on the Slovak economy and try to address the question of the possible impact of an independent monetary policy on the Slovak economy in 2009 and 2010. The paper is organized as follows: Section 2 gives a brief overview of the literature dealing with monetary policy using the VAR/VEC framework. Section 3 describes model specification and the estimation methodology. In Section 4 we discuss estimation results, study the effects of monetary policy shocks using impulse response

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functions and describe the results of robustness checks. Section 5 addresses the question of the possible impact of an independent monetary policy on the development of the Slovak economy, focusing mainly on GDP growth. We present the results of the counterfactual simulations and also the results of robustness checks. We conclude the paper in Section 6.

2. Analysis of Monetary Policy Using the VAR/VEC Framework

The importance of understanding monetary policy transmission, the consequences of the chosen monetary policy framework and the reaction of the real economy to monetary policy decisions and monetary policy shocks has led to a broad range of studies concentrating on these issues. For analysis of monetary policy shocks, a large number of studies have used the VAR or VEC framework since supporting evidence of the usefulness of such a framework was provided by Sims (1986).

While a quantitative comparison is relatively difficult, qualitative results of different studies of the effects of monetary policy shocks in different countries are quite comparable. In general, a contractionary monetary policy shock is followed by a drop in domestic economic activity and inflation (Christiano *et al.*, 1999; Peersman and Smets, 2001; Mojon and Peersman, 2001; Joiner, 2001; Borys *et al.*, 2009). On the other hand, there are several papers pointing to the existence of the price puzzle, which means after a contractionary monetary policy shock, inflation rises before it starts to decrease (see, for example, Hanson, 2004). The results are mixed for the exchange rate, as there are papers providing evidence of an appreciation after a contractionary shock in large economies (Eichenbaum and Evans, 1995) but also papers finding a gradual depreciation in response to a similar shock in open economies (Joiner, 2001; Borys *et al.*, 2009).

As in a lot of cases, the stationarity of the time series used for analysis of monetary policy transmission is questionable; there is a broad field of literature incorporating also the possible existence of a cointegrating relationship of the variables into the models (Holtemöller, 2003; Jang and Ogaki, 2004; Eleftheriou, 2009). In general, the equation for the change in the short-term interest rate or the cointegrating relationship between the short-term/policy rate and other macroeconomic variables is interpreted as the interest rate rule/monetary policy reaction function of the central bank.

While the literature focusing on monetary policy and the transmission of monetary policy is very broad and exhaustive, research still remains constrained in the case of Slovakia. In the most relevant papers, a contractionary monetary policy shock is followed by a drop in inflation and output (Jurašeková Kucserová, 2009; Horváth and Rusnák, 2009). While Horváth and Rusnák (2009) document an appreciating exchange rate after monetary policy tightening; this appreciation appears after an initial depreciation. Moreover, Horváth and Rusnák (2009) conclude that the ECB's monetary policy shock affects the development of prices more than that of the NBS. However, Jurašeková Kucserová (2009) uses time series from the period 1996–2008, which means the period of both qualitative and quantitative monetary policy is covered. Furthermore, the negative response of GDP is achieved only after eliminating technology shocks from the system and imposing sign restrictions on inflation. Even then, the reaction of GDP turns positive after a couple of months. Horváth and

Rusnák (2009) uses time series from the period 1999–2007, so the same criticism as before holds also for that paper. While achieving a negative response of the output gap, this response turns also positive after a few months and seems to be relatively insignificant. Moreover, as mentioned above, the paper documents a stronger reaction of the price level to the euro area monetary policy shock than to the domestic one.

On the other hand, there are papers providing more mixed evidence for Slovakia. Frömmel *et al.* (2011) try to estimate a Taylor-type monetary policy reaction function, but they do not find realistic inflation coefficients. The basic Taylor rule is also estimated in Polovková (2009), but the value of the coefficients for inflation and the output gap does not match the author's expectations and there is no adjustment of the policy rate. These mixed results of the literature can be partially related to the different methodologies, but partially also to the different time period with which the authors worked, as in the case of a small, open transition economy combining periods with different monetary policy regimes can lead to biased and non-realistic results. In our paper, we focus solely on the period when the NBS conducted qualitative monetary policy. Moreover, we take into account the possible cointegration between the domestic variables and, as our main contribution to the existing literature, in addition to the analysis of the monetary policy shocks we study the possible effects of alternative monetary policy decisions on the Slovak economy in the period 2009–2010.

3. Model Specification and Estimation Methodology

The main goal of the paper is to estimate the counterfactual benefits of having an independent monetary policy in the period 2009–2010, when the global economic recession translated into a sharp decline of the Slovak economy. As the first step, we have to know what reaction function such an independent monetary policy would have followed.

For studying monetary policy transmission, we used data from January 2000 to December 2008, i.e. from the period when the NBS conducted an independent qualitative monetary policy. The set of domestic endogenous variables that are included in the baseline specification are the Slovak interbank rate of one-month maturity ($BRIBOR1M_t$), core inflation (CPI_core_t, as a year-on-year percentage change of the price index), the year-on-year percentage change of real GDP (GDP_t), the EUR/ /SKK[1] exchange rate (the natural logarithm of the exchange rate EUR/SKK_ln_t) and the balance of trade as a share in nominal GDP (year-on-year changes in percentage points, BTA_t). The short-term interbank rate is included as the approximation of the policy rate, as a strong reaction of the Slovak interbank rates of shorter maturity to the changes of the NBS key interest rate can be documented (see, for example, Klacso, 2008, or NBS, 2008b).[2] Core inflation is included instead of the standard CPI or HICP inflation as there is evidence that in the case that the price level increased

[1] While the standard notation for the exchange rate is EUR/SKK, it expresses how many Slovak korunas can be purchased for EUR 1. This means that when the Slovak koruna appreciates, the EUR/SKK exchange rate decreases.

[2] Another advantage of approximating the key policy rate by the interbank rate is that the expectations of the banking sector about the future development of the key rate and possible asymmetric reactions to an increase or decrease of the policy rate are captured to some extent.

due to increasing regulated prices or tax increases that did not have a direct impact on core inflation, the central bank did not react with a restrictive monetary policy (this was the case in, for example, 2003; see NBS, 2004). The balance of trade is included additionally to the standard macroeconomic variables used for the analysis of monetary policy due to the fact that, mainly in the first years of the independent qualitative monetary policy, the central bank adjusted interest rates also based on the development of the trade deficit (mainly in 2001–2003; see NBS, 2002; NBS, 2003; and NBS, 2004).

The set of other variables representing the set of exogenous variables includes the EURIBOR interbank interest rate of one-month maturity in percentages to capture the monetary policy decisions of the ECB and their impact on the Slovak economy, the EUR/USD exchange rate (in natural logarithms), the index of Brent crude oil one month Forward per barrel (in natural logarithms) and the Standard & Poor's 500 stock index (in natural logarithms) as an indicator of the global economic and fiscal cycle and thus of foreign demand. As monetary and fiscal policies should be independent of each other and the aim of this paper is not to study fiscal policy, the inflation of regulated prices is considered to be an exogenous variable that affects the inflation rate. In the case of the interest rates and variables where year-on-year changes (in percentage or percentage points) enter the equation, data were not seasonally adjusted. In all other cases data were seasonally adjusted using Census X12.[3]

To get consistent and unbiased estimates, the VAR/VEC methodology requires macroeconomic data to be stationary or cointegrated. One of the basic assumptions stationary series have to fulfill is that the series started sufficiently long ago to get near their limiting mean value (see, for example, Enders, 1995, or Gerlach-Kristen, 2003). As there is not such a long history available for Slovak macroeconomic time series, our assumption is that these series behave as non-stationary. This assumption is supported in nearly all cases also by unit root tests (see *Appendix 1*), when at least one of the tests used does not reject the null hypothesis that the time series contain a unit root. There is one disputable case, GDP growth, where the interpolated series are integrated at least of order two based on the results of the Phillips-Perron test. However, as they can be treated as integrated of order one based on the ADF test and they are integrated of order one at most using quarterly series (based on the results of both the ADF and PP test), we treat GDP growth also as integrated of order 1. Therefore we dealt with the data as integrated of order 1.

Moreover, as cointegration tests do not reject the existence of a cointegrating relationship between the domestic variables (see *Appendix 3*, Baseline), we estimate a VEC model as this cointegrating relationship between the endogenous variables may contain important information, omission of which can lead to misleading estimation results. The general form of the VEC model can be written as

$$\Delta \mathbf{y_t} = \mathbf{A} + \mathbf{\Pi y_{t-1}} + \sum_{j-1}^{p} \mathbf{B_j \Delta y_{t-j}} + \mathbf{Cexog_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t},\ t = 1,...,T \tag{3.1}$$

where $\mathbf{y_t}$ is an $n \times 1$ vector of endogenous variables, $\mathbf{exog_t}$ is an $m \times 1$ vector of exo-

[3] To seasonally adjust data, we used Census X12 with the additive method and the default seasonal and trend filter in Eviews 7.

genous variables (trend and dummy variables may be also included), and $\mathbf{A}$, $\mathbf{B_i}$, $\mathbf{\Pi}$ and $\mathbf{C}$ are $n \times 1, n \times n$ and $n \times m$ matrices of parameters to be estimated, $\boldsymbol{\varepsilon}_\mathbf{t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$. For a better interpretation, the model can be written in a parameterized form:

$$\Delta \mathbf{y_t} = \mathbf{A} + \boldsymbol{\alpha}\boldsymbol{\beta}^\mathbf{T}\mathbf{y_{t-1}} + \sum_{j-1}^{p} \mathbf{B_j}\Delta\mathbf{y_{t-j}} + \mathbf{Cexog_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{3.2}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ matrices of rank r. The rank refers to the number of cointegrating relationships between the endogenous variables, the columns of matrix $\boldsymbol{\beta}$ represent the cointegrating vectors and the columns of matrix $\boldsymbol{\alpha}$ are the adjustment parameters.

As there are only nine years of observations when the NBS conducted qualitative monetary policy, it is necessary to use monthly data rather than quarterly. However, as there are time series that are available only with quarterly frequencies (GDP growth, balance of trade), we had to estimate/incorporate missing observations. There are several approaches to dealing with this problem in the context of monetary policy analysis. A straightforward way is to interpolate data observed at lower frequencies, as in Borys *et al.* (2009), where the authors use quadratic interpolation. Bernanke *et al.* (1997) use a form of state space model to interpolate quarterly GDP data. Jurašeková Kucserová (2009) constructs monthly GDP data based on the dependence of the quarterly data on quarterly receipts in selected branches of the economy.

We introduce a modified version of the Bayesian estimation method for mixed frequency VARs (BMF estimator) published in Eraker *et al.* (2011), which can be used to estimate VEC models with endogenous variables observed at mixed frequencies. In their paper, the authors compare their method to the basic approach of using only the lowest frequency for the estimation (this means that in the case of monthly and quarterly data, quarterly observations are used for the estimation). They show that the BMF estimator produces more accurate estimates of model parameters. They also argue in favor of the BMF estimator compared to the Kalman filtering approach: "[…] the Kalman filter approach is potentially cumbersome when the missing data occur at irregular frequencies, especially if there are multiple series with missing data at differing frequencies. In addition, the Kalman filter yields a likelihood function that is non-linear and non-Gaussian over a potentially very large parameter space; analyzing such likelihood functions often proves difficult both from frequentist and Bayesian viewpoints." (pp. 3–4) For further details, we refer to the paper.

This BMF estimator is an application of the Bayesian Gibbs sampler that draws the parameters (the objects of interest) in every iteration from the conditional posterior distributions of these parameters given their initial value and prior distributions. In the case of the VEC model, the parameters to be estimated (or the objects of interest) are the matrices $\mathbf{A}$, $\mathbf{B_j}$, $\mathbf{C}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\Sigma}$ and the missing observations of the endogenous variables.

Let us denote $\mathbf{y_{o,t}}$ the set of endogenous variables that are fully observed and $\mathbf{y_{u,t}}$ the set of endogenous variables with missing observations, so

$$\mathbf{y_t} = \begin{pmatrix} \mathbf{y_{o,t}} \\ \mathbf{y_{u,t}} \end{pmatrix}$$

For convenience, as it is also sufficient for the purposes of this paper, let's assume that there are only two frequencies at which the data are observed (monthly and quarterly in our case). Let $\hat{\mathbf{y}}_{\mathbf{u}}$ denote the set of observed and sampled data, $\hat{\mathbf{y}}_{\mathbf{u},\backslash\mathbf{t}}$ all elements of $\hat{\mathbf{y}}_{\mathbf{u}}$ except of the *t*-th ones and $\hat{\mathbf{Y}}^{\mathbf{i}}$ the complete set of observed and sampled data at iteration *i*. Given the initial values of the parameters and their prior distributions, the *i*-th iteration consists of the following steps:

– Step 1: for $t = 1,...,T$, draw missing data $\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}$ conditional on $\mathbf{y}_{\mathbf{o}}, \hat{\mathbf{y}}^{\mathbf{i-1}}_{\mathbf{u},\backslash\mathbf{t}}, \mathbf{A}^{\mathbf{i-1}}, \mathbf{B}^{\mathbf{i-1}}_{\mathbf{j}}$, $\mathbf{C}^{\mathbf{i-1}}, \boldsymbol{\alpha}^{\mathbf{i-1}}, \boldsymbol{\beta}^{\mathbf{i-1}}, \boldsymbol{\Sigma}^{\mathbf{i-1}}$, where $\hat{\mathbf{y}}^{\mathbf{i-1}}_{\mathbf{u},\backslash\mathbf{t}}$ denote the set of the most recently updated missing variables. That is, if the missing variables are updated in a consecutive order, $\hat{\mathbf{y}}^{\mathbf{i-1}}_{\mathbf{u},\backslash\mathbf{t}} = \left(\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,1}}, \hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,2}}, ..., \hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t-1}}, \hat{\mathbf{y}}^{\mathbf{i-1}}_{\mathbf{u,t+1}}, ..., \hat{\mathbf{y}}^{\mathbf{i-1}}_{\mathbf{u,T}}\right)$. $\mathbf{A}^{\mathbf{i-1}}, \mathbf{B}^{\mathbf{i-1}}_{\mathbf{j}}$, $\mathbf{C}^{\mathbf{i-1}}, \boldsymbol{\alpha}^{\mathbf{i-1}}, \boldsymbol{\beta}^{\mathbf{i-1}}, \boldsymbol{\Sigma}^{\mathbf{i-1}}$ are the latest draws of the parameter matrices;
– Step 2: draw $\boldsymbol{\beta}^{\mathbf{i}}$ conditional on $\hat{\mathbf{Y}}^{\mathbf{i}}, \mathbf{A}^{\mathbf{i-1}}, \mathbf{B}^{\mathbf{i-1}}_{\mathbf{j}}, \mathbf{C}^{\mathbf{i-1}}, \boldsymbol{\alpha}^{\mathbf{i-1}}, \boldsymbol{\Sigma}^{\mathbf{i-1}}$;
– Step 3: draw $\mathbf{A}^{\mathbf{i}}, \mathbf{B}^{\mathbf{i}}_{\mathbf{j}}, \mathbf{C}^{\mathbf{i}}, \boldsymbol{\alpha}^{\mathbf{i}}$ conditional on $\hat{\mathbf{Y}}^{\mathbf{i}}, \boldsymbol{\beta}^{\mathbf{i}}, \boldsymbol{\Sigma}^{\mathbf{i-1}}$;
– Step 4: draw $\boldsymbol{\Sigma}^{\mathbf{i}}$ conditional on $\hat{\mathbf{Y}}^{\mathbf{i}}, \mathbf{A}^{\mathbf{i}}, \mathbf{B}^{\mathbf{i}}_{\mathbf{j}}, \mathbf{C}^{\mathbf{i}}, \boldsymbol{\alpha}^{\mathbf{i}}, \boldsymbol{\beta}^{\mathbf{i}}$.

A detailed description of the drawing is provided in *Appendix 2*.

4. Estimation Results

In the baseline specification, we estimate a VEC model of order 1 containing five endogenous variables and five exogenous variables, also with one lag. Following Eleftheriou (2009), we interpret the cointegrating relationship as the monetary policy reaction function that represents the optimal value of the policy rate in relation to the value of other endogenous variables.[4] Therefore, the cointegrating vector is normalized such that the coefficient for the interbank rate is equal to one.

In the 2000–2008 period, monetary policy easing took place in general in an environment of decreasing inflation, an appreciating currency and an improving trade balance (NBS, 2001; NBS, 2002; NBS, 2003; NBS, 2004; NBS, 2005; NBS, 2006; NBS, 2007; NBS, 2008; and NBS, 2009). This means that if we write the co-integrating equation in the form:

$$BRIBOR1M_t = \beta_0 + \beta_1 CPI\_core_t + \beta_2 EUR/SKK\_\ln_t + \beta_3 BTA_t + \beta_4 GDP_t$$

we expect a positive sign of the coefficient for the inflation and the exchange rate and a negative sign of the coefficient for the trade balance. If GDP growth is in line with expectations, positive growth can be followed by monetary policy easing; however, potential overheating or extensive GDP growth can be followed by monetary policy tightening. Therefore, we have no *ex ante* expectations regarding the sign of the coef-ficient. In order to interpret the cointegrating equation as the monetary policy

[4] This approach has several shortcomings, however. It is hard to incorporate the channel of expectations in the model and it is also not possible to include leads or lags of the endogenous variables into the co-integrating relationship.

Table 1 Estimation Results—Baseline Specification

	BRIBOR1M	*CPI_core*	*EUR/SKK_ln*	*BTA*	*GDP*
Cointegrating coefficients	1.0000	1.0428	17.5903	0.0822	0.2326
Adjustment coefficients	-0.0149	0.0026	0.0005	0.1162	0.1352

reaction function, it is necessary for the adjustment coefficient for the interbank rate to have a negative sign so that if there is a deviation from the cointegrating relationship the policy rate reacts in the expected way.

In line with our expectations, the core inflation and the exchange rate enters the cointegrating equation with a positive sign, which means that a contractionary monetary policy followed an increasing inflation rate or a depreciating exchange rate.[5] Interestingly, the coefficient for the inflation is nearly one, which means that we cannot reject the hypothesis that the changes of core inflation were fully reflected in the monetary policy decisions. While we have not had an explicit expectation for the coefficient for GDP growth, the coefficient for the balance of trade is positive, contrary to our expectations. Based also on the small value of the coefficient, however, our explanation is that the balance of trade is merely of secondary importance in the reaction function. The negative value of the adjustment coefficient for the inter -bank rate means it is possible to interpret the cointegrating equation as the monetary policy reaction function. Based on the adjustment coefficient, the monthly correction of the interest rate in the case of a deviation from its equilibrium level is 1.5%. This relatively low adjustment rate can be explained by the possible changes of the weights of the respective domestic variables entering the reaction function in the period under review that is not captured by our model. Another possible explanation is that we included in the reaction function only domestic variables, while the monetary authority possibly reflected also the development of foreign variables (e.g. the development of the base rate of the ECB). Finally, the slow adjustment can reflect relatively strong inertia of the policy rate or the interest rate smoothing analyzed by, for example, Sack and Wieland (2000).

4.1 Impulse Responses

In this section we describe the effects of a monetary policy shock using impulse response functions. We focused on the reaction of the endogenous variables to a contractionary monetary policy shock. For identification of the shock we used the benchmark recursive assumption used also in Christiano *et al.* (1999), i.e. we assumed that the monetary policy shocks are orthogonal to the information set of the central bank and used a Cholesky decomposition of the variance-covariance matrix of the residuals. The order of the endogenous variables used in the decomposition is: GDP, core inflation, the exchange rate, the balance of trade and the interbank rate. The interbank rate is in last place to ensure the assumption of orthogonality, so that the monetary policy shock has no contemporaneous effects on the rest of the variables. As is shown in the above-mentioned study, the ordering of the rest of the variables does not alter their responses to the monetary policy shock.

[5] For the estimation we used 20,000 iterations with the first 10,000 iterations serving as burn-in. The parameters for the prior distributions were taken from a ML estimation of the VEC models using monthly approximation of quarterly data calculated by cubic interpolation.

As expected, the reaction of the interbank interest rate is immediate and fast. A contractionary monetary policy shock transmits into an increase of the short-term interest rate, while the peak of the response is after two months (*Appendix 4*). This result supports the functioning of the first stage of the transmission mechanism, i.e. the strong reaction of the interbank interest rates to the changes in the policy rates of the NBS.

Results are mixed in the case of the response of core inflation. The contractionary monetary policy shock is followed by an immediate increase of core inflation; the reaction turns negative after approximately four months while the cumulative response is negative after one year. This means that the price puzzle is present in the dynamics of core inflation. However, this reaction seems to be of negligible size and the result can be viewed as insignificant based on the 90% coverage interval. As during the period under review, monetary policy tightening took place in the case of an expected acceleration of the price dynamics; this result can reflect the fact that generally the monetary policy tightening was really followed by an increase of inflation. Another explanation of the insignificant result can be that during the period from 2000 to 2008 monetary policy worked mainly through its systemic impact on the economy and monetary policy shocks were of only minor relevance.

In the case of the exchange rate, there is an initial appreciation followed by a gradual depreciation. This result is in line with, for example, the findings in Borys *et al.* (2009) for the Czech economy. However, in contrast to their results, there is a cumulative appreciation of the exchange rate in the long run. Similarly to core inflation, these results are rather ambiguous based on the 90% coverage intervals.

The response of the balance of trade is in line with expectations, as after a contractionary monetary policy shock there is an increase of the trade balance. This means that after an increase of the key interest rates there is a positive development of economic imbalances. This result seems to be relatively significant; the shock diminishes approximately after one year.

The response of GDP is in line with the appreciating exchange rate, increasing inflation and the positive development of the trade balance. On the other hand, the results are in contradiction with the expected effect of a restrictionary monetary policy shock. GDP growth increases, reaching its peak after approximately one quarter. While the coverage intervals are relatively wide also in this case, the responses are more significant than in the case of the exchange rate or inflation.

While the appreciating exchange rate and the cumulative negative response of core inflation to monetary policy tightening is broadly in line with the economic theory, the rather weak response of the price level and the confusing reaction of the output is to a certain extent in line with the outcome of the literature studying the effects of monetary policy shocks in Slovakia described in Section 2. Based on the impulse response functions, however, the relevance of monetary policy shocks for the Slovak economy in the period under review is relatively low.

4.2 Robustness Checks

In the previous parts we described the baseline specification of our VEC model. However, as it is usually not entirely clear which macroeconomic variables enter the reaction function of the respective central bank; in this section we present

Table 2 Model Specifications—Endogenous Variables

	y_t				
Baseline	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ln_t	BTA_t	GDP_t
Specification 1	$BRIBOR1M_t$	CPI_t	EUR/SKK_ch_t	BTA_t	GDP_t
Specification 2	$BRIBOR1M_t$	CPI_t	EUR/SKK_ln_t	BTA_t	GDP_t
Specification 3	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ch_t	BTA_t	GDP_t
Specification 4	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ch_t	$BCCA_t$	GDP_t
Specification 5	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ln_t	$BCCA_t$	GDP_t
Specification 6	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ch_t		GDP_t
Specification 7	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ln_t		GDP_t
Specification 8	$BRIBOR1M_t$	CPI_t			GDP_gap_t
Specification 9	$BRIBOR1M_t$	CPI_core_t			GDP_gap_t
Specification 10	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ln_t		GDP_gap_t
Specification 11	$BRIBOR1M_t$	CPI_t	EUR/SKK_ln_t		GDP_gap_t
Specification 12	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ln_t	BTA_t	GDP_gap_t
Specification 13	$BRIBOR1M_t$	CPI_t	EUR/SKK_ln_t	BTA_t	GDP_gap_t
Specification 14	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ch_t	BTA_t	GDP_gap_t
Specification 15	$BRIBOR1M_t$	CPI_t	EUR/SKK_ch_t	BTA_t	GDP_gap_t
Specification 16	$BRIBOR1M_t$	CPI_core_t	EUR/SKK_ch_t		GDP_gap_t
Specification 17	$BRIBOR1M_t$	CPI_t	EUR/SKK_ch_t		GDP_gap_t

the result of the robustness check, where we compare different possible model specifications.

The set of endogenous variables potentially entering the monetary policy reaction function includes inflation (core inflation or CPI inflation, CPI_t), the indicator of economic development (GDP or the output gap[6] designated as GDP_gap_t), the EUR/SKK exchange rate (in levels or the year-on-year changes capturing the dynamics of the exchange rate designated as EUR/SKK_ch_t) and the indicator of economic imbalances (the balance of trade or the current and capital account designated as $BCCA_t$). The different specifications that were tested are presented in *Table 2*.[7] Johansen cointegration tests confirmed the existence of a cointegrating relationship between the endogenous variables, as in all cases at least one of the tests pointed to the existence of such a relationship (*Appendix 3*). We assumed one cointegrating relationship in all cases.

The estimated coefficients of cointegrating vectors and the related adjustment coefficients (*Appendix 5*) point to a relatively robust result: the negative sign of

[6] The output gap was estimated by detrending the monthly approximation of the real GDP growth rate using an HP filter with $\lambda = 14{,}400$.

[7] In the case where the year-on-year change of the exchange rate is included among the endogenous variables, it is the year-on-year change of the EUR/USD exchange rate that is included among the exogenous variables. In the case where CPI inflation is included among the endogenous variables, not just the lagged change but also the actual change of the inflation of regulated prices is included among the exogenous variables.

the adjustment coefficient for the interbank rate in more than half of the specifications confirms the interpretation of the cointegrating vector as the monetary policy reaction function. On the other hand, when it is the balance of the current and capital account or the GDP gap that is included in the specification, this adjustment coefficient has a wrong sign in several cases.

When comparing the specifications based on the Bayesian information criterion,[8] only specifications 7, 10 and 11 have a smaller value of this criterion than the baseline specification (*Appendix 6*). An interesting result is that in all four cases it is the logarithm of the exchange rate that is included in the list of endogenous variables.

When the specifications are compared based on the sum of squared residuals and thus the ability of the models to capture the development of the endogenous variables in the period 2000–2008 (*Appendix 7*), the results do not point to any specification significantly outperforming the baseline specification. The sum of squared residuals for the interbank rate is comparable across all specifications, which means that including the output gap instead of the GDP growth rate does not improve the estimation of the interest rate. An interesting result is that the development of the yearly change of the exchange rate is captured better when GDP growth is included. The development of GDP is captured better when the CPI is included instead of core inflation. A possible interpretation is that the CPI includes more information about the development of the real economy than does core information. However, in the case of the output gap there is no significant difference between the specifications including the CPI and core inflation.

The results are more explicit when comparing the forecasting ability of the models (*Appendix 8*).[9] When Slovakia became a member of the euro area at the beginning of 2009, monetary policy decisions were delegated to the ECB and the euro became the country's domestic currency. Therefore, we estimated only the rate of inflation, the indicator of economic imbalance and GDP growth, while the interbank interest rate was replaced by the EURIBOR interbank rate and the EUR/SKK exchange rate was kept constant and equal to its value at the end of 2008. We estimated the development of the three remaining endogenous variables for 2009 and 2010, when the impact of the economic recession peaked in Slovakia.

A basic result of the forecasts is that all of the specifications overestimate inflation up to March 2010. It seems that during this period inflation was at historically low levels due to external factors that are not included in the models (see, for example, NBS, 2010). Regarding the other two variables, the best estimates are clearly given by the baseline specification. In all other cases, the real values of the variables were out of the coverage intervals or the coverage intervals were too wide. In the case of the baseline specification, not only the development of the balance of trade in 2009 and 2010 is predicted relatively well, but also the drop in GDP at

[8] The Bayesian information criterion is calculated as $BIC = -2l/T + n\ln(T)/T$, where T is the sample size, n is the number of parameters and l is the natural logarithm of the maximized log-likelihood of the model.

[9] For simplicity, the forecasts are shown only for models where the cointegrating equation can be interpreted as the monetary policy reaction function, i.e. where the adjustment coefficient for the interbank rate is negative.

the beginning of the period and the gradual increase from the second half of 2009 are captured. The drop in GDP or the output gap is captured in nearly all specifications, supporting the inclusion of the oil price index and the stock index as an indicator of external demand. The comparison of the baseline specification and specification 3 points to the conclusion that the level of the exchange rates (EUR/SKK and EUR/USD) can better help to predict the development of the domestic economy than can their dynamic. The increase of the output gap is also captured in the models to a certain extent, but the coverage intervals point in favor of the baseline specification.

5. Counterfactual Effects of an Independent Monetary Policy

In this section we try to answer the question of whether monetary policy would have helped to mitigate the effects of the financial crisis and the global economic downturn on the Slovak economy had it been independent. Our main focus will be on the possible development of domestic GDP in the period 2009–2010. To do this, we conducted several counterfactual experiments that simulate the development of the economy under different possible paths of monetary policy.

While there are different approaches when conducting counterfactual policy experiments (see, for example, Bernanke *et al.*, 1997; Carlstrom and Fuerst, 2006; Sims and Zha, 2006), we followed to a certain extent the approach used by Sims and Zha (2006) and Bernanke *et al.* (1997). In the simulations, we have to distinguish between the systemic policy changes, i.e. changes that are expected by agents and the exogenous policy shocks, or changes that are unexpected. We assume that agents form their expectations based on the estimated policy rule and the adjustment of the interest rate in the case that there is a deviation from the estimated "optimal" level. This means that if we assume that monetary policy would have reacted based on the estimated policy rule, all the changes of monetary policy could be viewed as systemic and expected, so we would not pose any monetary policy shock on the system and the economy would have been affected solely by the systemic part of monetary policy.

The problem is more complex if we assume that monetary policy would not have reacted based on the historically observed policy rule. In this case, we assume that changes in monetary policy can be divided into a systemic part (the part of the change that would lead to the interest rate expected by agents) and an exogenous shock that explains the difference between the expected and the actual value of the interest rate. Moreover, we assume that within the relatively short period of our interest it would not be possible for the agents to adjust their expectations based on the new information in the form of monetary policy shocks. This means that when assuming an alternative monetary policy reaction, we complement the systemic part of the monetary policy reaction with an exogenous monetary policy shock with the parameters of the systemic part of monetary policy (i.e. the estimated coefficients of the VEC model and the variance-covariance matrix of the residuals) kept unchanged. As the systemic part of monetary policy, i.e. the way agents form their expectations, would not be adjusted during the two-year period, naturally the Lucas critique holds for this approach to a large extent. On the other hand, at least during the first months of 2009 (i.e. until May 2009), when the impact of the financial crisis and the global recession on the Slovak economy was the most pronounced, we think it is reasonable to assume an unexpected systemic part of monetary policy. Even in Sims and Zha (2006), the authors indicate that it is unreasonable to expect that "[…] policy change is immediately and fully understood and that the public has no doubt that it is permanent" (p. 27). This means that the Lucas critique is more biting in the recovery phase, from the beginning of the second half of 2009.

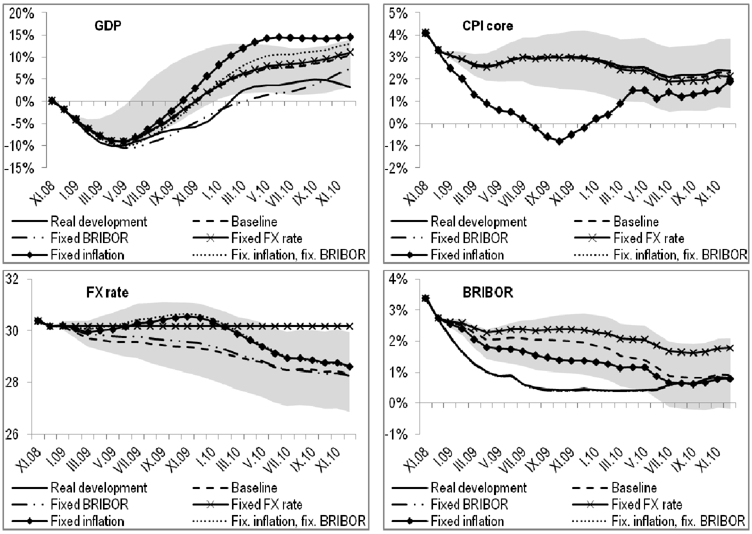
Monetary policy shocks are identified in the same way as in the previous section describing the impulse response functions. We used a Cholesky decomposition of the variance-covariance matrix of the residuals with the following order of the endogenous variables: GDP, core inflation, exchange rate, balance of trade and interbank rate. The interbank rate is in last place so that the monetary policy shock has no contemporaneous effects on the rest of the variables. This means that we assume the monetary policy shock works through the interest rate, so we are omitting other possible channels (such as the credit channel). This, together with the outcome of the analysis of impulse response functions (the overall weak impact of the monetary policy shocks) and the unchanged systemic part of monetary policy means that the results of the counterfactual experiments can represent a "lower bound" on the possible contribution of monetary policy to economic development.

5.1 Baseline Simulation

Within the "Baseline" simulation, we study the impact of an independent monetary policy (i.e. we simulate the development of the economy without Slovakia joining the euro area at the end of 2008) on the economy assuming it would follow the monetary policy rule estimated by the benchmark VEC model. Similarly as in the other simulations, we assume that the impact of the financial crisis and the global economic downturn is captured by the impact of the exogenous variables on the endogenous variables. This means that within this simulation we estimate the development of the endogenous variables with the value of the exogenous variables fixed for the period 2009–2010 at their real value and without posing any additional shock to the endogenous variables. The estimated development of the endogenous variables (*Figure 1*, designated as Baseline) is then compared to their real development in the period 2009–2010 (*Figure 1*, designated as Real development).

The "Baseline" simulation resulted in a higher interbank interest rate during the two-year period than its real value. This higher interbank interest rate can be related to the higher level of inflation, which does not differ significantly from the forecasted inflation in the baseline specification (presented in *Appendix 8*), and also to the fact that the reaction of the ECB to the financial crisis in 2009 was relatively fast and unprecedented. The drop in GDP in the first half of 2009 is present also under the "Baseline" simulation and is practically identical to the drop that occurred in reality. During the recovery phase, the simulation resulted in higher GDP growth compared to its real values. The outcome for inflation is in line with the findings in the previous section, i.e. monetary policy has only a small impact on inflation and that the price dynamics are driven mainly by other, domestic and external factors. The model expects a gradual appreciation of the exchange rate, which is probably driven (technically) by the strong appreciation trend observed in the years before Slovakia became a member of the euro area.

Figure 1 Counterfactual Simulations



Note: Mean values of the simulations and 70% confidence intervals for the Baseline simulation reported.

5.2 Possible Effects of an Alternative Monetary Policy

The next question is whether an alternative monetary policy reaction in the form of a more pronounced drop in interest rates in the first half of 2009 would have had a stronger effect on GDP without the exchange rate having been fixed. Within this simulation we impose monetary policy shocks on the VEC model to achieve the real development of the interest rates, so we replicate the monetary policy of the ECB with the exchange rate not fixed. This means that monetary policy shocks causing a lower interest rate than expected (monetary policy shocks are calibrated in such a way that they "explain" the difference between the interest rate under the "Baseline" simulation and the real interest rate) "surprise" agents during the two-year period.

The results of the simulation (*Figure 1*, designated as Fixed BRIBOR) underline the weak impact of the interest rates on inflation, as the price level does not differ significantly from the "Baseline" simulation. In the case of GDP, the results suggest that the external shock affected the Slovak economy to such an extent that even a much quicker reaction of the monetary policy would not have been able to dampen the impact significantly. While there are no significant differences, the development of GDP during the recovery phase under this simulation is closer to the real development than to the development under the "Baseline" simulation, whereas under the "Baseline" simulation GDP growth would be higher during the whole recovery phase. In the case of the exchange rate, under this simulation the appreciation would be less pronounced compared to the "Baseline" simulation, which can be related to the lower interest rate. However, there are no significant differences in the development of the exchange rate under the two simulations.

Based on the results of these two simulations, it would not have been possible to dampen the drop in GDP growth in the first half of 2009 when having an independent monetary policy. Monetary policy in the form of gradually decreasing interest rates affects economic development mainly during its recovery phase. A possible explanation is that the impact of external development was so strong and quick (real GDP growth turned negative in the last quarter of 2008 and reached its lowest value in the second quarter of 2009) that the monetary policy would not have been able to counteract this impact in such a short period. An interesting result is that GDP grows faster from the beginning of the second half of 2009 under the "Baseline" specification than under the specification designated as "Fixed BRIBOR" or when compared to the real development despite higher interest rates. We explain this development by the fact that in the "Fixed BRIBOR" specification the interest rate drops significantly during the strongest impact of the external shock, while during the recovery phase there is no space for further monetary policy easing. While the low interest rates do not prevent the economy from sliding into recession, monetary easing during the recovery can have a more significant impact.

5.3 The Importance of the Exchange Rate Channel

Within the next simulation (*Figure 1*, designated as Fixed FX rate) we address the question of whether it is the gradually decreasing interest rate or the appreciating exchange rate that causes higher GDP growth during the recovery phase under the "Baseline" simulation compared to the real development. Within this simulation we "switch off" the exchange rate channel, i.e. we impose on the VEC model exchange rate shocks calibrated in such a way that they offset the appreciation observed in the "Baseline" simulation compared to the real development.

Based on the fact that the development of GDP growth under the "Fixed FX rate" simulation is closer to the "Baseline" simulation and GDP growth under the "Fixed BRIBOR" simulation is closer to the real development, we conclude that higher GDP growth during the recovery phase is due more to the gradually decreasing interest rate. This result is also more in line with the economic theory that for a small open and export-oriented economy (like Slovakia) depreciation of the exchange rate can help more to boost production. While the Slovak koruna is relatively weaker under the "Baseline" simulation compared to the "Fixed FX rate" simulation, there is practically no difference in GDP growth. Based on the results of the simulation, we conclude that in the case of the "Fixed FX rate" simulation it is the higher interest rate (as a possible consequence of the weaker Slovak koruna) that offsets the positive impact of the weaker currency on output.

5.4 Simulations with Fixed Inflation

As a robust result of all the simulations is that inflation is affected only to a negligible extent, we present here two more simulations. The first simulates fixed inflation at its true values through the two-year horizon (*Figure 1*, designated as Fixed inflation), while the second one simulates also the development of the interbank rate identical to its real development (*Figure 1,* designated as Fixed inflation, fixed BRIBOR). In the first case we simulated the reaction of the NBS based on the estimated reaction function, while in the second we simulated the impact of a more pronounced monetary policy on GDP with inflation lower than that predicted by the model. In both simulations we imposed inflationary shocks and monetary policy shocks in a way similar to that described in the previous simulations.

In both simulations we get a much slower appreciation/depreciation of the exchange rate up to the end of 2009. In the "Fixed inflation" simulation we get a gradually decreasing interest rate that is lower than in the previous simulations, which is in line with the traditional monetary policy reaction to lower price dynamics. In line with the previous simulations, the drop in GDP in 2009 would not be dampened by the monetary policy. On the other hand, we get a stronger recovery in both cases. This stronger recovery can be related partially to the weaker exchange rate, which is in line with the results of the previous simulations. The stronger effect of gradually decreasing interest rates is also supported.

5.5 Robustness Checks

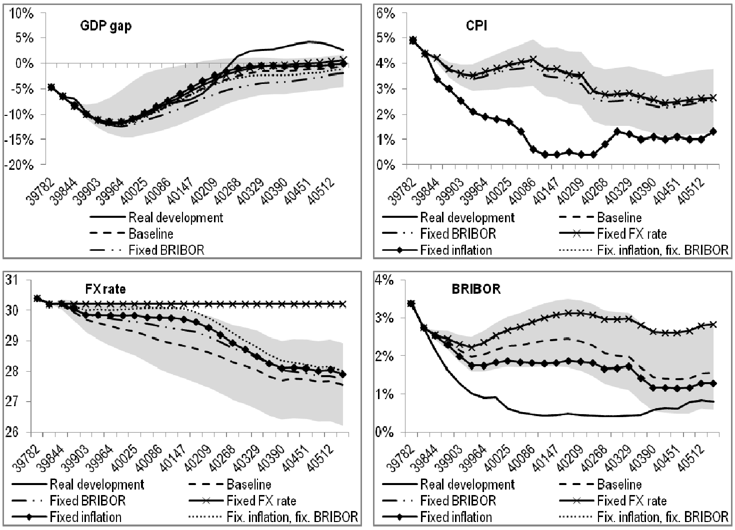
In the previous part we described the results of the counterfactual simulations using the baseline specification of the VEC model. The choice of this specification was confirmed by its forecasting ability and also by the relatively low value of the Bayesian information criterion compared to the other specifications. On the other hand, as this does not ensure that the model captures the true data generating process, in this part we present the results of additional counterfactual simulations using an alternative specification.

The alternative specification chosen for this part is specification 11. This is a more commonly used specification containing the interbank rate, CPI inflation, the exchange rate (in natural logarithms) and the output gap. In line with expectations, inflation, the exchange rate and the output gap enter the cointegrating equation with a positive coefficient, and the negative value of the adjustment coefficient for the interbank rate confirms that the cointegrating equation can be interpreted as the monetary policy reaction function (*Appendix 6*). While the forecasting ability of this specification is not as good as that of the baseline specification, it is better than that of the other specifications and the value of the Bayesian information criterion is lower than that of the benchmark specification.

For the counterfactual experiments, we used the same simulations as in the previous part. The monetary policy shocks were identified using the Cholesky decomposition of the variance-covariance matrix of the residuals with the endogenous variables in the following order: output gap, inflation, exchange rate and the interest rate. This means that, as in the previous part, we assume that the monetary policy shock has no contemporaneous effects on the rest of the variables.

The results of the counterfactual simulations are presented in *Figure 2*. While it is not possible to directly compare the results of these simulations to those using the baseline specification—the output gap is included instead of real GDP growth and CPI inflation instead of core inflation—the qualitative results are to a great extent the same. Based on the results, it would not be possible to dampen the drop in the output gap when having an independent monetary policy in place. Inflation would be higher than its real value also in this case. A small qualitative difference is that using specification 11, the simulated development of the output gap would remain under its real development also in the recovery phase in all specifications. This can be partially related to the result that there would not be depreciation of the exchange rate either in the "Fixed inflation" or in the "Fixed inflation, fixed BRIBOR" simulation. On the other hand, as we mentioned above, the results of the simulations can

Chart 2 Counterfactual Simulations Using an Alternative Specification



Note: Mean values of the simulations and 70% confidence intervals for the Baseline simulation reported.

represent a "lower bound" on the possible contribution of monetary policy to economic development. This means that the possible positive contribution of monetary policy during the recovery phase cannot be ruled out based on these results.

6. Conclusions

In this paper we have estimated a small macroeconomic VEC model for the Slovak economy including the real growth of GDP, core inflation, the EUR/SKK exchange rate in natural logarithms, the balance of trade as a share of GDP and the BRIBOR interbank rate with one-month maturity in order to study the possible effects of an independent monetary policy during the period of a marked decrease of economic activity in Slovakia. For the estimation, we used a modified version of a Bayesian estimation technique developed for models using data observed with different frequencies.

Based on impulse response functions and on forecasts, a robust result consists in the weak reaction of inflation to monetary policy decisions. A possible interpretation is that until 2004 the overall amount of loans granted to the real economy by banks was relatively small and the banking sector underwent significant changes and thus interest rate changes on client loans and deposits were not able to affect inflation. We suggest that after 2004 interest rate movements affected other parts of the economy that were not transmitted to inflation (residential real estate prices, for example). Therefore, we conclude that the qualitative monetary policy in 2000–2008 was conducted in a period of overall positive macroeconomic development that allowed the NBS to gradually harmonize its tools with those of the ECB and thus to prepare for Slovakia's entry into the European Union and the Euro area.

The main result of the counterfactual simulations is that the monetary policy would not have been able to mitigate the impact of the global economic recession that translated into a significant decline of domestic economic activity had it been independent. This result is robust across the different specifications and simulations used. Our explanation for this is that the impact of the financial crisis and the global economic recession was so strong and quick that the monetary policy would not have been able to counteract this impact in such a short period. It is more the recovery phase that would have been affected by an independent monetary policy, where a depreciating exchange rate and gradually decreasing interest rates could lead to a more pronounced increase of GDP.

APPENDIX 1

Tabel A1 Unit Root Tests of Slovak Macroeconomic Variables

	ADF		Phillips-Perron	
	Level	**1st difference**	**Level**	**1st difference**
BRIBOR 1M	0.562	0.000	0.001	0.000
CPI	0.123	0.000	0.115	0.000
CPI core	0.115	0.000	0.102	0.000
EUR/SKK y-o-y	0.037	0.000	0.131	0.000
log(EUR/SKK)	1.000	0.000	1.000	0.000
Balance of current and capital account	0.021	0.015	0.205	0.003
Balance of trade*	0.006	0.007	0.067	0.010
GDP growth*	0.696	0.000	0.151	0.134
GDP gap*	0.642	0.000	0.561	0.132
Balance of trade**	0.314	0.018	0.003	0.000
GDP growth**	0.081	0.000	0.081	0.000
GDP gap**	0.049	0.000	0.032	0.000

Notes: Respective *p*-values listed in table.
In all cases an intercept was included into the test equation.
* monthly data obtained by cubic interpolation
** quarterly data

APPENDIX 2

Estimation Methodology

Step 1

We can rewrite (3.1) in the following form (for simplicity, let's assume $p = 1$):

$$\begin{pmatrix}\Delta\mathbf{y}_{\mathbf{o,t}}\\ \Delta\mathbf{y}_{\mathbf{u,t}}\end{pmatrix}=\begin{pmatrix}\mathbf{A_o}\\ \mathbf{A_u}\end{pmatrix}+\begin{pmatrix}\mathbf{\Pi_{oo}} & \mathbf{\Pi_{ou}}\\ \mathbf{\Pi_{uo}} & \mathbf{\Pi_{uu}}\end{pmatrix}\begin{pmatrix}\mathbf{y}_{\mathbf{o,t-1}}\\ \mathbf{y}_{\mathbf{u,t-1}}\end{pmatrix}+\begin{pmatrix}\mathbf{B_{oo}} & \mathbf{B_{ou}}\\ \mathbf{B_{uo}} & \mathbf{B_{uu}}\end{pmatrix}\begin{pmatrix}\Delta\mathbf{y}_{\mathbf{o,t-1}}\\ \Delta\mathbf{y}_{\mathbf{u,t-1}}\end{pmatrix}+\begin{pmatrix}\mathbf{C_o}\\ \mathbf{C_u}\end{pmatrix}\mathbf{exog_{t-1}}+\begin{pmatrix}\boldsymbol{\varepsilon}_{\mathbf{o,t}}\\ \boldsymbol{\varepsilon}_{\mathbf{u,t}}\end{pmatrix}$$

$$\begin{pmatrix}\boldsymbol{\varepsilon}_{\mathbf{o,t}}\\ \boldsymbol{\varepsilon}_{\mathbf{u,t}}\end{pmatrix}\sim N\left(\mathbf{0},\begin{pmatrix}\mathbf{\Sigma_{oo}} & \mathbf{\Sigma_{ou}}\\ \mathbf{\Sigma_{uo}} & \mathbf{\Sigma_{uu}}\end{pmatrix}\right),\ t=1,\ldots,T$$

Please recall that $\mathbf{\Pi}=\boldsymbol{\alpha\beta}^{\mathbf{T}}$. The conditional distribution of the unobserved data,

$$p\left(\Delta\mathbf{y}_{\mathbf{u,t}} \mid \mathbf{y}_{\mathbf{0}}, \hat{\mathbf{y}}_{\mathbf{u},\backslash\mathbf{t}}, \boldsymbol{\Theta}\right)$$

is proportional to the product of densities of the changes in endogenous variables between *t* + 1 and *t* given data at *t*, $p\left(\Delta\mathbf{y}_{\mathbf{o,t+1}}, \Delta\mathbf{y}_{\mathbf{u,t+1}} \mid \Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}}, \mathbf{y}_{\mathbf{o,t}}, \mathbf{y}_{\mathbf{u,t}}, \boldsymbol{\Theta}\right)$ and the changes in endogenous variables between *t* and *t*-1 $p\left(\Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}} \mid \Delta y_{\mathbf{o,t-1}}, \Delta\mathbf{y}_{\mathbf{u,t-1}}, \mathbf{y}_{\mathbf{o,t-1}}, \mathbf{y}_{\mathbf{u,t-1}}, \boldsymbol{\Theta}\right)$:

$$p\left(\Delta\mathbf{y}_{\mathbf{u,t}} \mid \mathbf{y}_{\mathbf{o}}, \hat{\mathbf{y}}_{\mathbf{u},\backslash\mathbf{t}}, \boldsymbol{\Theta}\right) \propto$$
$$p\left(\Delta\mathbf{y}_{\mathbf{o,t+1}}, \Delta\mathbf{y}_{\mathbf{u,t+1}} \mid \Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}}, \mathbf{y}_{\mathbf{o,t}}, \mathbf{y}_{\mathbf{u,t}}, \boldsymbol{\Theta}\right) p\left(\Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}} \mid \Delta\mathbf{y}_{\mathbf{o,t-1}}, \Delta\mathbf{y}_{\mathbf{u,t-1}}, \mathbf{y}_{\mathbf{o,t-1}}, \mathbf{y}_{\mathbf{u,t-1}}, \boldsymbol{\Theta}\right)$$

The densities are conditionally normal:

$$p\left(\Delta\mathbf{y}_{\mathbf{o,t+1}}, \Delta\mathbf{y}_{\mathbf{u,t+1}} \mid \Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}}, \mathbf{y}_{\mathbf{o,t}}, \mathbf{y}_{\mathbf{u,t}}, \boldsymbol{\Theta}\right) \propto \exp\left\{-\frac{1}{2}\begin{bmatrix}\mathbf{v}_{\mathbf{1,t+1}} \\ \mathbf{v}_{\mathbf{2,t+1}}\end{bmatrix}^T \begin{bmatrix}\boldsymbol{\Sigma}^{\mathbf{oo}} & \boldsymbol{\Sigma}^{\mathbf{ou}} \\ \boldsymbol{\Sigma}^{\mathbf{uo}} & \boldsymbol{\Sigma}^{\mathbf{uu}}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{\mathbf{1,t+1}} \\ \mathbf{v}_{\mathbf{2,t+1}}\end{bmatrix}\right\}$$

where

$$\mathbf{v}_{\mathbf{1,t+1}} = \Delta\mathbf{y}_{\mathbf{o,t+1}} - \mathbf{A}_{\mathbf{o}} - \boldsymbol{\Pi}_{\mathbf{oo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{ou}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{oo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{B}_{\mathbf{ou}}\Delta\mathbf{y}_{\mathbf{u,t}} - \mathbf{C}_{\mathbf{o}}\mathbf{exog}_{\mathbf{t}}$$
$$\mathbf{v}_{\mathbf{2,t+1}} = \Delta\mathbf{y}_{\mathbf{u,t+1}} - \mathbf{A}_{\mathbf{u}} - \boldsymbol{\Pi}_{\mathbf{uo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{uu}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{uo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{B}_{\mathbf{uu}}\Delta\mathbf{y}_{\mathbf{u,t}} - \mathbf{C}_{\mathbf{u}}\mathbf{exog}_{\mathbf{t}}$$

$$p\left(\Delta\mathbf{y}_{\mathbf{o,t}}, \Delta\mathbf{y}_{\mathbf{u,t}} \mid \Delta\mathbf{y}_{\mathbf{o,t-1}}, \Delta\mathbf{y}_{\mathbf{u,t-1}}, \mathbf{y}_{\mathbf{o,t-1}}, \mathbf{y}_{\mathbf{u,t-1}}, \boldsymbol{\Theta}\right) \propto \exp\left\{-\frac{1}{2}\begin{bmatrix}\mathbf{v}_{\mathbf{1,t}} \\ \mathbf{v}_{\mathbf{2,t}}\end{bmatrix}^T \begin{bmatrix}\boldsymbol{\Sigma}^{\mathbf{oo}} & \boldsymbol{\Sigma}^{\mathbf{ou}} \\ \boldsymbol{\Sigma}^{\mathbf{uo}} & \boldsymbol{\Sigma}^{\mathbf{uu}}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{\mathbf{1,t}} \\ \mathbf{v}_{\mathbf{2,t}}\end{bmatrix}\right\}$$

where

$$\mathbf{v}_{\mathbf{1,t}} = \Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{A}_{\mathbf{o}} - \boldsymbol{\Pi}_{\mathbf{oo}}\mathbf{y}_{\mathbf{o,t-1}} - \boldsymbol{\Pi}_{\mathbf{ou}}\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{B}_{\mathbf{oo}}\Delta\mathbf{y}_{\mathbf{o,t-1}} - \mathbf{B}_{\mathbf{ou}}\Delta\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{C}_{\mathbf{o}}\mathbf{exog}_{\mathbf{t-1}}$$
$$\mathbf{v}_{\mathbf{2,t}} = \Delta\mathbf{y}_{\mathbf{u,t}} - \mathbf{A}_{\mathbf{u}} - \boldsymbol{\Pi}_{\mathbf{uo}}\mathbf{y}_{\mathbf{o,t-1}} - \boldsymbol{\Pi}_{\mathbf{uu}}\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{B}_{\mathbf{uo}}\Delta\mathbf{y}_{\mathbf{o,t-1}} - \mathbf{B}_{\mathbf{uu}}\Delta\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{C}_{\mathbf{u}}\mathbf{exog}_{\mathbf{t-1}}$$
$$\begin{pmatrix}\boldsymbol{\Sigma}_{\mathbf{oo}} & \boldsymbol{\Sigma}_{\mathbf{ou}} \\ \boldsymbol{\Sigma}_{\mathbf{uo}} & \boldsymbol{\Sigma}_{\mathbf{uu}}\end{pmatrix}^{-1} \equiv \begin{pmatrix}\boldsymbol{\Sigma}^{\mathbf{oo}} & \boldsymbol{\Sigma}^{\mathbf{ou}} \\ \boldsymbol{\Sigma}^{\mathbf{uo}} & \boldsymbol{\Sigma}^{\mathbf{uu}}\end{pmatrix}$$

Multiplying the two expressions gives

$$p\left(\Delta\mathbf{y}_{\mathbf{u,t}} \mid y_{\mathbf{0}}, \hat{\mathbf{y}}_{\mathbf{u},\backslash\mathbf{t}}, \boldsymbol{\Theta}\right) \propto \exp\left\{-\frac{1}{2}\left(\Delta\mathbf{y}_{\mathbf{u,t}} - \mathbf{V}_1^{-1}\mathbf{V}_2\right)^{-1} \mathbf{V}_1\left(\Delta\mathbf{y}_{\mathbf{u,t}} - \mathbf{V}_1^{-1}\mathbf{V}_2\right)\right\}$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are defined as follows:

$$\mathbf{V}_1 = \boldsymbol{\Sigma}^{\mathbf{uu}} + \mathbf{B}_{\mathbf{ou}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{oo}}\mathbf{B}_{\mathbf{ou}} + \mathbf{B}_{\mathbf{ou}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{ouo}}\mathbf{B}_{\mathbf{uu}} + \mathbf{B}_{\mathbf{uu}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{uo}}\mathbf{B}_{\mathbf{ou}} + \mathbf{B}_{\mathbf{uu}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{uu}}\mathbf{B}_{\mathbf{uu}},$$
$$\begin{aligned}\mathbf{V}_2 = {} & -\boldsymbol{\Sigma}^{\mathbf{uo}}\left(\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{A}_{\mathbf{o}} - \boldsymbol{\Pi}_{\mathbf{oo}}\mathbf{y}_{\mathbf{o,t-1}} - \boldsymbol{\Pi}_{\mathbf{ou}}\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{B}_{\mathbf{oo}}\Delta\mathbf{y}_{\mathbf{o,t-1}} - \mathbf{B}_{\mathbf{ou}}\Delta\mathbf{y}_{\mathbf{u,t-1}} - \mathbf{C}_{\mathbf{o}}\mathbf{exog}_{\mathbf{t-1}}\right) + \\ & + \mathbf{B}_{\mathbf{ou}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{oo}}\left(\Delta\mathbf{y}_{\mathbf{o,t+1}} - \mathbf{A}_{\mathbf{o}} - \boldsymbol{\Pi}_{\mathbf{oo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{ou}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{oo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{C}_{\mathbf{o}}\mathbf{exog}_{\mathbf{t}}\right) + \\ & + \mathbf{B}_{\mathbf{ou}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{ou}}\left(\Delta\mathbf{y}_{\mathbf{u,t+1}} - \mathbf{A}_{\mathbf{u}} - \boldsymbol{\Pi}_{\mathbf{uo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{uu}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{uo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{C}_{\mathbf{u}}\mathbf{exog}_{\mathbf{t}}\right) + \\ & + \mathbf{B}_{\mathbf{uu}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{uo}}\left(\Delta\mathbf{y}_{\mathbf{o,t+1}} - \mathbf{A}_{\mathbf{o}} - \boldsymbol{\Pi}_{\mathbf{oo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{ou}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{oo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{C}_{\mathbf{o}}\mathbf{exog}_{\mathbf{t}}\right) + \\ & + \mathbf{B}_{\mathbf{uu}}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{uu}}\left(\Delta\mathbf{y}_{\mathbf{u,t+1}} - \mathbf{A}_{\mathbf{u}} - \boldsymbol{\Pi}_{\mathbf{uo}}\mathbf{y}_{\mathbf{o,t}} - \boldsymbol{\Pi}_{\mathbf{uu}}\mathbf{y}_{\mathbf{u,t}} - \mathbf{B}_{\mathbf{uo}}\Delta\mathbf{y}_{\mathbf{o,t}} - \mathbf{C}_{\mathbf{u}}\mathbf{exog}_{\mathbf{t}}\right)\end{aligned}$$

This means that in each iteration the unobserved data are sampled from a multivariate normal distribution. Sampling from a normal distribution is straightforward, as algorithms are already available. In the case of monthly and quarterly data, for the series with quarterly frequency we have observations in iteration i for $\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}, t = 3l, l \in N$.
For $t \neq 3l$, we draw $\Delta\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}$ and calculate $\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}$ as $\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}} = \hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t-1}} + \Delta\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}$.
For $t = 3l$ we calculate $\Delta\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}}$ simply as $\Delta\hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}} = \hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t}} - \hat{\mathbf{y}}^{\mathbf{i}}_{\mathbf{u,t-1}}$.

Step 2
For convenience, we can rewrite (3.2) in a condensed matrix format:

$$\mathbf{Y} = \mathbf{X\Gamma} + \mathbf{Z\beta\alpha}^{\mathbf{T}} + \mathbf{E} \tag{A2.1}$$

where **Y**, **X**, **Z** and **E** are $T \times n, T \times k, T \times n, T \times n$ matrices, the *t*-th rows of which are given by (assuming p = 1): .., $\left(1, \Delta\mathbf{y}^{\mathbf{T}}_{\mathbf{t-1}}, \mathbf{exog}^{\mathbf{T}}_{\mathbf{t-t}}\right)$, $\mathbf{y}^{\mathbf{T}}_{\mathbf{t-1}}$ and $\boldsymbol{\varepsilon}^{\mathbf{T}}_{\mathbf{t}}$. In that case that an intercept is included in the cointegrating equation, the *t*-th row of **Z** is given by $\left(\mathbf{y}^{\mathbf{T}}_{\mathbf{t-1}}, 1\right)$. Γ, β and α are the respective parameters to be estimated. Let's suppose there is only one cointegrating equation, so that β is a vector ($n \times 1$ without and $n+1 \times 1$ with intercept). We can normalize the vector for the first variable, so that $\beta_1 = 1$. In that case, the number of restrictions on the values of the cointegrating vector will be r = 1. Let's assume the prior density of the cointegrating vector has the form: $p(\boldsymbol{\beta}) \propto 1$.

In this case, as is shown in Bauwens and Lubrano (1994), the marginal posterior conditional distribution of the cointegrating vector has the form:

$$p(\boldsymbol{\beta} \mid \mathbf{Y}, \mathbf{X}, \mathbf{Z}) \propto \left|\boldsymbol{\beta}^{\mathbf{T}}\mathbf{W_0}\boldsymbol{\beta}\right|^{\mathbf{l_0}} / \left|\boldsymbol{\beta}^{\mathbf{T}}\mathbf{W_1}\boldsymbol{\beta}\right|^{\mathbf{l_1}}$$

where

$$\mathbf{M_X} = \mathbf{I_T} - \mathbf{X}\left(\mathbf{X}^{\mathbf{T}}\mathbf{X}\right)^{\mathbf{-1}}\mathbf{X}^{\mathbf{T}}$$

$$\mathbf{W_0} = \mathbf{Z}^{\mathbf{T}}\mathbf{M_X}\mathbf{Z}$$

$$\mathbf{W_1} = \mathbf{Z}^{\mathbf{T}}\mathbf{M_X}\left[\mathbf{I_T} \text{ - } \mathbf{Y}\left(\mathbf{Y}^{\mathbf{T}}\mathbf{M_X}\mathbf{Y}\right)^{\mathbf{-1}}\mathbf{Y}^{\mathbf{T}}\right]\mathbf{M_X}\mathbf{Z}$$

$$l_0 = (T - k - r - n)/2$$

$$l_1 = (T - k - r)/2$$

Given the assumption this is a kernel of a 1-1 *poly-t* density that is "integrable and has finite moments of order equal to the order of overidentification of β" (Bauwens and Lubrano, 1994, p. 13).

In the case that the cointegrating vector contains an intercept, it is recommended that only the first n elements of β are estimated (except the first element, which is restricted to 1). The last element corresponding to the intercept term can be calculated, e.g. as the mean value of the residuals from the cointegrating equation (thus adjusting the mean of the residuals to zero).

Although the posterior conditional distribution of the cointegrating vector is known and is integrable, drawing the elements of the cointegrating vector is not straightforward. Let's denote the elements of $\boldsymbol{\beta}$ except for the j-th one as $\boldsymbol{\beta}_{\backslash\mathbf{j}}$. A possible way to draw the elements of the cointegrating vector is to estimate (calculate numerically) for each element of the vector its marginal posterior conditional distribution. In each iteration, we use a gridy Gibbs sampler to obtain

$$p\left(\boldsymbol{\beta}_{\mathbf{j}}^{\mathbf{i}} \mid \boldsymbol{\beta}_{\backslash\mathbf{j}}^{\mathbf{i\text{-}1}}, \hat{\mathbf{Y}}^{\mathbf{i}}, \mathbf{A}^{\mathbf{i\text{-}1}}, \mathbf{B}_{\mathbf{j}}^{\mathbf{i\text{-}1}}, \mathbf{C}^{\mathbf{i\text{-}1}}, \boldsymbol{\alpha}^{\mathbf{i\text{-}1}}, \boldsymbol{\Sigma}^{\mathbf{i\text{-}1}}\right), j = 2, 3, ..., n,$$

where $\boldsymbol{\beta}_{\backslash\mathbf{j}}^{\mathbf{i\text{-}1}}$ is the vector of the most recently updated coefficients, $\boldsymbol{\beta}_{\backslash\mathbf{j}}^{\mathbf{i\text{-}1}} = \left(\beta_1^i, \beta_2^i, ..., \beta_{j-1}^i, \beta_{j+1}^{i-1}, ..., \beta_n^{i-1}\right)$. Having calculated the conditional posterior density it is possible to sample from this distribution using the cumulative distribution function and sampling from a uniform distribution on the unit interval.

Step 3

Let's take the equation for the t-th row of $\boldsymbol{Y}$ in (A2.1):

$$\mathbf{Y_t} = \mathbf{X_t}\boldsymbol{\Gamma} + \mathbf{Z_t}\boldsymbol{\beta}\boldsymbol{\alpha}^{\mathbf{T}} + \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} \tag{A2.2}$$

$$\tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} \sim N\left(\mathbf{0}, \tilde{\boldsymbol{\Sigma}}\right)$$

where

$$\mathbf{Y_t} = \Delta\mathbf{y_t^T}, \ \mathbf{X_t} = \left(1, \Delta\mathbf{y_{t\text{-}1}^T}, \mathbf{exog_{t\text{-}t}^T}\right), \ \mathbf{Z_t} = \mathbf{y_{t\text{-}1}^T}, \ \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} = \boldsymbol{\varepsilon}_{\mathbf{t}}^{\mathbf{T}}$$

When estimating coefficient matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\alpha}$ and the variance-covariance matrix $\tilde{\boldsymbol{\Sigma}}$, it is important to notice that (A2.2) becomes a linear model conditional on the estimated values of $\boldsymbol{\beta}$. Thus, we can rewrite (A2.2) as

$$\mathbf{Y_t} = \mathbf{X_t}\boldsymbol{\Gamma} + \tilde{\mathbf{Z}}_{\mathbf{t}}\boldsymbol{\alpha}^{\mathbf{T}} + \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} = \hat{\mathbf{X}}_{\mathbf{t}}\hat{\mathbf{B}} + \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}}$$

where

$$\tilde{\mathbf{Z}}_{\mathbf{t}} = \mathbf{Z_t}\boldsymbol{\beta}, \ \hat{\mathbf{X}}_{\mathbf{t}} = \left(\mathbf{X_t}, \tilde{\mathbf{Z}}_{\mathbf{t}}\right) \text{ and } \hat{\mathbf{B}} = \left(\boldsymbol{\Gamma}^{\mathbf{T}}, \alpha\right)^{\mathbf{T}} \tag{A2.3}$$

Finally, based on (A2.3) we can rewrite (A2.1) in a compact form as

$$\mathbf{Y_t} = \tilde{\mathbf{X}}_{\mathbf{t}}\tilde{\mathbf{B}} + \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}}, \ t = 1, 2, ..., T \tag{A2.4}$$

where

$$\tilde{\mathbf{X}}_{\mathbf{t}} = \mathbf{I_n} \otimes \hat{\mathbf{X}}_{\mathbf{t}} \text{ and } \tilde{\mathbf{B}} = \mathbf{vec}\left(\hat{\mathbf{B}}\right)$$

$\tilde{\mathbf{X}}_{\mathbf{t}}$ is a $n \times n(1 + n + r + h)$ matrix, where r is the rank of $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$, h is the number of exogenous variables included in the model and $\tilde{\mathbf{B}}$ is a $n(1 + n + r + h) \times 1$ vector of parameters. Let's assume an independent Normal-inverse Wishart prior for (A2.4):

$$p\left(\tilde{\mathbf{B}}\right) \sim N\left(\boldsymbol{\mu}_{\boldsymbol{\Omega}}, \boldsymbol{\Sigma}_{\boldsymbol{\Omega}}\right)$$

$$p\left(\tilde{\boldsymbol{\Sigma}}\right) \sim iW\left(\boldsymbol{\Psi}, m\right)$$

where $\mathbf{\Psi}$ is the mean and m is the degrees of freedom for the variance-covariance matrix. The conditional posterior distribution for $\tilde{\mathbf{B}}$ then has the form

$$p\left(\tilde{\mathbf{B}} \mid \mathbf{Y}, \tilde{\mathbf{X}}, \tilde{\mathbf{\Sigma}}\right) \sim N\left(\left(\mathbf{\Sigma}_{\mathbf{\Omega}}^{-1} + \sum_{t=1}^{T} \tilde{\mathbf{X}}_{\mathbf{t}}^{\mathbf{T}} \tilde{\mathbf{\Sigma}}^{-1} \tilde{\mathbf{X}}_{\mathbf{t}}\right)^{-1} \left(\mathbf{\Sigma}_{\mathbf{\Omega}}^{-1} \boldsymbol{\mu}_{\mathbf{\Omega}} + \sum_{t=1}^{T} \tilde{\mathbf{X}}_{\mathbf{t}}^{\mathbf{T}} \tilde{\mathbf{\Sigma}}^{-1} \mathbf{Y}_{\mathbf{t}}\right), \left(\mathbf{\Sigma}_{\mathbf{\Omega}}^{-1} + \sum_{t=1}^{T} \tilde{\mathbf{X}}_{\mathbf{t}}^{\mathbf{T}} \tilde{\mathbf{\Sigma}}^{-1} \tilde{\mathbf{X}}_{\mathbf{t}}\right)^{-1}\right)$$

(see, for example, Eraker *et al.*, 2011). As the conditional posterior of $\tilde{\mathbf{B}}$ has normal distribution, drawing from this distribution in *i*-th iteration is straightforward.

Step 4

Based on the prior described in Step 3, the posterior conditional distribution of the variance-covariance matrix has the form:

$$p\left(\tilde{\mathbf{\Sigma}} \mid \mathbf{Y}, \tilde{\mathbf{X}}, \tilde{\mathbf{B}}\right) \sim iW\left(\mathbf{\Psi} + \sum_{t=1}^{T} \left(\mathbf{Y} - \tilde{\mathbf{X}}\tilde{\mathbf{B}}\right)\left(\mathbf{Y} - \tilde{\mathbf{X}}\tilde{\mathbf{B}}\right)^{T}, T + m\right) \tag{A2.5}$$

(see, for example, Eraker *et al.*, 2011).

The posterior conditional distribution is also inverse Wishart, and as algorithms for drawing from this distribution are already available, drawing from (A2.5) is also straightforward.

APPENDIX 3

Table A3 Johansen Cointegration Tests

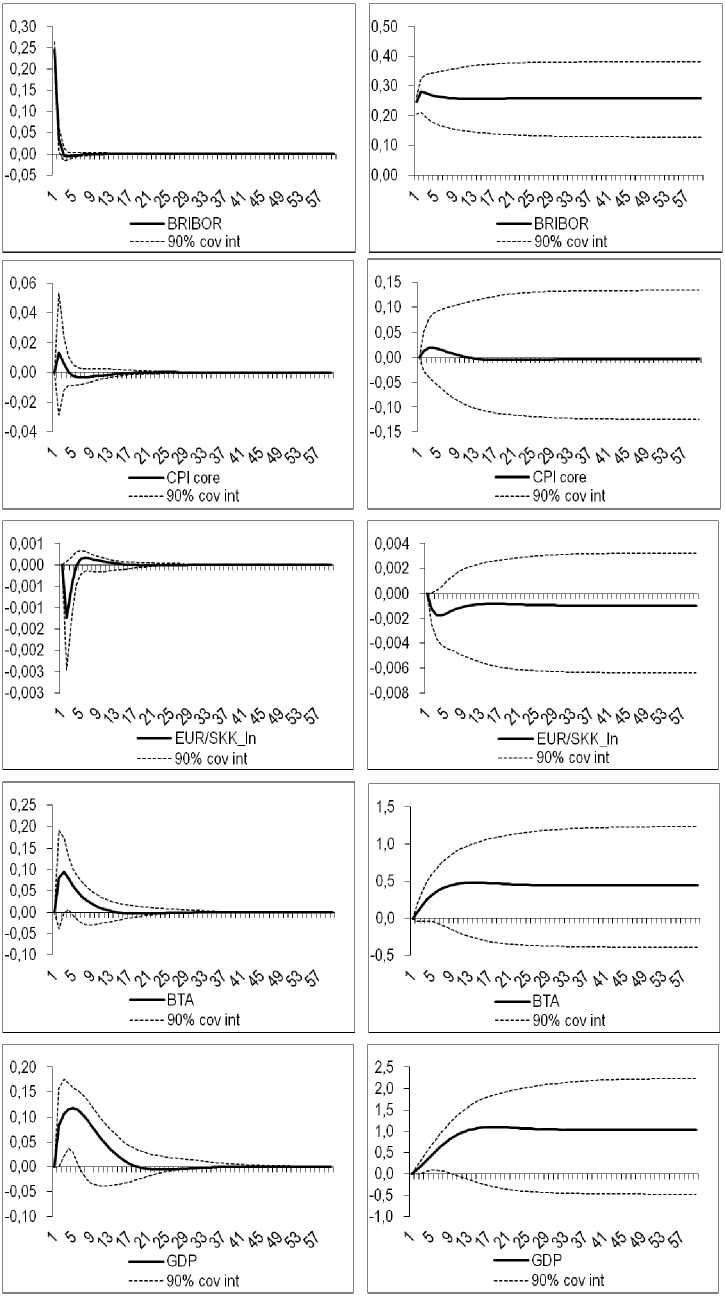
	Trace test					Max eigenvalue test				
	None*	**At most 1***	**At most 2**	**At most 3**	**At most 4**	**None***	**At most 1***	**At most 2**	**At most 3**	**At most 4**
Baseline	0.002	0.048	0.383	0.748	0.589	0.016	0.053	0.285	0.709	0.589
Specification 1	0.000	0.024	0.121	0.413	0.379	0.009	0.106	0.140	0.407	0.379
Specification 2	0.001	0.038	0.339	0.855	0.734	0.010	0.047	0.184	0.811	0.734
Specification 3	0.000	0.066	0.394	0.557	0.417	0.001	0.076	0.430	0.548	0.417
Specification 4	0.000	0.035	0.578	0.739	0.560	0.001	0.014	0.527	0.705	0.560
Specification 5	0.004	0.056	0.519	0.693	0.275	0.040	0.036	0.488	0.763	0.275
Specification 6	0.036	0.782	0.840	0.616		0.006	0.718	0.810	0.616	
Specification 7	0.081	0.918	0.830	0.378		0.009	0.928	0.857	0.378	
Specification 8	0.001	0.103	0.319			0.003	0.098	0.319		
Specification 9	0.003	0.784	0.437			0.001	0.788	0.437		
Specification 10	0.084	0.921	0.736	0.152		0.009	0.967	0.898	0.152	
Specification 11	0.026	0.515	0.784	0.205		0.012	0.410	0.897	0.205	
Specification 12	0.001	0.034	0.326	0.676	0.165	0.021	0.043	0.264	0.834	0.165
Specification 13	0.000	0.006	0.193	0.785	0.216	0.015	0.011	0.098	0.891	0.216
Specification 14	0.000	0.033	0.250	0.464	0.582	0.000	0.062	0.295	0.409	0.582
Specification 15	0.000	0.004	0.021	0.361	0.448	0.008	0.090	0.020	0.333	0.448
Specification 16	0.017	0.620	0.770	0.798		0.004	0.557	0.705	0.798	
Specification 17	0.008	0.269	0.466	0.475		0.009	0.321	0.433	0.475	

Notes: Respective p-values listed in table.
An intercept was included in the cointegrating equation and VAR in all cases.
The lag interval is 1 to 4 in all cases.

APPENDIX 4

Figure A4 Impulse Response Functions

Changes Cumulative responses



APPENDIX 5

Table A5 Estimated Coefficients of Cointegrating Vectors and Adjustment Coefficients

Cointegrating coefficients	*BRIBOR1M*	*CPI*	*CPI_core*	*EUR/SKK_ch*	*EUR/SKK_ln*	*BTA*	*BCCA*	*GDP*	*GDP_gap*
Baseline	1.0000		1.0428		17.5903	0.0822		0.2326	
Specification 1	1.0000	-21.3659		0.1393		3.5734		-19.0678	
Specification 2	1.0000	-19.2609			1.4290	3.6960		-19.2726	
Specification 3	1.0000		0.1692	0.1739		-0.0530		-0.2997	
Specification 4	1.0000		-0.2198	-0.1195			-0.6697	-0.4364	
Specification 5	1.0000		1.0040		18.5601		0.4069	0.1566	
Specification 6	1.0000		0.6551	0.5058				0.0460	
Specification 7	1.0000		1.3273		17.2139			0.4617	
Specification 8	1.0000	0.8791							1.1412
Specification 9	1.0000		0.2934						0.8306
Specification 10	1.0000		0.7926		7.9981				0.8077
Specification 11	1.0000	0.5814			2.4400				0.8238
Specification 12	1.0000		0.7868		12.7371	0.1150			0.4161
Specification 13	1.0000	0.6907			-0.8432	-0.1352			1.0494
Specification 14	1.0000		0.7672	0.5538		0.3572			0.3234
Specification 15	1.0000	0.5937		0.1932		0.1315			0.9046
Specification 16	1.0000		0.5780	0.3607					0.6841
Specification 17	1.0000	0.7170		-0.0275					0.8774

Adjustment coefficients	*BRIBOR1M*	*CPI*	*CPI_core*	*EUR/SKK_ch*	*EUR/SKK_ln*	*BTA*	*BCCA*	*GDP*	*GDP_gap*
Baseline	-0.0149		0.0026		0.0005	0.1162		0.1352	
Specification 1	-0.0011	0.0011		-0.0055		0.0019		-0.0721	
Specification 2	-0.0002	0.0012			-0.0001	-0.0074		-0.0547	
Specification 3	-0.0008		-0.0018	0.0051		0.0088		0.0132	
Specification 4	0.0025		-0.0011	0.0109			0.1910	0.0255	
Specification 5	0.0021		0.0186		-0.0003		0.3892	0.1320	
Specification 6	0.0009		0.0019	0.0038				-0.0371	
Specification 7	-0.0080		-0.0226		-0.0013			0.1184	
Specification 8	0.0000	-0.0015							0.1068
Specification 9	-0.0033		-0.0058						0.0629
Specification 10	-0.0052		-0.0032		0.0001				0.0701
Specification 11	-0.0003	-0.0005			0.0001				0.0718
Specification 12	-0.0061		-0.0031		0.0002	0.1612			0.1528
Specification 13	0.0203	0.0256			-0.0001	0.0140			0.0298
Specification 14	0.0372		0.0078	-0.1328		0.1973			-0.0026
Specification 15	0.0020	0.0152		0.0007		-0.0098			0.0701
Specification 16	-0.0018		-0.0033	-0.0079					0.0469
Specification 17	0.0021	0.0004		-0.0303					0.0745

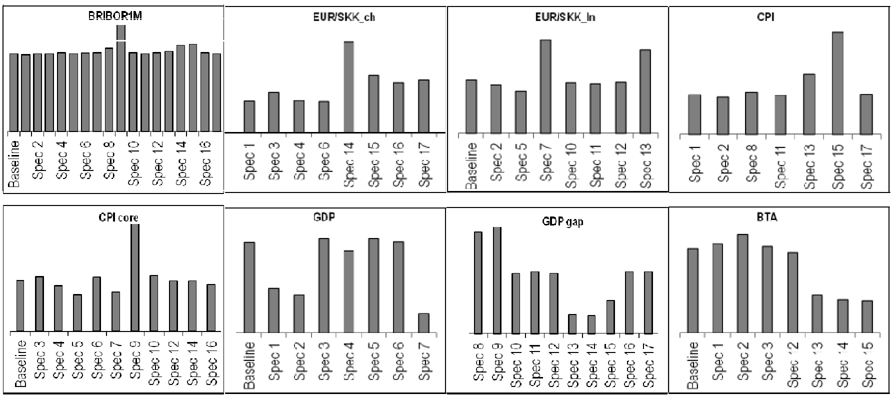
APPENDIX 6

Tabel A6 Values of the Bayesian Information Criterion

	BIC		BIC
Baseline	5.365	Specification 9	79.704
Specification 1	17.792	Specification 10	2.223
Specification 2	7.918	Specification 11	2.252
Specification 3	18.436	Specification 12	8.354
Specification 4	16.674	Specification 13	6.657
Specification 5	5.816	Specification 14	16.237
Specification 6	12.222	Specification 15	16.991
Specification 7	0.921	Specification 16	12.112
Specification 8	8.140	Specification 17	12.137

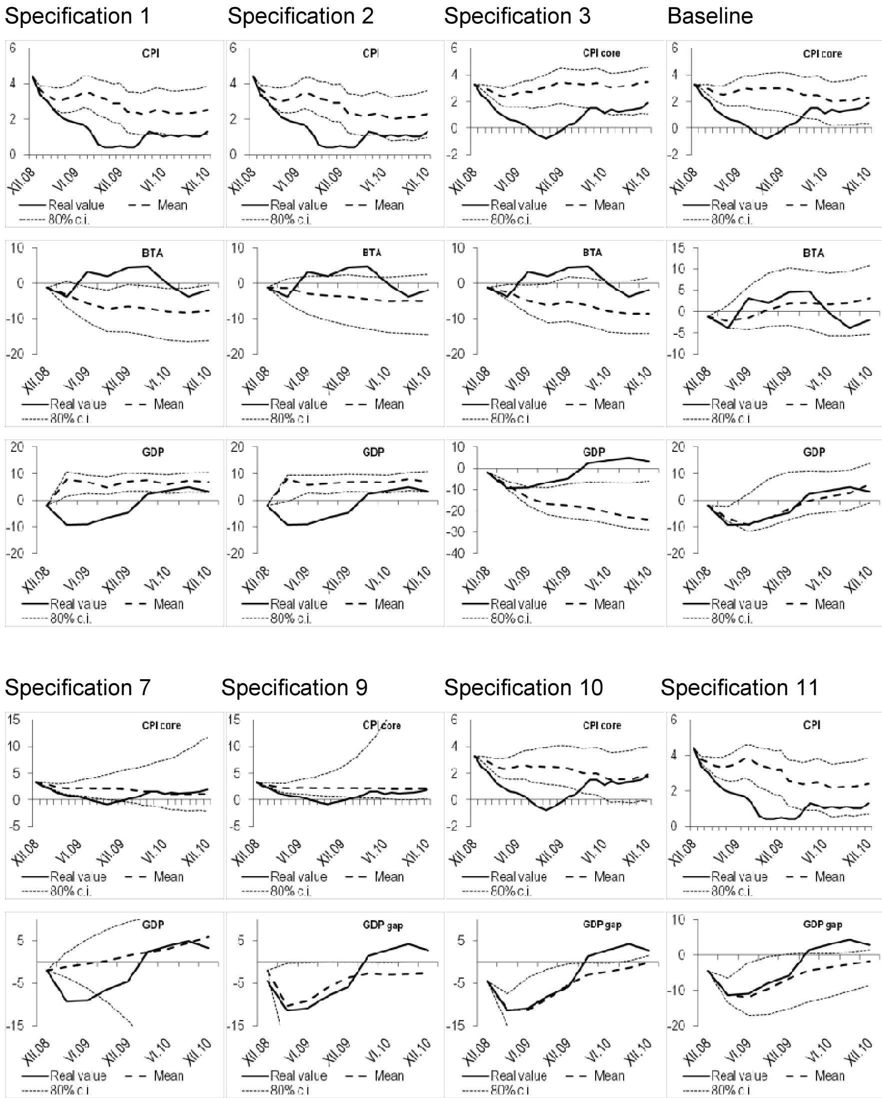
APPENDIX 7

Figure A7 Sum Of Squared Residuals

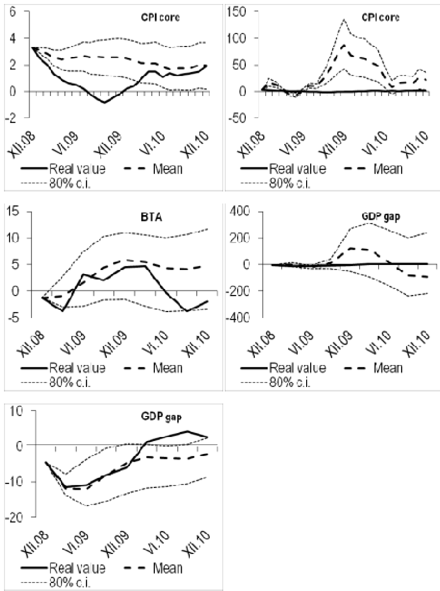


APPENDIX 8

Figure A8 Forecasts of the Endogenous Variables



Specification 12 Specification 16



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