# CVaR Portfolio Models for Electricity Generating Capacities

Dizertačná práca

Mgr. Jana Szolgayová

Univerzita Komenského v Bratislave
Fakulta matematiky, fyziky a informatiky
Katedra aplikovanej matematiky a štatistiky

9.1.9 Aplikovaná matematika

Školiteľ: prof. RNDr. Pavol Brunovský, DrSc.

Bratislava, 2010

**Abstrakt**

Táto práca sa zaoberá analýzou rôznych modelov voľby portfólia reálnych aktív, s využitím konceptu podmienenej hodnoty rizika. Motiváciou je aplikácia formulovaných modelov na problém optimálnej skladby investícií do nových kapacít v oblasti energetiky pod vplyvom neistej ceny emisií. Navrhnuté sú tri modely zohľadňujúce potrebné špecifiká uvažovaného problému. Dané modely sú analyzované a vzájomne porovnané pre reálne vstupné dáta. Práca sa takisto venuje všeobecnému porovnaniu klasickej Markowitzovej teórie portfólia s jedným z navrhnutých modelov v prípade normálne rozdelených výnosov reálnych aktív.

**Kľúčové slová:** podmienená hodnota rizika, teória portfólia, investície pod vplyvom neistoty

## Abstract

The focus of this thesis is on the application of conditional Value-at-Risk to the optimal portfolio selection problem. In particular, portfolios of real assets are analyzed, the motivation being the investment into new electricity generating capacities under climate policy uncertainty. Three different models are formulated, accounting for the specifics of the underlying problem. For real data, the results of the individual models are presented and compared. In addition, the difference between the standard Markowitz portfolio framework and portfolio optimization based on conditional Value-a-Risk is analyzed in case of normally distributed assets profit.



**Keywords:** conditional Value-at-Risk, portfolio theory, investment under uncertainty

# Preface

*"The most fun and perhaps the greatest value of doing something is in doing it. The results may well go up in smoke, be wrong, become obsolete and forgotten, but some new ideas may have emerged in pursuing them, and some of them may somewhere, sometime, bear fruit."*

V. Klemeš

Currently, the discussion about whether climate policy should be implemented has shifted towards a different focus, centering now around the type and extent of policy instruments that should be implemented to achieve the desired reduction in emissions. Still, there is no global agreement on the needed legislative, which makes the future climate policy one of the increasingly significant uncertainties connected with the investment into new power generating capacities. However, considerable investment is due in the OECD countries in the coming years, some of which will need to be carried out before this uncertainty is resolved. As the investment in new electricity generating capacities is long lived and is characterized by large up-front sunk costs, the decisions are mostly irreversible and their effect will persist in the following decades. The focus of this thesis in on the formulation of an optimization framework that can be applied to analyze the effects of the climate policy uncertainty on the optimal composition of investment into new electricity generating capacities.

The way towards this thesis has been a long one, requiring a lot of patience and perseverance, not only from my side but also from others. Therefore, I would like to express my gratitude towards them. First of all, I would like to thank my supervisor, Pavol Brunovský, who has probably suffered the most, for his support, encouragement and understanding. Not the least, he is to a large extent responsible for my collaboration with the International Institute of Applied Systems Analysis (IIASA).

The motivation for the work presented in this thesis stems from my congenial experience at IIASA that was possible mainly thanks to Zuzana Chladná, and Michael Obersteiner. I am indebted also to my colleagues, Sabine Fuss and Nikolay Khabarov. Not only for the fruitful collaboration, results of which are presented on the next pages, but also for their friendship and support when I needed it most.

Last but not least I would like to thank my family and friends, first and foremost my husband Michal Mikuš, for their continuing sympathy and understanding. Thank you all.

# Contents

# List of Figures

# List of Tables

# Introduction

The main motivation for the analysis presented in this thesis is the following problem: "Assuming that the operations and investments at the plant level are carried out optimally, what is the optimal energy mix in case of a uncertain climate change policy?"

Structurally, the thesis is divided into two parts. Whereas the first part provides a more detailed overview of the topic that is discussed, the second part presents our original contribution.

The first part comprises the first three chapters, each addressing a different aspect of the subject under consideration. In Chapter 1 the motivation for the problems studied in this thesis is given, explaining the basic structure and characteristics of the electricity sector. The aim and contribution of the thesis are stated in Chapter 2, starting with an overview of the current state-of-the-art, explaining the specifics that should be addressed by any new modeling framework. It further presents a general idea and outline of the approach suggested, providing also a brief overview of the structure of the thesis, together with a description of the data and assumptions. The first part concludes with a synopsis of the state-of-the-art with respect to the chosen methodology (Chapter 3).

Chapters 4 to 7 comprise our contribution to the discussed topic. Chapter 4 is devoted to the optimization of the investor behavior on the plant level, describing how these results can be used to derive input distributions used by the portfolio models, which are formulated in Chapters 5 to 7. The application of these models is presented with respect to investment in new electricity generating capacities.

This thesis was motivated mainly by my experience at the International Institute of Applied Analysis (IIASA) in Laxenburg. All models presented in this thesis originated by collaboration with my colleagues at IIASA. They were presented at several conferences and were used in applications either published or submitted to various journals. In the following my contribution is explained more in detail.

The Real Options model presented in Chapter 4 has been developed and implemented by me in MATLAB. It has been applied and extended primarily with my colleague S. Fuss to analyze different aspects of uncertainty for investment in the energy sector, e.g. [29, 68, 24, 26].

The original motivation of analyzing the effect of uncertainty on the investment in the energy sector has been proposed by M. Obersteiner, who also suggested the direction of CVaR portfolio applications. The general concept common to all models, i.e. the combination of optimization on the plant level and on the aggregate level by two separate models, originated in the cooperation with two co-authors - N. Khabarov and S. Fuss.

The first result along these lines was the combination of the Real Options model with the basic CVaR portfolio model for discrete distributions formulated by Uryasev in [59], presented in Chapter 5. The implementation in GAMS and MATLAB was was done mainly by me with help of N. Khabarov. Two applications of this model have been already published [27, 22]. The numeric results presented in Chapter Section 5.3.3, however, are new and independent of these publications. They are my own contribution, being motivated by the need to provide a benchmark for the results derived with the extended models from Chapter 6.

The contribution of Chapter 5 lies not only in the numeric results, though. It also provides a coherent comparison of the CVaR portfolio model to the classic mean-variance framework for portfolios without short positions. First the analytical results, for normally distributed assets profit (Section 5.2). Second, the numerical results for the distributions stemming from the Real Options model, using the analytical solution for the mean-variance framework for three assets (Section 5.3.2, Appendix C). All these results and computations presented are my own work.

The idea of the robust portfolio model has originated in discussions with S. Fuss and N. Khabarov. The formulation of the robust portfolio model from Chapter 6 (Theorem 6.2.1) was performed by me, the model has been implemented mainly thanks to Nikolay Khabarov. An application of this approach has been already published [25]. An independent application has been submitted to Energy Policy [28].

The first application of the robust portfolio model presented in Section 6.3 is similar in concept to the submission to Energy Policy. However, the numeric results presented in this thesis are different, since the underlying data and price parameters were chosen so that the assumptions and specifications for applications presented in Sections 5.3.3, 6.3 and 6.4 are the same and the results are mutually comparable. The concept of time structure, i.e. the idea of the second application from Chapter 6 is originally mine as is its implementation. The results presented in Section 6.4 are submitted to Applied Energy [67].

The main result of Chapter 6 is not in the separate applications, rather it is the analysis of the differences between the results of these individual models. This comparison constitutes my own and original contribution.

The dynamic portfolio model from Chapter 7 has been formulated by me and implemented in cooperation with N. Khabarov. An illustrative application of the model has been published in [65]. The analysis presented in Chapter 7, however, is more thorough and is mainly my contribution, accepted in the European transactions on Electrical Power [66].

# Chapter 1

# The Electricity Sector

## 1.1 Electricity Sector and Climate Policy

### 1.1.1 Introduction

The latest IPCC report [35] has uttered concern and demanded stringent policies concerning the accumulation of Greenhouse Gas (GHG) emissions in the atmosphere. Whether we agree with the conclusions of IPCC or not, the policy makers have become alert, as can be observed on the example of the current EU 20–20 target. The target refers to a desired 20% cut in GHG emissions and a minimum renewables share of 20% to be achieved by the year 2020. The aim of such action is to limit global warming to 2°C, a critical threshold beyond which dangerous climatic consequences can be expected (see e.g.[53] and [5]).Also, individual European countries have taken measures to reduce their $CO_2$ emissions. These range from feed-in tariffs to obligations to produce a minimum amount of electricity from renewables.

The power generation is a significant contributor to the total $CO_2$ emissions (see Figure 1.1), having a share of over 40%. Therefore the success of the policies depends to a large extent on their effect on the electricity sector. Still, two thirds of the electricity generated nowadays comes from fossil fueled capacities. The existing power plant stock in OECD countries is ageing and will need substantial replacement over the next 10 – 20 years [37]. As the investment in new electricity generating capacities is long lived and is characterized by large up-front sunk costs, the decisions are mostly irreversible and their effect will persist in the following decades. Moreover, because of the liberalization of the electricity sector the investment into power generation is connected with an increased level of uncertainty. Therefore, analyzing the effect of a climate policy on the resulting energy mix is not only a crucial question, but also calls for an appropriate accounting of the uncertainties involved.

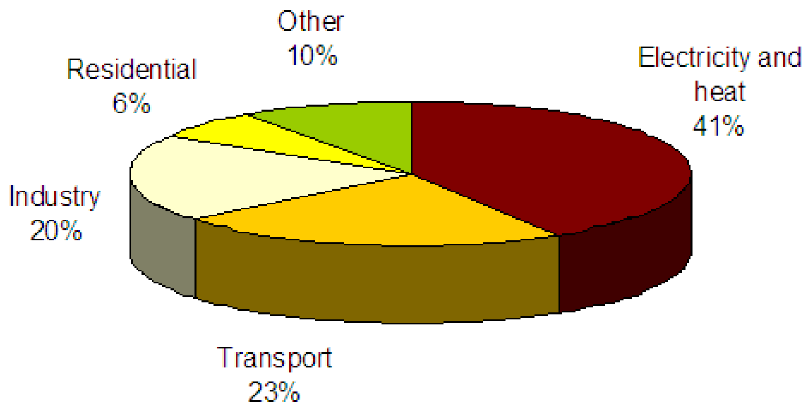


Figure 1.1: *World $CO_2$ emissions by sector, 2007*, Source: IEA, 2009

The purpose of this chapter is to provide a brief overview of the electricity sector, concentrating on the part that is relevant for the scope of this thesis. First we will provide a short description of the sector itself focusing on power generation, its characteristics and uncertainties connected with investment into new capacities. In the next sections we will introduce the climate policy aspect and present the characteristics of electricity generating technologies that are considered in this thesis.

### 1.1.2 Electricity Sector overview

A liberalized electricity sector is a complex and integrated system, usually including a large array of stakeholders that provide services through electricity generation, transmission, distribution and marketing for industrial, commercial, public and residential customers.

The electrical power industry is commonly split up into four processes (see Figure 1.2). These are electricity generation, electric power transmission, electricity distribution and electricity retailing. In many countries, electric power companies own the whole infrastructure from generating stations to transmission and distribution infrastructure. For this reason, electric power is viewed as a natural monopoly. Especially in the past, the power industry has been generally heavily regulated, often with price controls and is frequently government-owned and operated.

The electricity sector has experienced huge changes [34]. The market for electricity is being liberalized, greenhouse gases need to be reduced; therefore,

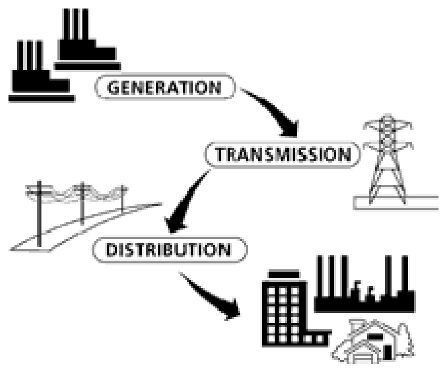


Figure 1.2: *Electrical power industry structure*, Source: Canadian Clean Power Coalition, 2004.

the increasing role of electricity makes even more necessary the need for precise and reliable data on production, generating capacity as well as on consumption of electricity in order to manage future development and ensure security of supply in the most efficient way.

Prior to the liberalization of energy markets, energy firms were able to operate as integrated monopolies. They were able to pass on all costs of investments to energy consumers. For example, in the electric power sector, utilities could expect the cost of their prudently incurred investments in power generation, including an adequate rate of return, to be recovered from consumers. Many firms were state-owned and could borrow money backed implicitly or explicitly by the government's guarantee. In view of that guaranteed rate of return, utilities could finance their investment with a low share of equity and borrow at interest rates close to government debt yields. There was no market risk. The main risk was the risk of unfavorable regulatory decisions and cost overruns due to bad project management.

In such an environment, most of the risks associated with such investments were not directly a concern of the energy company. Increased costs, if demonstrated to be prudently incurred, could be passed on as increased prices. In other words, it was not that risks did not exist in this situation, but merely that risks were transferred from investors to consumers or taxpayers. In this situation, there was little incentive for companies to take account of such risks when making investment decisions. The introduction of liberalization in energy markets is removing the regulatory risk shield. Investors now have additional risks to consider and manage. For example, generators are no longer guaranteed the ability to recover all costs from power consumers. Nor is the future power price level known. Investors now have to consider not only the profits, but also the risks that are associated with them . The natural question arising in these circumstances is how risks affect the choice of generating technologies.

As the focus of this thesis is on the investment in power generation capacities, we will concentrate on the characteristics of this part of the power industry. A detailed overview of other sectors is provided by [11]. Among the major characteristics of investment into new power capacities are both capital intensity and long economic lifetime of the investment. Therefore the decisions are usually considered to a substantial degree as irreversible, which will necessarily also influence the optimization problem. Investment in power generation comprises a large and diverse set of risks. A good summary of optimization methods for electric utility resource planning has been performed by [32]. According to [32] most utility planners use deterministic methods (such as deterministic equivalents and scenario analysis) to assess different expansion plans under uncertainty. More advanced methods for stochastic optimization under uncertainty are rarely used, due to the complexity of the problem and the computational requirement involved. Still, there are a few suggestions how to better deal with uncertainty and flexibility for the regulated industry (e.g. [50], [72]). The electricity supply system is complex and so is the planning process. Due to computational reasons it is impossible to solve the total power system planning problem in one large operation. Some decomposition is therefore necessary and traditionally we distinguish between different levels, stages and objectives. The decisions involved can be categorized according to the time horizon involved into *investment planning* with a planning horizon of decades, and long- , medium- and short-term *scheduling* with planning horizon ranging from 1-2 years to 1-2 weeks [73] .

For investment planning, the literature [10, 32] identifies three main categories of the underlying risk - *market uncertainty* concerning both demand and input and output prices; *capital cost uncertainty* resulting from technical change, relevant mainly for new technologies (e.g. wind, solar) and *regulatory uncertainty*, i.e. carbon policy.

The most fundamental change affecting the electricity sector in liberalized markets is the inherent uncertainty about the electricity price. However, technologies producing the same level of power are by the electricity price uncertainty affected equally. The power generators trade their power output usually either through an organized power exchange or via bilateral contracts [34]. The electricity price uncertainty can be hedged against for example by entering into bilateral long-term contracts with distribution companies, or trading futures instead of selling electricity on the spot market [11]. Another important uncertainty is the electricity demand. Similar to the case of electricity price uncertainty, this can be shifted to distribution companies through the bilateral long-term contracts. In case the power producer is selling electricity on the spot market, both the demand and electricity price uncertainty fall within the problem of scheduling optimization.

Changes in fuel prices can influence the operating costs of a power generation capacity directly. The fuel uncertainty is relevant only for some technologies, e.g. it is significant for gas-fired plants because of the extreme volatility of gas prices, whereas for some renewables as wind it is not present at al. Therefore fuel uncertainty can affect the decision about the technology of a new power generating capacity. [4] provide an extensive empirical literature overview of this topic, claiming that the losses resulting from fuel volatility could be reduced by including renewables into the energy mix.

Investment costs are also to an extent uncertain. This is particularly the case for emerging technologies such as solar panels, wind farms and other renewable technologies where cost reductions are likely, but there is no certainty to the extent of such reductions.

According to [10], probably the greatest uncertainty for investors in new power plants will be the regulatory controls on future carbon dioxide emissions. Unknown value of carbon emissions permits and the mechanism chosen to allocate permits will become a very large and potentially critical uncertainty in power generation investment. This uncertainty will grow in the future, particularly as future restrictions on levels of carbon dioxide emissions beyond the first commitment period of the Kyoto Protocol are unknown. A general overview of the climate policy uncertainty is presented in the next section.

## 1.2 Climate Policy Overview

The long-term stated goal of Article 2 of the UNFCC is the "stabilization of greenhouse gas concentrations in the atmosphere" at a level that would "prevent dangerous anthropogenic interference with the climate system ." According to recent studies of long-term scenarios for stabilizing atmospheric concentrations of greenhouse gases (GHG) , stabilizing atmospheric concentrations of carbon dioxide ($CO_2$) at 450-650 parts per million (ppm) significantly reduces the expected change in global average surface temperature and associated impacts relative to the baseline projections for the increased GHG concentrations [52].

As more evidence about the contribution of anthropogenic GHG emissions to the rate of global warming and the associated damages is brought forward, the debate of whether climate policy should be implemented has shifted towards a different focus. More precisely, the debate now centers around the type and extent of policy instruments that should be implemented to achieve the desired reduction in emissions before irreversible damages accumulate. Still, there is no global agreement on the needed legislative, which makes the future climate policy one of the increasingly significant uncertainties connected with the investment into new power generating capacities.

The tools and instruments currently under consideration can be distinguished by two features. Firstly, it is the target of the policy, i.e. the part of the sector that is influenced by the policy. There are both instruments punishing every technology for each ton of $CO_2$ emitted, and ones that support the adoption and diffusion of specific renewable technologies directly,for example by obligations or targeted subsidies. Secondly, the policy can be based on either a government mechanism, such as a $CO_2$ tax or a subsidy, or a market system (such as current European cap and trade mechanism). The cap and trade policy is a mechanism where a central authority sets a limit on the amount of a $CO_2$ that can be emitted. Companies or other groups are issued emission permits and are required to hold an equivalent number of allowances which represent the right to emit a specific amount. The total amount of allowances and credits cannot exceed the cap, limiting total emissions to that level. Companies that need to increase their emission allowance must buy credits from those who pollute less. The transfer of allowances is referred to as a trade. In effect, this mechanism results in a carbon market where the buyer is paying a charge for polluting, while the seller is being rewarded for having reduced emissions by more than was needed.

There are already such policies in force all across Europe. For instance, UK applied a combination the so-called Climate Change Levy - a tax imposed on the use of fossil fuels, with a renewable obligation - a regulation setting a minimum share of electricity that has to be generated by renewable sources. The European countries introduced the a cap and trade system - European Trading Scheme (ETS). This mechanism is currently under consideration by the US legislative - The American Clean Energy and Security Act, an energy bill that would establish a variant of a cap-and-trade plan for greenhouse gases, was approved by the House of Representatives in 2009 and is still in consideration in the Senate.

Although one of the aims of this thesis is to account for the effect of climate policy on investment decision in the electricity generating capacities, the focus is not on the comparison of different policy instruments. For the analysis in the thesis, we represent the climate policy by a $CO_2$ price, i.e. a stochastic carbon penalty for each ton of $CO_2$ produced. This corresponds to the mechanism of a carbon market, i.e. a cap and trade scheme. Although

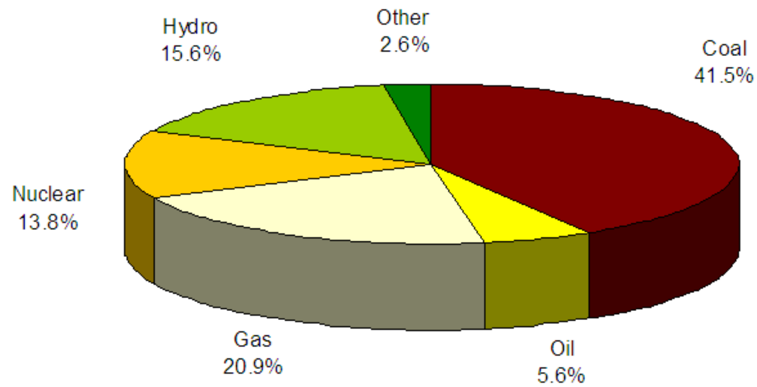


Figure 1.3: *Electricity generation by fuel, 2009*, Source: Key World Energy Statistics, 2009 .

this assumption is a simplification, we believe it is justifiable, indeed, the current situation suggests that this type of instrument will prevail in the future, eventually resulting in a global carbon market. Concrete assumptions on the future development of the carbon policy are presented in the next section.

## 1.3 Electricity Generating Technologies

Major sources for electricity generation comprise nuclear, fossil fuels (e.g. gas, oil, coal), and renewables (e.g. hydro, wind and biomass). The leaders are currently (see Figure 1.3) coal, gas, nuclear and hydro, with biomass and wind being recognized as the most promising technologies for the future (World Energy Outlook 2007). The focus of this thesis will be on four technologies only - coal, gas, wind and biomass. The reasoning behind this choice will be explained in the next chapter. Here we present a brief description of the characteristics of the chosen technologies, based on the reports from the International Energy Agency [36] [37]. They provide both technical and cost data based on surveys of existing power plants in various countries and projections how they will develop in the future. Table 1.1 gives an overview and explanation of common abbreviations and terminology used and can serve as a glossary for the following chapters.

Currently, almost two-thirds of the world's electricity is produced from fossil fuels. For the future, the IEA predicts a great expansion of coal-fired capacities in the developing countries in a case of no climate policy. Even in

| *Term* | *Unit* | *Definition* |
|---|---|---|
| Availability factor | [%] | Ratio of available time (when the power plant is able to produce electricity) to the calendar period. Availability characterizes the reliability of a plant. |
| Capital Costs | [€ /kWe] | Construction costs per kilowatt electric capacity |
| Capacity factor | [%] | Ratio of the electricity output of the power plant to theoretically possible electricity output in the period under report. |
| CCS | | Carbon Capture and Storage |
| Combustible renewables | | Examples include biomass, but also certain waste products |
| Efficiency | [%] | Ratio of the electricity output of the power plant and the fuel input, in energy terms. |
| Fossil fuels | | Include (exhaustible reserves of) oil, coal and gas. All of them emit GHG gases during their combustion. |
| GHG | | Greenhouse gas emissions |
| Operating and Maintenance cost (O&M) | [€ /y] | Yearly costs relating to the standard operating, maintenance and administrative activities of the utility. |
| Renewables | | Energy resources where energy is derived from natural processes that are replenished constantly. They include geothermal, solar, hydropower, wind, tide, wave, biomass and biofuels. |

Table 1.1: *Basic terminology*

a $CO_2$ constrained world the coal and gas can continue to play an important role (World Energy Outlook 2007). Moving on the pathway to clean and effective use of fossil fuels, the $CO_2$ capture and storage technologies (CCS) are considered to be the most promising ones in the near future.

### 1.3.1 CCS

$CO_2$ capture and storage (CCS) is one of the most promising options for mitigating emissions from coal-fired power plants and other industrial facilities. CCS is a three-step process involving the capture of $CO_2$ emitted by large-scale stationary sources and the compression of the gas and its transportation (usually via pipelines) to a storage site. $CO_2$ may also be used for enhanced oil or gas recovery. CCS processes can currently capture more than 85% of $CO_2$ that would otherwise be emitted by a power plant, but they reduce the plant's thermal efficiency by about 8 to 12 percentage points and, thus, decrease the electricity output for a given fuel input.

This option can be applied to both coal and gas-fired power plants. In combination with biomass, the use of $CO_2$ capture and storage would go even further and actually remove $CO_2$ from the atmosphere, rather than just avoiding its release from fossil fuels. The process of capturing $CO_2$ generally represents the largest component of CCS costs.

All the individual elements needed for CCS have been demonstrated, but there is still an urgent need for an integrated full-scale demonstration plant. At present there are four large-scale CCS projects in operation around the world, each involving around 1 Mt of $CO_2$ per year, two in Norway and the others in Algeria and Canada. In addition to these projects in the oil and gas sector, around 20 other major projects in the power sector have been announced.

### 1.3.2 Renewables

There are various forms of renewable energy, deriving directly or indirectly from the sun, or from heat generated deep within the earth. They include energy generated from solar, wind, biomass, geothermal, hydropower and ocean resources, solid biomass, biogas and liquid biofuels. The most common source of renewable power generation is currently hydropower, leading the market especially in Nordic countries.

Since 1990, renewable energy sources in the world have grown at an average annual rate of 1.7%. Growth has been especially high for "new" renewables (wind, solar), which grew at an average annual rate of 19%, and the bulk of the increase happened in OECD countries, with large wind energy programmes in countries such as Denmark and Germany. The discussions on climate change have undoubtedly stimulated the development of renewable energy in order to reduce the emissions of greenhouse gases.

The IEA outlook predicts that it is biomass and wind that can start to make a substantial contribution in the next decade, as many of the technology options for these two renewable sources are already cost competitive in many markets. Regarding renewable energy sources for electricity generation, the survey indicates, that in case of real investments the wind power plants are the most often considered option, solar and combustible renewables remaining marginal.

Combustion of biomass for power generation is a well-proven technology. It is commercially attractive where quality fuel is available and affordable. Capital costs of biomass fred power plants are rather high when compared to fossil fueled capacities, but their combination with CCS can make them attractive in case of a rising $CO_2$ policy.

Wind is currently next to hydro power the second eminent renewable resource used power generation. The availability factor of the wind power plants is relatively low, reported between 17% and 38% for onshore plants, and between 40% and 45% for offshore plants. Economic lifetimes of the wind power plants range between 20 and 40 years for all plants.

Most wind power plants are onshore but there are also some examples of offshore wind power plants (for example in Denmark, Germany or Netherlands). Capacities of the individual wind units are usually small with plants consisting of multiple units, generally comprising up to 100 for one plant. Capital costs for the majority of wind power plants vary between 1000 and 2000€ /kWe.

The costs of onshore and offshore wind have declined sharply in recent years, mostly due to mass deployment, the use of larger blades and more sophisticated controls. The learning effect is expected to continue in the coming years, potentially bringing additional cost reductions. The best onshore sites are already competitive with other power sources. Offshore installations are more costly, especially in deep water, but are expected to be commercial after 2030. However, because of its low availability factor, in situations where wind will have a very high share of generation, it will need to be complemented by sophisticated networks, back-up systems, or storage, to accommodate its intermittency [10].

### 1.3.3 Fossil Fuels

Today natural gas is experiencing significant growth as a fuel for power generation. Gas offers many advantages in this sector compared to other fossil fuels: high efficiency, relatively low capital costs, and cleanliness. Gas is the cleanest fuel among fossil fuels and its demand will be favored for environmental reasons. For gas-fired plants, most countries report shorter technical lifetimes, between 20 and 30 years. In recent years, natural gas consumed for electricity generation has accounted for almost 20% of global electricity production (up from 13% in 1973), and accounts for approximately half of the world production of heat generated in heat and combined heat and power plants.

Over the last 30 years, the share of coal in global total primary energy supply has been stable at around 25%. Coal is now mainly used for electricity production and to a lesser extent by industry. Coal-fired power plants are more capital intensive than gas-fired ones. Combustion of pulverized or powdered coal to raise steam in boilers has been the mainstay of coal-based power generation worldwide for almost a hundred years. The efficiency of the current generation of pulverized coal units has steadily improved and today ranges between 30% and 45% (on a lower heating-value basis) depending on the quality of coal used, ambient conditions and the back-end cooling employed. More efficient technologies for coal combustion are already available or in an advanced stage of development. These include high-temperature pulverized coal and integrated coal-gasification combined-cycle (IGCC) plants. Capital costs for coal are almost twice as high as the capital costs for gas fired power plants, ranging between 1000 and 1500 € /kWe, with IGCC plants lying even above the 1500 € /kWe limit.

# Chapter 2

# Conceptual Overview

## 2.1 Portfolio Optimization into new power generation capacities

Portfolio theory is rooted in finance. Its systematic dates back to the pioneering work by Nobel laureate Harry Markowitz as early as 1952 ([45]). His work was followed by a vast number of authors, notably Merton [47, 48, 49], Samuelson [63] and Fama [20]. Economists soon realized that the method also provided considerable insight into decision making concerning capital investment. Therefore, the applications of this theory to real investments followed soon after. Still, in case of investment into power generating capacities, the literature is not as numerous.

Even though the first attempts date back to the seventies [6], more pronounced interest in the topic has arisen only lately [3, 2]. In general, these models consider a mean-variance framework to test whether the portfolios of electricity generating capacity used by certain regions or countries are efficient. [33] refine the approach of [6] by building a GARCH-type model, which allows the covariance matrix of the underlying assets to be systematically updated in time. [62] apply a similar framework to UK diversification in electricity sector investment, including also the carbon price risk. All these studies apply the mean-variance framework in the style of Markowitz [45, 46]. Recent research on different measures of risk (explained more in detail in Chapter 3) has provided the portfolio theory with new insights which led to the development of more advanced portfolio approaches. However, their application to the power generation portfolios is still missing. It should be noted, that there are some examples of their applications in the electricity sector [17, 21, 70, 64]. The authors are investigating the value of flexibility of optimal electricity supply scheduling, applying the concept of conditional-value-at-risk, mostly in term of constraints entering the optimization. However, the focus of these papers is quite different from the one studied in this thesis.

Although the portfolio theory has been increasingly used to study investments in new power generation capacities, most analysis has been based on the static mean-variance approach. Such an approach entails various shortcomings. It is important to understand these, both to comprehend the need for a better framework and to identify the key points this new framework should address. The shortcomings are caused mainly by the assumptions of the mean-variance framework, which are in contradiction to the specifics of the investment into real assets, in our case the power generation capacities. The characteristics of investments into financial and real assets are distinct. The differences lie mainly in following factors: irreversibility of investment (usually represented by high sunk costs), demand constraints and longevity of the investment. The shortcomings of the static mean-variance approach that are implied by these differences can be divided into three main categories.

The first is concerned with properties of the profit distribution of the assets, the second is centered around the assumed risk preference of the investor. The last considers the static setup of the framework.

Returns of real assets are generally not normally distributed [18], with fat tails and entail potentially high losses (due to the irreversibility of investment and high sunk costs), however, the mean-variance framework considers the return on the assets as normally distributed random variables.

Moreover, since in case of real assets the returns are characterized by potentially high losses, the risk preference of the investor is very important for the result of the optimization. Since capital is typically long lived in the applications, and a part of the decisions is irreversible, the investor has to be particularly sensitive to the downward tail risk. Therefore, the use of variance as a proxy for risk is usually not appropriate.

The last major drawback is the fact that the static framework fails to account for the effects irreversibility has on optimal dynamic behavior. [49] has developed an inter-temporal version of the portfolio approach, however, due to its complexity the applications were not numerous and it did not really displace the Markowitz framework at that time [23]. Only recent advances [12] enabled it to be applied, e.g. in pension fund management [43]. However, when considering a portfolio of long-lived real assets, static Markowitz portfolio can not be extended to a dynamic setting the way it has been done for financial assets. The reason for this is the special feature of irreversibility - once resources have been committed (for example to install a new power plant), this asset can hardly be removed from the generating portfolio at zero transaction cost. Rebalancing of the portfolio in the classic sense is not feasible in case of real asset investment. However, the dynamic feature of the decision should not be unaccounted for, as the effect can be significant. Since the investor usually faces demand constraints, and also needs to replace the capacity at restricted time instants known ahead, he will make decisions at several time points. These decision can hardly be assumed to be independent.

In this thesis these general issues of portfolio optimization for real assets are studied in the particular case of investment into new power generation capacities under climate policy uncertainty. Considering the climate policy as the major source of uncertainty raises another issue that has not been covered in any of the previous literature on energy planning. Whereas most input/output prices can be modeled as fluctuating processes, uncertainty about climate change is still qualitatively different. It has been currently agreed that there are two main factors determining the climate policy in the future - the required level of stabilization of GHG concentration and the future socio-economic conditions. However, there is not enough information to determine either of them. Therefore, different scenarios of climate policy have been developed for different scenarios of the underlying factors. The resulting policies depend significantly on the scenarios. As some investment will need to carried out before this uncertainty is resolved, the investors will naturally prefer decision that will behave well under each of the possible scenarios, trying to find a robust strategy. This constitutes the last concern we try to address in our approach.

The issue of robustness with connection to portfolio theory has been studied more closely only recently. In the mean-variance framework, this topic has been investigated for example by [14] and [30]. Assuming different types of uncertainties in the mean or covariance matrix, they transformed the to semidefinite, or second-order cone programming problems, which can be efficiently solved by interior-point algorithms developed in recent years. [15] use a minimax approach to analyze an optimal mean-variance portfolio selection problem, where the expected return of each underlying asset varies in an estimated interval while the covariance between any two asset returns is given and fixed.

## 2.2 Contribution of this Thesis

Chapter 1 has outlined the importance of the decisions about the new electricity generating capacities with respect to the emerging climate policy. The uncertainties entering into the investment problem have been also listed. This motivation led us to ask the following question:

Assuming that the operations and investments (e.g. retrofitting or refurbishments) at the plant level are carried out optimally, what is the optimal energy mix in case of a uncertain climate change policy?.

In other words, if we analyze the situation from point of view of the generator, what is his best response to the $CO_2$ policy? A considerable number of power generators will favor some kind of generation portfolio with a mix of different types of generation. These types of generators likely form the core of generation capacity in most countries. Several companies interviewed maintain guidelines for the overall portfolio mix that they wanted to achieve [10]. It should be noted however, that these are usually used to indicate the strategic direction rather than acting as "hard" targets. In this thesis we try to find the optimal portfolio for a risk averse investor who is facing uncertain fuel and $CO_2$ prices.

While optimization of the investment and management of a power plant (e.g. an incremental investment such as CCS) is performed by the individual producer, large investors will typically want to invest in a technology portfolio rather than concentrate on a single technology or chain (e.g.coal with a possibility to add CCS). We propose an optimization framework that derives the optimal behavior on both levels.

We do it by separating these decisions, forming two levels of decision making. On the plant level the operation and management are chosen optimally to maximize the expected revenue. Assuming that the operations and investments (e.g. retrofitting or refurbishments) at the plant level are carried out optimally, the second level answers the question how the energy mix should be composed. At the larger scale, the objective of the investor is different. It has been explained that the impacts of the decisions about new power generating capacities are long-lasting and the uncertainties present not negligible. The resulting profit is therefore highly uncertain with potentially high losses. On the larger scale the investor needs to base his decisions not only on the expected profit, but also on the risks. This is reflected in the portfolio framework suggested for the second level.

This thesis presents a optimization framework for the investment into new power generation capacities. It suggests to use a combination of real options and portfolio optimization. The real options model is used to derive the optimal management strategy on the power plant level for each electricity generating technology considered. Following this strategy implies a distribution of profit flows resulting from investment into a power plant of a given technology. These profit distributions are used as an input for the portfolio model.

There are several portfolio models suggested. First is the basic version that defines the optimal portfolio as such a combination of technologies that maximizes the expected profit given a constraint on risk. For the measure of risk the conditional Value-at-Risk was chosen, which in contrary the the variance features several favorable characteristics. This portfolio leads to a problem of linear programming. This model is further modified to account for the specifics mentioned in the previous section which leads to the robust and dynamic models which are suggested in the second half of the thesis.

The main contribution of the thesis can be seen in three key points.

First, it is proposed combined framework. Although both real options and portfolio theory are established and commonly used for applications, the suggested combined framework is original. It accounts for the possibility to optimize the management also on the plant level, which is mostly disregarded. For the optimization on the larger scale, portfolio theory is applied. Whereas due to its characteristics the classic mean-variance framework is not appropriate, the conditional Value-at-Risk was adopted as the measure of risk. Three different portfolio models are discussed. The basic portfolio model from [60] is employed to provide a benchmark for the suggested modification of the portfolio problem. The proposed modifications present extensions of the basic portfolio model, both in the direction of robustness and dynamics. They are shown to preserve the advantageous characteristics of portfolio optimization using conditional value-at-risk, namely that it leads to linear programming problems. The proposed framework is able to formulate quite a complex modeling problem in an effective way, which is a original contribution to the literature on portfolios of real assets.

Second, it is the applications of the proposed models to analyze problems that are currently relevant in the energy sector. By testing the models with real-world data, we can verify the validity of our conclusions for actual investments, even though the models remain still highly stylized. The results should be therefore taken as an illustration rather than a precise numerical prediction. Still, they enable us to study the relation between the investor's assumptions about future climate policy and the resulting optimal energy mix. This can be used to derive policy implications and to identify the key drivers for investment into low-carbon technologies. The results also suggest an explanation of the observed behavior in power generation investment. Although the applications are still only limited with the respect to the number of technologies considered, they are still able to illustrate the importance of choosing an appropriate framework for portfolio optimization.

Not the least, we provide a coherent comparison of the portfolio selection problem for real assets when minimizing risk in terms of conditional Value at risk to the classic Markowitz portfolio framework minimizing variance for the case of normally distributed assets profits.

## 2.3 Research Outline

A broad outline of the thesis with a short overview of the individual chapters is given in Table 2.1.

The thesis is comprised of two parts. Up to now the first part provided the motivation for the problems addressed (Chapter 1), explaining the specifics that need to be addressed by the modeling framework (Chapter 2). The aim and contribution of the thesis are stated this chapter. The first part concludes with a synopsis of the state-of-the-art with respect to the chosen methodology (Chapter 3).

The second part presents own results. It starts out by a description of the real options framework (Chapter 4) that optimizes the behavior of the investor on the plant level. These results are further used as an input into the portfolio model. The formulation of the general basic CVaR portfolio model is shown in Chapter 5. First, its results are put into perspective with respect to the classic mean-variance framework in case of normally distributed assets profit. Further we formulate the CVaR portfolio model in case the assets profit distributions stem from the real options model. We analyze first the difference between the proposed and the classic mean variance portfolio model. We conclude with the sensitivity of the optimal portfolios with respect to climate policy uncertainty, which constitutes also the motivation for the next chapter.

In Chapter 6 we propose an extension of the basic CVaR model that is able to identify a portfolio performing well across a set of scenarios.the basic model is further. We present also two applications of the suggested model, each addressing a different issue that was neglected in the analysis so far. The first concentrates on the uncertainty in climate policy, trying to find an optimal energy mix that is robust across different climate policy scenarios. The second investigate decisions that would lead to profit flows which are more stable over time.

The portfolio analysis remained inherently static insofar as the large investor considered only the current investment, ignoring possible future investments. The extension from Chapter 7 seeks to remedy this deficiency by taking into account the possibility to diversify not only over assets, but also over time. More specifically, we look at the dynamics of the optimal technology mix over a future time period conditional on the initial distribution of technologies, such that given energy demand is met.

Chapter 8 provides a summary of the results of the previous chapters with

| Chapter | Outline |
|---|---|
| **1** The Electricity Sector | Motivation. Overview of the Electricity Sector, focus on power generation and uncertainties connnected with investment into new capacities. |
| **2** Conceptual Overview | Litearure review - Portfolio theory and its applications in energy sector. Problem definition. Main contribution. Research Outline. Data description. |
| **3** CVaR and Portfolio optimization | Theory overview - CVaR as a risk measure and its use in portfolio optimization. |
| **4** Profit distributions analysis | Real options model for optimization on the plant level with stochastic $CO_2$ and fuel prices. Resulting profit distribution analysis. |
| **5** CVaR vs. M-V | Comparison of portfolio optimization using MV and CVaR. Case of normally distributed assets profit. Basic framework : combination of a CVaR portfolio model with the real options model leading to a linear programming (LP) problem. Efficient frontier. Comparison with the classic mean-variance approach and climate policy sensitivity. |
| **6** Robust Portfolios | Portfolio model deriving decisions robust across a set of scenarios. Formulation as a LP problem. Applications : Assessing the impact of climate policy uncertainty, Time structure of profit. Comparison to the basic framework. |
| **7** Dynamic Framework | Formulation of a portfolio model allowing for diversification across time. Analyzing the impact of this extension on the resulting energy mix. |

Table 2.1: *Outline of the thesis.*

respect to the evolution of results considering different portfolio models. It also investigates the implications derived from the presented analysis for the climate change policy. It concludes with a synopsis of the contribution and identifies areas for further research.

## 2.4 Assumptions

This thesis presents an integrated real options and CVaR portfolio model to assess the impact of climate policy on the investment decisions in power generation. In the previous chapter the overview of the energy sector with concentration on the power generation was provided, analyzing the underlying uncertainties and ivestment options. In this section, we list the assumptions and simplifications considered in the modeling framework.

**Technology options choice.** For the analysis we consider only four technologies - wind farms, coal-fired, gas-fired and biomass-fired power plants. The choice of the technologies is based both on current composition of energy mix and on the projection for the future. As presented in the previous chapter, coal and gas-fired power plants are the major representatives of fossil fuel technology and form the core of electricity production, promising to stay significant also in the next decades. Biomass technology has the unique property that in combination with the CCS module it is actually able to remove the $CO_2$ from the atmosphere. Wind farms were chosen as a representation of the "standard" renewables with zero carbon dioxide emissions. Although it is not the most prominent electricity source from renewables, with hydropower having a larger share in the current energy mix, it has been chosen for two reasons. First, wind power is regarded as a promising renewable technology for the future, with more scope for new installations, as hydropower is already a well-established technology. Second, since both technologies exhibit the same characteristics (no emissions, no fuel cost uncertainty), there was no fundamental reason of introducing both technologies into the analysis. Therefore, wind acts as a proxy for this type of renewables, and could represent hydropower as well.

**Uncertainties considered.** We abstract from demand and electricity uncertainty. Since in this analysis we use normalized data for electricity generation, the fluctuations in electricity price have the same effect on all technologies considered and don't have any impact on the investor's decisions. Since the focus of this thesis is the analysis of the energy mix, this ultimately means comparison of different electricity generating technologies. Therefore the focus in the analysis should be on parameters where the individual technologies differ, where the electricity price uncertainty is the least significant for our analysis. This assumption can be represented by a situation where the producer enters into a bilateral contract with the distribution company. This contract binds him to fulfil a supply constraint ensuring a fixed electricity price at the same price.

**Scalability.** We abstract from electricity supply scheduling and assume that electricity generation does not feature significant economies of scale. This is corroborated by [73]. That means that the size of the installed capacities is not important. The power plants considered are scaled to produce the same amount of electricity per year, equal to the supply constraint. This is probably the most significant simplification, influencing mainly the wind technology. Wind is characterized by a low capacity factor, meaning the power plant can not operate all the time through the year. Assuming it is sufficient that the yearly output is met overestimates the profitability of this technology. This effect is limited in the analysis by the introduction of constraints on the wind share in the energy mix.

**Separation** We assume the decisions on the plant level are independent of the investments on the aggregate level.

**Profit and risk aversion.** For the decisions on the aggregate level, we assume the investor risk averse, i.e. he is also concerned about the risk associated with the investments. This is not the case on the plant level, where the decisions are driven only by expected profit maximization. In addition, except for the dynamic portfolio model, we assume the profitability of investment on the aggregate level is measure by profit, not return on investment. This can be justified in case of a utility with contracted supply, where the investor has to deliver the contracted amount and is concerned with the net profit he can gain. This is in contrary to the investments in financial assets, where the return per unit of investment is usually the measure of profitability.

**Data assumptions.** These are explained in detail in the next section.

## 2.5 Data

### 2.5.1 Power Generation Technology Parameters

For each technology, i.e. coal, gas, biomass and wind, Table 2.2 summarizes the data needed for the analysis performed in this thesis. The data needed

| *Parameters* | *Efficiency* [%] | Capacity *factor* [%] | $CO_2$ *emissions* [kg/kWh] | *O&M costs* [€ /kW/y] | *Capital costs* [€ /kW] |
|---|---|---|---|---|---|
| Coal | 46 | 89 | 0.74 | 68.297 | 1,182 |
| Coal+CCS | 36 | 85 | 0.111 | 101.465 | 1,525 |
| Gas | 58 | 89 | 0.348 | 15.281 | 500 |
| Gas+CCS | 49 | 85 | 0.052 | 34.042 | 843 |
| Biomass | 35 | 89 | 0 | 43.269 | 1,537.19 |
| Biomass+CCS | 27 | 85 | -1.41 | 64.282 | 1,880.19 |
| Wind | na | 40 | 0 | 76 | 1,800 |

Table 2.2: *Power Plant Data* (Source: derived from van den Broek et al. (2008), biomass-fired technology parameters stem from International Energy Agency, (2005).

in the analysis for each technology considered are: construction (i.e. capital) costs, operating and maintenance costs, fuel costs and $CO_2$ costs. The table presents the capital costs and O&M cost per 1 kW of installed capacity. Individual power plants have been scaled so that the yearly electricity output is the same, equal to the output of a coal fired power plant of 1kW installed capacity. Fuel costs depend on fuel efficiency and price scenario considered. The precise information on price assumptions is provided in the next section.

All technologies except wind are considered capture-ready, i.e. the CCS module can be added to the power plants during their lifetime. The capital costs of this upgrade are given by the difference between the capital costs of the power plant with and without the CCS module. It should be noted that investment into such a module is connected not only with significant reductions in $CO_2$ emissions, but also with large capital investment, higher O&M costs and efficiency loss. The efficiency loss comes from the need of electricity to operate the module. Since we consider the electricity output as fixed, this electricity needs to be imported and is accounted for in the O&M costs.

The lifetime of all of the considered options is assumed to be equal to thirty years. This assumption on equal lifetimes simplifies the analysis and is in line with the ranges for economic lifetimes reported by the IEA [36]. It is a simplification, however, since chosen lifetime constitutes the upper bound for both wind and gas-fired power plants, whereas it is the lower bound for the coal technology.

Several differences between the technologies can be deduced from the

| *Scenario* | 'A2r' | 'B1' | 'B2' |
|---|---|---|---|
| Population size | High | Medium | Low |
| Income | Low | Medium | High |
| Resource-use efficiency | Low | Medium | High |
| Technology dynamics, fossil | Medium | Medium | Low |
| Technology dynamics, non-fossil | Low | Medium | High |
| Required emission reduction | High | Medium | Low |

Table 2.3: *Assumptions of the individual socio-economic scenarios on the key drivers, measured relative to each other* Source: GGI database, 2009

presented table. The gas-fired plant is a more clean and less capital intensive alternative, but suffers from large and volatile fuel costs. Coal, on the other hand, is more costly (both in capital and in O&M costs), but the resulting fuel costs are relatively low and stable. The biomass technology features lower efficiency and is even more capital intensive than coal. On the other hand, it offers the largest potential in case of a strict climate policy. A wind farm is an example of a technology with the highest capital, but stable and relatively low operating costs. It is the only technology that is indifferent to the fluctuations both in $CO_2$ and fuel prices. We also see that biomass has a special position, since its emissions when equipped with the CCS module are negative [69].

### 2.5.2 Price Parameters

The climate policy and fuel costs data used in this thesis are provided by the GGI (Greenhouse Gas Initiative) Scenario Database generated by the MESSAGE model developed at IIASA. The MESSAGE model is a large-scale bottom-up, cost-minimizing energy systems model, for more detail about the model, the reader is referred to [57]. The GGI scenario database documents the results of a set of greenhouse gas emission scenarios that were created using the IIASA Integrated Assessment Modeling Framework. Beside its principal results that comprise the estimation of technologically specific multi-sector response strategies it also reports the projections of future carbon prices for a range of alternative climate stabilization targets for each of three scenarios considered.

The three scenarios - labeled 'A2r','B1' and 'B2' - are distinguished by different assumptions on socio-economic development of the world. The scenarios are derived from (and also use the naming conventions of) the scenarios

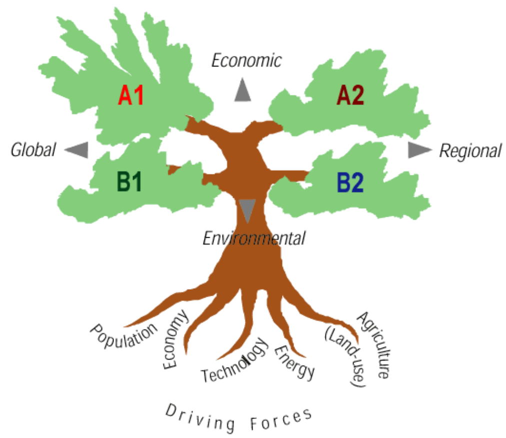


Figure 2.1: *Schematic illustration of SRES scenarios.* Source: IPCC Special Report on Emissions Scenarios, 2000.

presented in the IPCC Special Report on Emissions Scenarios (SRES) [51], see Figure 2.1[1]. However, the original scenarios presented in SRES have been revised to incorporate the results of new scenario analyzes with the aim to improve scenario consistency. The scenario 'A1' has been omitted and the scenario 'A2r', while maintaining its main structural and qualitative characteristics, has been markedly revised to that reflects the most recent long-term demographic outlook.

The assumptions of each scenario can be summarized as follows, a brief characterization of the assumption on the key-drivers is given also in table 2.3.

The 'A2r' storyline and scenario family describes a very heterogeneous world. The underlying theme is self-reliance and preservation of local identities. Fertility patterns across regions converge very slowly, which results in high population growth. Economic development is primarily regionally oriented and per capita economic growth and technological change are more fragmented and slower than in other storylines. Therefore, stabilization is not achieved easily and GHG shadow prices for more ambitious target increase tremendously over the course of the projection period.

The 'B1' storyline and scenario family describes a convergent world with a

[1]The main differences in the scenarios lies in two dimensions. First is the global vs. regional scenario orientation, the second the development and environmental orientation. In reality, the four scenarios share a space of a much higher dimensionality given the numerous driving forces and other assumptions needed to define any given scenario in a particular modeling approach.

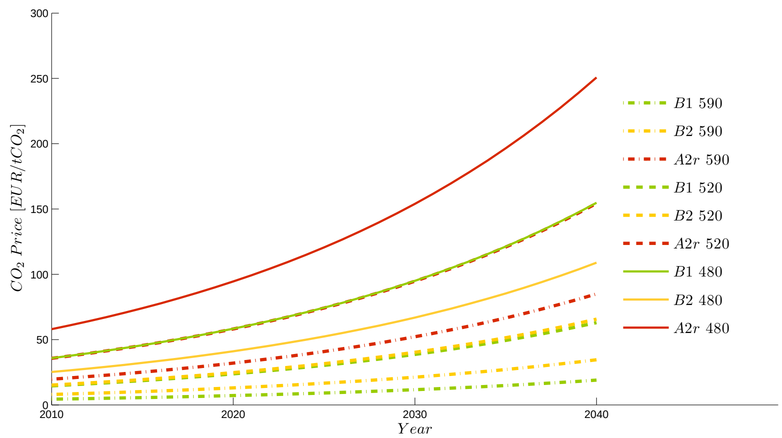


Figure 2.2: *$CO_2$ shadow price projections from the GGI Scenario Database*
Source: GGI Database.

low population growth and with rapid changes in economic structures toward a service and information economy, with reductions in material intensity, and the introduction of clean and resource-efficient technologies. The emphasis is on global solutions to economic, social, and environmental sustainability, including improved equity, but without additional climate initiatives. GHG (shadow) prices actually even decrease towards the end of the projection period, but this long-term view does not fall in the range of the planning period of the is thesis.

The ‘B2’ storyline and scenario family describes a world in which the emphasis is on local solutions to economic, social, and environmental sustainability. It is a world with moderate population growth, intermediate levels of economic development, and less rapid and more diverse technological change than in the B1 and A1 storylines. While the scenario is also oriented toward environmental protection and social equity, it focuses on local and regional levels.

In other words, the different socio-economic scenarios capture the fact that the the future $CO_2$ emissions depend also on population growth, the transition of new technologies to the developing countries, etc. Therefore to decrease the emissions in a world with a growing population is more costly than in the world where the population stabilizes and the transition of the less emission intensive technologies to the developing countries is prompt.

However, there is still uncertainty about the actual $CO_2$ concentration target which should be achieved. Therefore, for each scenario the database provides predictions for both fuel and $CO_2$ prices for different stabilization targets. In this thesis we will analyze three of them, ranging between 480 and 590 ppm, which results in nine different alternatives for the price predictions. These alternatives are defined by a scenario-target combination. A stricter target, i.e. the need to stabilize at a lower concentration implies a higher $CO_2$ price, and vice-versa. The discount rate is considered the same in all scenarios, equal to 5%.

Figure 2.2 shows the developments of $CO_2$ prices for each scenario and target combination considered in this thesis. The database predicts an exponential rise for the $CO_2$ price, where the trend and starting values differ across the scenarios and stabilization targets. This result justifies our assumption to model the $CO_2$ price as a geometric Brownian motion $CO_2$, where the data for trend and starting value of these processes have been supplied by the GGI database. The yearly carbon price volatility is assumed to be 5%.

Parameters for the fuel price are derived similarly. Fuel prices are modeled as a geometric Brownian motion, where the trend and starting value for each scenario and stabilization target are given by the GGI database. The volatilities of the fuel prices are taken from [55], where yearly volatilities of both gas and coal prices are estimated from historical data based on the assumption of the prices following a geometric Brownian motion. Their results show the gas as the fuel with highest volatility and coal the lowest one. Biomass price volatility is assumed to be slightly higher than that of coal, but significantly lower than that of gas [71]. Data for the electricity price used to calculate the technology returns for the model from Chapter 7 stem also from the GGI database. The table presenting the overall set of electricity, fuel and $CO_2$ price parameters for each scenario and stabilization target is presented in Appendix A.

# Chapter 3

# Conditional Value-at-Risk

## 3.1 Introduction

One of the principal components of computational finance is portfolio optimization. Historically, this problem has been approached in two ways. The earlier, expected utility maximization, is nowadays being employed mainly in theoretical studies. The more recent is the concept of the trade-off between risk and return. It is currently widely being used both in theory and practice. It is based on the assumption that investors are as a rule risk-averse. That is, they refrain to a certain extent to buy assets that exhibit a large variance in their returns. Typically, a risk averse investor would therefore compose his portfolio of a combination of assets. It would consist of both assets with lower but relatively certain expected rates of return and assets with a high expected but less certain rate of return. It is the trade-off that matters. The classical framework for this idea is the mean-variance approach introduced by Markowitz [45] minimizing portfolio variance subject to a given lower bound on expected return (or, vice versa, maximum return subject to a constraint on variance). In this case risk is represented by variance. Even though the mean-variance approach is capable of explaining diversification and the risk-return trade-off in a very straightforward manner, it exhibits a number of shortcomings. Attempts to remedy them lead to numerous alterations and extensions of the basic model over the last decades.

One of the criticized assumptions concerns measuring risk preference by quadratic utility. This assumption implies that the investor is indifferent to other properties of the return distribution (such as higher order moments, e.g. skewness and kurtosis). For this thesis, the assumption of joint normal distribution of the asset returns is unacceptable as well. It is frequently observed that returns in equity and other markets are not normally distributed, the same holds also for the distributions derived in the applications in the following chapters.

For the above reason, alternative risk measures have been introduced. Since the mid nineties, risk management in financial institutions has been employing another (downside) measure of risk - VaR. Unlike variance, value-at-risk (VaR) captures extreme - and thus potentially dangerous - events by providing information on the tail of the distribution. VaR has been recognized by international regulatory bodies: the requirements of the Basel committee on Banking Supervision ([7, 8]) are geared towards the use of VaR. Although a step in the right direction, VaR still suffers from several defects. First, VaR as a risk measure lacks several properties desirable for applications in portfolio optimization. In case of general distributions (especially discrete) it is not subadditive and, consequently, it is not coherent in the sense of [1]. Moreover, when applied to portfolio optimization, it can exhibit multiple local extrema for discrete distributions, leading to problems of non-convex optimization. Another shortcoming concerns the economic interpretation of VaR. Being defined as a percentile of the distribution it does not contain any information about the losses beyond that threshold. Therefore it does not capture the downside risk in the worst cases, which may be pertinent for many investors.

Beginning later nineties, conditional value-at-risk (CVaR) has been studied as an alternative measure of risk. Its application to financial optimization has been first developed in [60]. CVaR, which is essentially the mean of the tail of the distribution exceeding VaR has been proved not to suffer from the above-mentioned caveats. It has been shown to have better properties than VaR (see [1], [19]). [54] has proved that CVaR is a coherent risk measure with additional desirable properties (e.g. positive homogeneity, convexity). In addition, the powerful results in [60, 59], made computational optimization on CVaR readily accessible: they proved, that as a rule, CVaR minimization leads to convex, or even linear optimization problems. Due to the mentioned properties CVaR became attractive not only as a subject of research but also for applications in practice. Let us note, though, that similar concepts have been used in the stochastic programming literature before, albeit not in the context of financial mathematics. The conditional expectation constraints and integrated chance constraints (see [56]) may serve the same purpose as CVaR.

In the following chapter we define CVaR as a risk measure and summarize its fundamental properties. We focus on the properties that are essential for the applications presented in next chapters, and the implications for losses with discrete distributions in particular. Later we introduce a portfolio approach using CVaR (either in the objective or in the form of underlying constraints). The approach provides an optimization shortcut making (by linear programming techniques) otherwise infeasible large-scale calculations possible. For a more comprehensive and integrated treatment on CVaR with complete proofs see [60, 59].

## 3.2 Conditional value-at-risk

### 3.2.1 Definition and basic properties

In decision making under uncertainty, in particular when dealing with potential losses, measures of risk play an important part. The potential loss is usually considered in a form of $z = f(x, y)$, where $x \in X \subset \mathbb{R}^n$ is the decision vector and the random vector $y \in Y \subset \mathbb{R}^m$ represents the uncertain factor. Assuming the probability distribution of $y$ is known, $z$ is a random variable with its distribution dependent on decision $x$. Assuming the decision maker is concerned not only about the expectation of $z$, but also about the risk associated with decision $x$, the choice of the risk measure can crucially influence the character of the problem. The conditional Value-at-Risk is advantageous not only because its use leads to convex optimization problem, but also because of the straightforward economic interpretation.

Let us consider a random vector $y$ defined by a probability measure $P$ on a measure space $Y$. By $f(x, y)$ we denote a loss function associated with the event $y$, depending on a parameter $x \in X \subset \mathbb{R}^n$. We assume $f$ continuous in $x$ and measurable in $y$ and such that $E[|f(x, y)|] < \infty$ for each $x \in X$. Let $\Psi(x, \cdot)$ denote the resulting distribution function of the loss, i.e.

$$\Psi(x, \xi) = P\{y|f(x, y) \leq \xi\}. \tag{3.2.1}$$

Let us consider a confidence level $\alpha \in (0, 1)$ (in applications usually chosen at the levels 0.95, 0.99). The Value-at-Risk (VaR) is at this level defined as follows.

**Definition 3.2.1.** *The $\alpha$-VaR of the loss associated with a decision $x$ is the value*

$$\xi_\alpha(x) = \min\{\xi|\Psi(x, \xi) \geq \alpha\}. \tag{3.2.2}$$

It should be noted that the minimum is attained since $\Phi(x, \xi)$ is nondecreasing and right-continuous in $\xi$. The CVaR can then be defined as

**Definition 3.2.2.** *The $\alpha$-CVaR of the loss associated with a decision $x$ is the value $\phi_\alpha(x)$ equal to the mean of the $\alpha$-tail distribution of $z = f(x, y)$,* *where the distribution in question is the one with the distribution function* $\Psi_\alpha(x,\cdot)$*, defined by*

$$\Psi_\alpha(x,\xi) = \begin{cases} 0 & \textit{for } \xi < \xi_\alpha(x) \\ [\Psi(x,\xi) - \alpha]/[1-\alpha] & \textit{for } \xi \geq \xi_\alpha(x) \end{cases} \tag{3.2.3}$$

Please note that since $\Psi_\alpha(x,\cdot)$ is nondecreasing, right-continuous and $\Psi_\alpha(x,\xi) \to 1$ for $\xi \to \infty$, it is a distribution function. Thus, the $\alpha$-tail distribution is well defined.

The subtlety of the previous definition lies in the fact that it defines CVaR well also in case of non-continuous loss distributions. It can be easily noted, that from the definition $\alpha$-CVaR dominates $\alpha$-VaR in the sense that $\phi_\alpha(x) \geq \xi_\alpha(x)$.

Intuitively, CVaR can be described as the expected value of losses exceeding the $\alpha-$VaR, which is effectively the $\alpha^{th}$ percentile of the loss distribution (see Figure 3.1). Although correct and consistent for continuous distributions, this definition was shown to be cumbersome and ambiguous for general distributions (especially discrete distributions). This is the reason why the definition of CVaR is not as straightforward, although in most case it yields the same results.

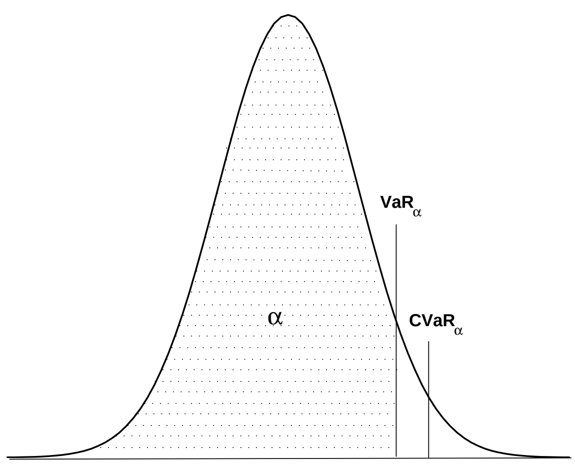


Figure 3.1: *VaR and CVaR of a loss distribution*

In case the loss distribution is discrete (e.g. it is empirical, derived numerically) the definition of CVaR can be further simplified. Let us suppose the probability measure $P$ is supported by finitely many points $y_k,\ k = 1,\ldots,N$

of $Y$, so that for each $x \in X$ the distribution of the loss $z = f(x, y)$ is likewise concentrated in $N$ finitely many points, and $\Psi(x, \cdot)$ is a step function with jumps at those points.

**Lemma 3.2.1.** *Fixing $x$, let those corresponding loss points be ordered as $z_1 < z_2 < \cdots < z_N$, with the probability of $z_k$ being $p_k > 0$. Let $k_\alpha$ be the unique index such that*

$$\sum_{k=1}^{k_\alpha} p_k \geq \alpha > \sum_{k=1}^{k_\alpha - 1} p_k. \tag{3.2.4}$$

*The $\alpha$-VaR of the loss is given then by*

$$\xi_\alpha(x) = z_{k_\alpha}, \tag{3.2.5}$$

*whereas the $\alpha$-CVaR is given by*

$$\phi_\alpha(x) = \frac{1}{1-\alpha}[(\sum_{k=1}^{k_\alpha} p_k - \alpha) z_{k_\alpha} + \sum_{k=k_\alpha+1}^{N} p_k z_k]. \tag{3.2.6}$$

One of the most important findings of [59], derived for general loss distributions, including the discrete ones, is the way how the $\alpha$-VaR and $\alpha$-CVaR of the loss $z$ associated with a choice of $x$ can be calculated simultaneously by solving an elementary optimization problem of convex type in one dimension. Let us denote

$$F_\alpha(x, \xi) = \xi + \frac{1}{1-\alpha} E[(f(x, y) - \xi)^+], \tag{3.2.7}$$

where $x^+ = \max\{0, x\}$. Then following holds

**Theorem 3.2.1.** *As a function of $\xi \in \mathbb{R}$, $F_\alpha(x, \xi)$ is finite and convex (hence continuous), with*

$$\phi_\alpha(x) = \min_\xi F_\alpha(x, \xi) \tag{3.2.8}$$

*and moreover*

$$\xi_\alpha(x) = \min\{\arg\min_\xi F_\alpha(x, \xi)\}, \tag{3.2.9}$$

*where* $\arg\min$ *is the set of those $\xi$ for which the minimum is attained. In this case it is a nonempty, closed, bounded interval (possibly reducing to a single point).*

The proof of Theorem 3.2.1 for general loss distributions can be found in [59]. The Theorem sheds light on the cardinal difference between CVaR and VaR. It reveals the fundamental reason why CVaR is much easier to deal with than VaR in applications to optimal portfolio choice: the minimal of $\phi_\alpha$

as a function of parameters is much better behaved than the optimal solution set arg min interval having $\xi_\alpha$ as its lower endpoint.

The minimization formula of Theorem 3.2.1 is particularly important in the circumstance of Lemma 3.2.15. In this case $F_\alpha(x, \xi)$ is piecewise linear with discontinuities at the loss values $z_k$. Therefore, the arg min has to consist either of a single point $z_{k_\alpha}$, or an interval $[z_{k_\alpha}, z_{k_{\alpha+1}}]$ between successive corner points.

Furthermore, as proved in [59], CVaR has several favorable properties, we summarize the ones that are relevant for the suggested portfolio applications.

**Lemma 3.2.2.** *If $f(x, y)$ is convex with respect to $x$, then $\phi_\alpha(x)$ is convex with respect to $x$ as well. Indeed, in this case $F_\alpha(x, \xi)$ is jointly convex in $(x, \xi)$. Likewise, if $f(x, y)$ is sublinear[1] with respect to $x$, then $\phi_\alpha(x)$ is sublinear with respect to $x$ and $F_\alpha(x, \xi)$ is jointly sublinear in $(x, \xi)$.*

### 3.2.2 CVaR portfolio model

The implications of Theorem 3.2.1 and Lemma 3.2.2 are of particular importance for portfolio optimization. Let us consider the case where $f$ is given by

$$f(x, y) = -(x_1 y_1 + \cdots + x_n y_n). \tag{3.2.10}$$

Then $f(x, y)$ represents a loss function equal to the negative portfolio profit, where $x$ are the shares invested into the assets with profit $y$. It should be noted that since in this case $f(x, y)$ is linear with respect to $x$, then $f$ is also sublinear and hence convex. Therefore, not only the $\alpha$-CVaR connected with the decision $x$ can be derived by the minimization of $F_\alpha(x, \xi)$ according to $\xi$ (Theorem 3.2.1), but Lemma 3.2.2 ensures that the minimized function is sublinear. Moreover, it also implies that the $\alpha$-CVaR as a function of $x$ is sublinear, so that for X compact the problem of finding the decision minimizing the risk in terms of CVaR is a convex optimization problem. The implications of using CVaR in portfolio optimization are explained in more detail in this section.

In the problems of optimization under uncertainty, there are two central approaches how to use CVaR. Firstly, it can enter the objective, where the optimization problem is to find a portfolio with a minimum CVaR (usually with a given minimum constraint on expected profits). Alternatively, it can be incorporated in the constraints. The optimization problem in the latter case is to find the portfolio maximizing expected profits provided a constraint

[1] A function $h(x)$ is sublinear if $h(x + x') \leq h(x) + h(x')$ and is positively homogeneous. A function is positively homogeneous if $h(\lambda x) = \lambda h(x)$ for $\lambda > 0$. Sublinearity is equivalent to the combination of convexity with positive homogeneity; see [58].

on the CVaR is satisfied. A considerable advantage of CVaR over VaR in that context is the preservation of convexity as seen in Lemma 3.2.2. In numerical applications, joint convexity of $F_\alpha(x,\xi)$ with respect to both $x$ and $\xi$ is even more valuable than convexity of $\phi_\alpha(x)$ in $x$. The reason can be seen in the following results:

**Theorem 3.2.2.** *Minimizing $\phi_\alpha(x)$ with respect to $x \in X$ is equivalent to minimizing $F_\alpha(x,\xi)$ over all $(x,\xi) \in X \times \mathbb{R}$, in the sense that*

$$\min_{x\in X} \phi_\alpha(x) = \min_{(x,\xi)\in X\times\mathbb{R}} F_\alpha(x,\xi), \tag{3.2.11}$$

*where moreover*

$$(x^*,\xi^*) \in \arg\min_{(x,\xi)\in X\times\mathbb{R}} \Leftrightarrow x^* \in \arg\min_{x\in X} \phi_\alpha(x), \xi^* \in \arg\min_{\xi\in\mathbb{R}} F_\alpha(x^*,\xi). \tag{3.2.12}$$

**Corollary 3.2.1.** *If $(x^*,\xi^*)$ minimizes $F_\alpha$ over $X \times \mathbb{R}$, then not only does $x^*$ minimize $\phi_\alpha$ over $X$, but also*

$$\xi_\alpha(x^*) \le \xi^*. \tag{3.2.13}$$

*In fact, $\xi_\alpha(x^*) = \xi^*$ if $\arg\min_\xi F_\alpha(x^*,\xi)$ reduces to a single point.*

The fact that minimization of CVaR does not have to be carried out numerically by repeated calculation of $\phi_\alpha(x)$ for various decisions $x$ means a powerful attraction to work with CVaR.

In the case when $\arg\min_\xi F_\alpha(x^*,\xi)$ does not consist of a single point (which could easily happen in case of $y$ being discretely distributed), the joint minimization does not immediately yield the $\alpha$-VaR associated with $x^*$. It should be noted, though, that in those circumstances $\arg\min_\xi F_\alpha(x^*,\xi)$ is the interval between two consecutive points $z_k$ in the discrete distribution of losses. In that case, therefore, $\xi_\alpha(x^*)$ can easily be obtained from the joint minimization as the highest $z_k \le \xi^*$.

The following results allow for the use of linear programming techniques for the double minimization in the case of portfolio optimization when $y$ has a discrete distribution. Let us consider a case where the decision vector $x = (x_1, x_2, \ldots, x_n)$ represents a portfolio of assets with $x_i$, i.e. $x_i \ge 0$ for $i = 1, 2 \ldots, n$ and $\sum_{i=1}^n x_i = 1$. By $y = (y^1, y^2, \ldots, y^n)$ we denote the random vector of returns of the assets $1, 2 \ldots, n$. The profit of the portfolio is then the sum of the profit of the individual assets multiplied by the proportions, expressed in terms of losses as

$$f(x,y) = -[x_1y_1 + \ldots x_ny_n] = -x^T y. \tag{3.2.14}$$

Let us assume the $y$ has a discrete probability space with elements $y_k$ (e.g. the distribution is empirical, scenario based), $k = 1, 2 \ldots, N$ having probabilities $p_k$. Then the problem of minimizing CVaR is equivalent to minimizing

$$F_\alpha(x, \xi) = \xi + \frac{1}{(1-\alpha)} \sum_{k=1}^{N} p_k[-x^T y_k - \xi]^+, \qquad (3.2.15)$$

over $X \times \mathbb{R}$. This means that the problem of finding the optimal portfolio that minimizes the CVaR can be reduced to linear programming. In terms of auxiliary variables $u_k$ for $k = 1, \ldots, N$ it is equivalent to minimizing

$$\xi + \frac{1}{(1-\alpha)} \sum_{k=1}^{N} p_k u_k \qquad (3.2.16)$$

subject to the linear constraints $x_i \geq 0$ for $i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} x_i = 1$ where $u_k \geq 0$ and

$$x^T y_k + \xi + u_k \geq 0 \qquad (3.2.17)$$

for $k = 1, 2, \ldots, N$.

As already mentioned, CVaR can enter the optimal portfolio selection not only in the objective. An alternative way is to introduce the CVaR in the form of a constraint in the portfolio optimization.

**Theorem 3.2.3.** *Let $g : X \mapsto \mathbb{R}$ and let $\alpha_i \in \mathbb{R}$, $\omega_i \in \mathbb{R}$, $i = 1, \ldots, l$. The problem to minimize $g(x)$ over $x \in X$ subject to the constraints $\phi_{\alpha_i} \leq \omega_i$, for $i = 1, 2, \ldots, l$ is equivalent to the problem to minimize $g(x)$ over $(x, \xi_1, \ldots, \xi_l) \in X \times \mathbb{R}^l$ satisfying $F_{\alpha_i}(x, \xi_i) \leq \omega_i$ for $i = 1, 2, \ldots, l$. In fact, $(x^*, \xi_1^*, \ldots, \xi_l^*)$ solves the second problem if and only if $x^*$ solves the first problem and the inequality $F_{\alpha_i}(x^*, \xi_i^*) \leq \omega_i$ holds for $i = 1, 2, \ldots, l$.*

*Moreover one has $\phi_{\alpha_i}(x^*) \leq \omega_i$ for every $i$ and actually $\phi_{\alpha_i} = \omega_i$ for each $i$ such that $F_{\alpha_i}(x^*, \xi_i^*) = \omega_i$ (i.e. such that the corresponding CVaR constraint is active).*

When $X$ and $g$ are convex, and $f(x, y)$ is convex in $x$, we know that the portfolio optimization is a problem of convex programming. In comparison, analogous problems where risk is represented in terms of VaR instead of CVaR could lead to non-convex optimization problems.

These results are relevant for portfolio application of CVaR, CVaR being defined for a loss distribution associated with a decision $x$. Sometimes, however, it is also of interest to report the CVaR associated with a single distribution. We will do so especially when comparing profit of different assets, reporting not only their mean and variance, but also the risk when measured by CVaR. Given a random variable $y$ representing profit, this measure is defined as the $\phi_\alpha(1)$, where the loss function is $f(x, y) = -y$. Similarly the VaR of a profit distribution is the $\alpha$-th percentile of the distribution $-y$. In cases when it is obvious we are interested in statistics of a single distribution, let us refer to these values as $CVaR(y)$, $VaR(y)$.

Therefore, the –VaR associated with a profit distribution $y$ represents the profit that can be secured at the confidence level $\alpha$ and the –CVaR the mean of the profits lower than –VaR. This explains why for profit distributions we will report –VaR and –CVaR (and for cost distribution their VaR and CVaR).

Another important feature of CVaR as a risk measure should be noted here. Whereas the variance of a profit distribution is independent of its mean, this is not the case of CVaR. In other words, let us consider a random variable $y$ representing profit. Then for any constant $k \in \mathbb{R}$, the variance of $y + k$ is equal to the variance of $y$. However, since CVaR is in principle the mean of the $\alpha$-tail of $-y$, then $CVaR(y + k) = CVaR(y) - k$. (This also follows from [61], where the $CVaR(X - EX)$ was shown to be an example of a general deviation measure of $X$). Therefore, risk when measured by CVaR reflects also the mean of the underlying distribution, a higher expected profit translating to a lower CVaR and vice-versa.

# Chapter 4

# Profit distributions analysis

In the previous chapter the main short-comings of the standard mean-variance portfolio framework have been described. We stated, that one of the major weaknesses lies in the assumption of the assets' returns being normally distributed. However, we did not provide any evidence to the contrary, i.e. that the profits of individual electricity generating technologies exhibit a distribution other than a normal one. The goal of this chapter is twofold - to describe the method for deriving the profit distributions and to investigate these profit distributions to provide a justification for the need of a different portfolio framework. The method presented in this chapter will be used to derive the distributions used in the portfolio models in this thesis. Consequently, we analyze the properties of the derived distributions, trying to show that these are not necessarily normal. This would corroborate the need, in case of portfolios of electricity generating technologies, of a portfolio framework possible to account also for non-normal distributions.

The framework described in the next section is primarily intended to derive the profit distribution created by the investment into a power plant of a specified technology.

Let us assume the investor decides to build a power plant of a given technology with a given capacity. The resulting total cost of the whole investment through the lifetime of the power plant depend not only on several uncertain factors, but also on the investor's response to them. In most cases the investor has some flexibility to optimize the operation of the power plant. Therefore, the derived cost should reflect the cost of an individual technology in case of optimal management under the underlying uncertainties. To account for this fact, we propose to derive it as a solution to an optimal investment and operation plan for a single representative cost-minimizing electricity producer.

# 4.1 Real Options model

## 4.1.1 Formulation

For the real options optimization we consider three technologies based on the fuel used - gas, coal and biomass. As already mentioned, each technology considered is analyzed separately.

We consider a producer who has to deliver a certain amount of electricity over the course of the planning period and faces a stochastic carbon dioxide price $P$. The technology used to produce electricity is fixed. The problem of the investor is to optimize the operation of the given power plant. Independent of the technology, the possible actions the producer can consider and optimize are the same - the investment into and further operation (switching on/off) of a CCS (carbon capture and sequestration) module. We assume the decisions can be taken on a yearly basis. Although this is a simplification, in case of real investments, as is the case of investment into a CCS module, this is not a major distortion of reality. The investor's problem can be formulated as the following optimal control problem:

$$\left.\begin{array}{lll} \min\limits_{a_t} & \mathbb{E}[\sum\limits_{t=0}^{T}\frac{1}{(1+r)^t}\pi(x_t,a_t,P_t)+c(a_t)] & \\ \text{s. t.} & x_{t+1}=x_t+a_t & \text{for } t=0,\cdots,T \\ & \ln(P_{t+1}/P_t)\sim N(\mu-\frac{\sigma^2}{2},\sigma^2) & \text{for } t=0,\cdots,T \\ & x_0=1 & \\ & P_0^c=P^0 & \\ & a_t\in A(x_t) & \text{for } t=0,\cdots,T, \end{array}\right\} \quad (4.1.1)$$

where $x_t$ is the state variable, $a_t$ the control variable, $\pi$ the yearly costs, $c$ the costs associated with the undertaken action, $r$ the discount rate, $\mu$ the drift and $\sigma$ the volatility parameter of the $CO_2$ price. The $CO_2$ price is assumed to follow a geometric Brownian motion. The control is considered a Markov control in the form of a feedback sequence [31], i.e. in the form of $a_t = a_0(t, x_t, P_t)$ for some function $a_0 : \mathbb{R}^3 \to \mathbb{R}$. The possible values of the control variable with the resulting costs are following:

| $a_t$ | description | $c(a_t)$ |
|---|---|---|
| 0 | take no action | zero costs |
| 2 | install the CCS module | costs of the CCS module |
| 1 | switch the module on | costs for switching |
| $-1$ | switch the module off | costs for switching |

The specific values of the costs depend on the technology analyzed and are specified in Section 2.2. The state variable describes whether the CCS module has been built and whether it is currently running, with $A(x_t)$ denoting the set of feasible controls for the given state:

| $x_t$ | description | $A(x_t)$ |
|---|---|---|
| 0 | the CCS module has not been installed yet | $\{0, 2\}$ |
| 1 | the CCS module has been installed but is not running | $\{0, 1\}$ |
| 2 | the CCS module has been installed and is running | $\{-1, 0\}$ |

The yearly costs consists of the cost of fuel, $CO_2$ expenses, operations and maintenance (O&M) costs

$$\pi(x, a, P) = q^f P^f + q^c(x+a)P + O\&M(x+a), \tag{4.1.2}$$

where $P^f$ is the fuel price and $q^c$, $q^f$ are the annual quantities of $CO_2$ emitted and fuel combusted, respectively. For all the technologies considered, we assume the planning horizon $T$ to be equal to thirty years, i.e. the the lifetime of the plant (that means the power plant is new at the beginning).

### 4.1.2 Solution Methods

As formulated, the problem is a discrete stochastic optimal control problem on a finite horizon. Because the performance criterion is bounded below and the development of the stochastic variable is independent of the state and control, the optimal control of the problem 4.1.1 exists and can be derived by dynamic programming [9, Corrolary 3.5.1]. That means the optimal control can be derived recursively by the Bellman equation

$$\begin{array}{rcl} V(T, x, P_t) & \equiv & 0 \\ V(t, x, P_t) & = & \min\limits_{a \in A(x)} \{\pi(x, a, P_t) + c(a) + \\ & + & (1+r)^{-1}\mathbb{E}_t[V(t+1, x+a, P)|x+a, P_t]\} \end{array} \tag{4.1.3}$$

as

$$\begin{array}{rl} a_0(t, x, P_t) & = \operatorname*{argmin}\limits_{a \in A(x)} \{\pi(x, a, P_t) + c(a) + \\ & +(1+r)^{-1}\mathbb{E}_t[V(t+1, x+a, P)|x+a, P_t]\}\}. \end{array} \tag{4.1.4}$$

The Bellman equation enables us to derive the value function backwards, determining the optimal actions at the same time; the first part of the value to be minimized are the immediate costs one would obtain upon undertaking action a, while the second part of the sum is the so-called continuation value, which represents the costs of the power plant from time $t$ until the end of the planning horizon, when it is managed optimally. There are several approaches how to calculate the continuation value in the Bellman equation 4.1.3. Since we assume the decisions can be done only at pre-specified points in time, the state does not change between them. Also, the distribution of the $CO_2$price is independent of the state and action chosen. Therefore, there is no problem with path-dependence and the continuation value can be derived numerically by the discretization of carbon price and Monte Carlo simulation.

An alternative approach is based on the financial option pricing theory. We can use the fact, that the value function between the two consecutive decision nodes can be calculated as a solution to a partial differential equation. The equation can be derived by applying the Ito Lemma to the value function as a function of carbon price between two consecutive decision nodes. Following [16] and [13], we can derive the partial differential equation, assuming there is no cashflow during the year except at the moment of choosing an action. Since the decision can carried out only at the pre-specified points in time, for a fixed state and action chosen at time $t$, the value function $V(t, x_{t+1}, P_t)$ is on the interval $(t, t+1)$ a function of time and carbon price only. Let us, for a fixed state $x$ and $\tau \in (t, t+1]$, denote $W_x(\tau, P) = V(\tau, x, P)$. The properties of $W_x(\tau, P)$ between on [t,t+1] are described by equation

$$rW_x = \frac{dW_x}{d\tau} + \mu P \frac{dW_x}{dP} + \frac{1}{2}\sigma^2 P^2 \frac{d^2 W_x}{dP^2} \tag{4.1.5}$$

with initial condition

$$W_x(t+1, P) = V(t+1, x, P) \tag{4.1.6}$$

and boundary condition

$$W_x(\tau, 0) = (1+r)^{\tau - t - 1} V(t+1, x, 0) \tag{4.1.7}$$

$$\tag{4.1.8}$$

for $\tau \in [t, t+1)$.

The boundary condition can be explained in the following way. Because the carbon price follows a GBM process, in case it is equal to zero, it is equal to zero at any point in time. Therefore, the costs in each decision node are deterministic and there is no effect of the carbon price on the value function. Consequently, the value function in that case depends on time only through discounting.

This means that starting with the terminal condition $V(T, x, P) = 0$ the optimal control can be computed recursively, where in each step (i.e. each decision node) the described partial differential equation has to be solved numerically for each state and action feasible and the Bellman equation is used to determine the value function in the previous decision node as

$$V(t,x,P_t) = \min_{a\in A(x)} \{\pi(x,a,P_t) + c(a) + W_{x+a}(t,P_t)\} \tag{4.1.9}$$

and the resulting optimal actions as

$$a_0(t,x,P_t) = \operatorname*{argmin}_{a\in A(x)} \{\pi(x,a,P_t) + c(a) + W_{x+a}(t,P_t)\}. \tag{4.1.10}$$

Both methods rely on numerical estimates of the continuation value. Both methods were tested and they delivered the same results. Although the method of using partial differential equations along with appropriate boundary conditions is mathematically the most elegant way, this approach has proven – once numerically implemented – computationally intensive and numerically unstable for higher values of carbon price volatility. A relatively fine price grid (for the discretization of the prices) is needed in order to obtain precise results. Moreover, this approach is less flexible to variations and extensions, indeed when using a different process for the prices a new differential equation needs to be derived. The advantage of the Monte Carlo approach is that it is relatively easy to alter and it can be used to look at less standard processes. Also, it has proven to remain efficient in this framework for a rather high degree of complexity and delivers the same results as the partial differential equations approach.

### 4.1.3 Types of Results

The output of the recursive optimization part is the optimal "strategy", i.e. the optimal control in the form of feedback sequence defined by the derived function $a_0(t,x,P)$. It is a multidimensional table, which lists the optimal action for each decision node,, for each possible state and for each possible carbon price in that period.[1] The output table can be regarded as a kind of "recipe" for the producer, so that in each decision node he knows what to do for each possible state occurring and for each possible realized price.

The optimal strategy does not show what properties do the realized decision have. For the analysis of the final outcome, we can then simulate

[1]Note that the price will be discretized, so if we talk about possible instances of the price, we mean each point in a grid between a pre-defined maximum and minimum price, where the latter are set in such a way that they encompass 95% of all simulated price paths.

(10,000) possible $CO_2$ price paths and extract the corresponding decisions from the output matrix (or the "recipe").

Eventually, we are interested in deriving the profit distribution (in terms of negative costs) representing the profitability of the investment into a power plant given that it is operated optimally in face of stochastic $CO_2$ and fuel prices. The real options model presented assumed only stochastic $CO_2$ prices. However, the fuel requirements of the power plant for a given technology are the same both for the power plant with and without the CCS module. Therefore, the costs for fuel are independent of the actions chosen. That means that the derived results are optimal for the case of stochastic fuel prices as well. This fact enables us to generate the cost distributions for an investor facing both stochastic $CO_2$ and fuel prices. We simulate 10,000 fuel price paths (assuming they follow a geometric Brownian motion with parameters specified in the previous section), which together with the optimal decisions $a_t$ are used to compute the total discounted cost for each simulation. The profit of each simulation is thus calculated as the negative of the sum of these costs and the capital cost needed for the installation of the power plant. The profit distribution used as the input for the portfolio model is given by the sample of profit for the 10,000 simulations.

In this way, the profit distributions for coal, gas and biomass technology are derived (for given parameters on fuel and $CO_2$ prices), the costs of the wind plant are independent of both stochastic processes and therefore the profit is deterministic, computed as the negative of the sum of capital cost and discounted operations and maintenance costs.

These distributions are used as input to portfolio models presented in Chapter 5 and Chapter 6, Section 6.3. For the models suggested in later sections, the distributions had to be adjusted.

### Distributions for the Robust portfolio model

For the portfolio model in section 6.4, the distributions need to distinguish the time structure of the profit flows. Whereas in the basic framework a technology was represented by a single distribution, in this case we generate a sequence of 5-year discounted profit distributions over the lifetime of the plant for each scenario and each technology under the assumption of annualized capital costs for all installations (i.e. the plant itself and also any retrofitted equipment such as the CCS module). These distributions are derived in the same way as the total cost distributions. To calculate these distributions, we use annualized capital costs, i.e. we assume the capital costs are distributed over the whole planning horizon, so that the sum of the discounted yearly payments is equal to the capital costs. The cost for each subperiod is discounted to the beginning of the subperiod, i.e. the first subperiod is discounted to year 0, the second to year 5 etc. In this way, the magnitudes between the periods become comparable, each representing the net profit of the operation of the power plant in the respective interval. In this case, one technology is represented by 6 profit distributions, each given by the distributions of profit in the individual subperiods.

**Distributions for the Dynamic portfolio model**

The dynamic portfolio model presented in Chapter 7 requires some additional modification in the input distributions. First, we derive return distributions instead of profit distributions. The return is calculated as the profit on a unit of investment, i.e. it is the quotient of the sum of the discounted operations profit (discounted to the time of investment) over the capital costs. The operations profit consists of the profit of producing electricity minus the operations costs.

Second, some adjustment of the parameters of the real options model is necessary. The planning horizon is extended, investigating the effect of investment that will be undertaken in future. The solution of the real options model is computed not only for each technology for the case presented (i.e. where $T = 30$), but also for cases where $T = 35$ and $T = 40$, where the power plant is installed only in year 5 and 10 respectively. In this way, we have a return distribution for each technology and each installation time.

## 4.2 Profit Distribution Analysis

The method for deriving the profit distribution of a given electricity generating technology was described in the previous sections. Using the data presented in Table 2.2 the method can be used to produce the profit distributions of the individual technologies. In this section we can finally analyze the outcome and present the properties of the derived distributions.

The results presented are for the coal, gas and biomass technology, using the B2 590ppm scenario for the carbon price (see Table A.1). The descriptive statistics of the distributions together with the estimated correlation are presented in Tables 4.1 and 4.2. A comprehensive summary of the statistics of the individual technology profit distributions for each socio-economic scenario and target can be found in Appendix B.

Two important facts can be observed in the presented tables. Firstly, for this scenario the most profitable technology is gas, followed by biomass and coal. Biomass is the technology with the largest variance of the profits, coal

| *Parameters* | *Biomass* | *Coal* | *Gas* |
|---|---|---|---|
| Mean | -4349.12 | -4656.28 | -3001.08 |
| Standard Deviation | 883.69 | 314.25 | 694.90 |
| Sample variance | 780899.64 | 98752.47 | 482890.15 |
| Kurtosis | 1.79 | 1.48 | 4.72 |
| Skewness | -0.90 | -0.81 | -1.61 |
| –CVaR | -6591.51 | -5443.00 | -4970.82 |

Table 4.1: *Descriptive statistics of the distributions derived by the Real Options model for the B2 590ppm scenario*

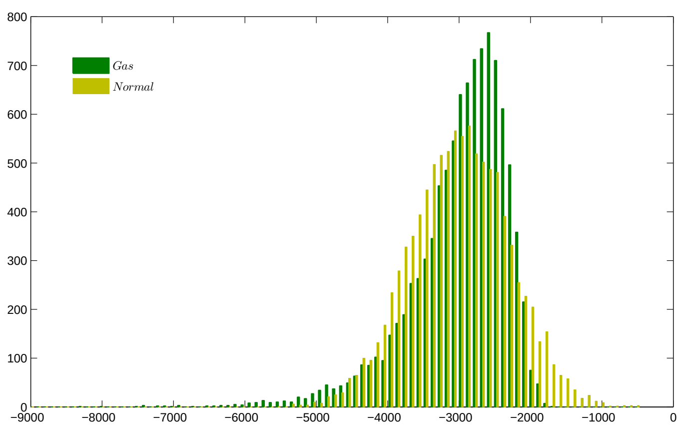


Figure 4.1: *Gas distribution for the B2 590ppm compared to a normal distribution with the same mean and standard deviation*

with the lowest and gas being in between. However, if we measure risk by the conditional Value-at-Risk, the relationship is different, with gas being the least risky one. This already suggests that the optimal combination of technologies based on CVaR could lead to significantly different conclusions as would be the case if based on variance. Secondly, the correlation between the individual technology chain profits is relatively small with biomass being negatively correlated with the fossil-fueled technologies. Both can be explained by the fact that the biomass power plant, in contrary to the other technology chains can actually gain from a stricter climate policy. This is due to the special feature of biomass being a zero-emission technology in the first place and a negative-emission technology upon addition of a carbon capture module, which will then capture a larger amount of CO2 than

| | *Biomass* | *Coal* | *Gas* |
|---|---|---|---|
| *Biomass* | 1 | -0.0904 | -0.0429 |
| *Coal* | -0.0904 | 1 | 0.0432 |
| *Gas* | -0.0429 | 0.0432 | 1 |

Table 4.2: *Estimated correlation between the distributions for the B2 590ppm scenario*

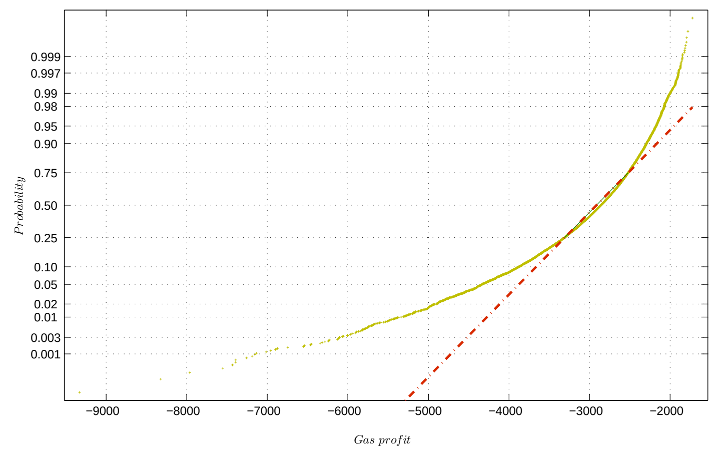


Figure 4.2: *Normal probability plot for the gas distribution for the B2 590ppm scenario*

generated by the combustion of biomass minus the amount sequestered by planting biomass as fuel. The rising $CO_2$ prices therefore lead not only to increasing profits, but also to the increasing volatility of the profit streams. On the contrary, the investment into the CCS module in case of a fossil-fueled technology leads to a sharp decrease in net emissions and therefore to smaller fluctuations caused by the carbon price. Since the effect of the $CO_2$ price on the biomass and fossil-fueled technology chains is contrasting, also the resulting correlation between them is negative. This correlation is not very significant, due to two reasons. Firstly, the fossil fueled technology can smooth out a significant amount of the carbon price uncertainty by the investment into the CCS module and secondly, the profit volatility also being caused by the uncertain fuel prices, which are assumed to be independent.

Already the skewness and kurtosis figures from Table 4.1 suggest that these distributions are not normally distributed. This suspicion is further corroborated by the normal probability plots of the individual distribution.

| | *Jarque-Bera test* | *Lillilifors test* |
|---|---:|---:|
| *Biomass* | 2683.70 | 0.0604 |
| *Coal* | 2008.7 | 0.0510 |
| *Gas* | 13552 | 0.0950 |
| Critical value | 5.98 | 0.0091 |

Table 4.3: *Normality test statistics for the distributions derived by the Real Options model for scenario B2 590ppm scenario.*

For a normal distribution the the normal probability plot should be a linear function, the result for the gas distribution is shown in Figure 4.2. Alternatively, a direct comparison of the gas distribution with a normal distribution with the same mean and standard deviation is presented in Figure 4.1. The normality was tested for all distributions using the standard normality tests - a Jarque-Bera [42] and Lilliefors test [44]. The Jarque-Bera test is a moment test of the null hypothesis that the sample comes from a normal distribution with unknown mean and variance, against the alternative that it does not come from a normal distribution. The Lilliefors test is a 2-sided goodness-of-fit test suitable when a fully-specified null distribution is unknown and its parameters must be estimated. It is an empirical distribution function test of the default null hypothesis that the sample in vector x comes from a distribution in the normal family, against the alternative that it does not come from a normal distribution. In all cases the null hypothesis was clearly rejected in both tests at the 5% significance level. The results of the performed tests are summarized in Table 4.3.

## 4.3 Conclusion

In this section we presented a method of how to derive the profit distributions for individual electricity generating technology chains. The distributions are in terms of negative costs. They represent the value of investment into a power plant of a given capacity, when the power plant is operated optimally. We assume the investor is facing stochastic carbon and fuel prices. Four different technologies are considered, coal and gas as the representatives of the fossil-fueled capacities and biomass and wind as the representatives of renewable technologies. The distributions are tested for normality and found to be non-normally distributed. This substantiates the need to use a different framework than the mean-variance for constructing energy portfolios.

# Chapter 5

# CVaR vs. M-V

## 5.1 Problem specification

First, let us formulate the general portfolio problem to be analyzed in this chapter. Let us consider $n$ assets, investment into asset $i$ yielding profit $y_i$. Here $y = (y_1, y_2, \dots, y_n)^T$ is a random vector with known distribution. Let us further denote $x_i$ the share of the asset $i$ in the portfolio. Let us consider short positions not to be allowed, therefore, the share invested into each asset cannot be negative. The problem is to find the optimal composition of investment, with risk as the objective and expected profit as constraint.

A *portfolio* is hence any $x \in \mathbb{R}^n$ satisfying $\sum_{i=1}^n x_i = 1$ and $x \geq 0$. The term *feasible portfolio* is used in this thesis always with respect to a specific portfolio problem and represents an arbitrary portfolio satisfying the constraint on the expected profit that is present in the portfolio problem considered.

We will analyze two different problems, depending on the measure of risk chosen. The first is the standard mean-variance Markowitz framework, where the objective is to minimize variance:

$$\left.\begin{array}{lll}
\min\limits_{x_i} & \sigma^2(x^T y) & \\
\text{s.t.} & \mu(x^T y) \geq R & \\
& \sum\limits_{i=1}^{n} x_i = 1 & \\
& x_i \geq 0 & i = 1, \dots, n,
\end{array}\right\} \tag{5.1.1}$$

where $\mu$ is used to denote the expected value, and $\sigma^2$ the variance.

The second framework uses the risk measure introduced in the previous chapter, conditional value-at-risk. Using the same notation, where the loss function of the portfolio is $-x^T y$, the problem can be formulated in the following way :

$$\left.\begin{array}{ll} \min\limits_{x_i} & \phi_\alpha(x) \\ \text{s.t.} & \mu(x^T y) \geq R \\ & \sum\limits_{i=1}^{n} x_i = 1 \\ & x_i \geq 0 \qquad\qquad i = 1, \ldots, n, \end{array}\right\} \tag{5.1.2}$$

where the CVaR is calculated for a given confidence level $\alpha$. It is important to note that the set of feasible portfolios is the same for both problems. Moreover, the set of feasible solutions is nonempty if and only if $R \leq \max_i \mu_i$. Therefore, in the following we will always assume $R \leq \max_i \mu_i$.

## 5.2 Optimal portfolios for normally distributed assets profit

The main focus of this thesis is on energy portfolios, where the profits of underlying assets were shown to be non-normally distributed. However, let us first investigate the case of normally distributed asset profits. This example is useful to demonstrate the difference between the CVaR and mean-variance portfolio approaches. In this section we will ultimately analyze the difference in the solutions $\bar{x}_{MV}$, $\bar{x}_{CVaR}$ of problems (5.1.1) and (5.1.2) with respect to the set of portfolios constituting the efficient frontier.

However, we do not do so immediately. First, we investigate the solutions to slightly modified problems, where the constraint on the expected profit is given by equality:

$$\left.\begin{array}{ll} \min\limits_{x_i} & \sigma^2(x^T y) \\ \text{s.t.} & \mu(x^T y) = R \\ & \sum\limits_{i=1}^{n} x_i = 1 \\ & x_i \geq 0 \qquad\qquad i = 1, \ldots, n, \end{array}\right\} \tag{5.2.3}$$

$$\left.\begin{array}{ll} \min\limits_{x_i} & \phi_\alpha(x) \\ \text{s.t.} & \mu(x^T y) = R \\ & \sum\limits_{i=1}^{n} x_i = 1 \\ & x_i \geq 0 \qquad\qquad i = 1, \ldots, n, \end{array}\right\} \tag{5.2.4}$$

The justification of this step will be evident later on. Now we will analyze the optimal portfolios for problems (5.2.3) and (5.2.4). Please note that the set of feasible portfolios is the same for both problems. Therefore, it is sufficient to refer to the elements of this set as feasible portfolios. Moreover, the set of feasible portfolios is nonempty if and only if $R \in [\min_i \mu_i, \max_i \mu_i]$. Therefore, when analyzing the solution of problems (5.2.3) and (5.2.4) we assume $R$ satisfies $R \in [\min_i \mu_i, \max_i \mu_i]$.

Consider all feasible portfolios for an arbitrary fixed profit $R$. An optimal portfolio is then defined as the one with the lowest variance among all portfolios with the same $R$. Let us denote the solution of the problem (5.2.3) as $\hat{x}_{MV}(R)$ and the resulting portfolio variance as $\overline{V}(R) = \sigma^2(\hat{x}_{MV}(R))$ respectively. It is important to realize that the $\overline{V}(R)$ is a convex function of $R$.

Similarly, let us denote the solution of (5.2.4) for given $R$ as $\hat{x}_{CVaR}(R)$ and the value of the objective attained in $\hat{x}_{CVaR}(R)$ as $\overline{CVaR}_\alpha(R) = \phi_\alpha(\hat{x}_{CVaR}(R))$.

Let us denote the mean and covariance matrix of $y$ by $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ and $\Sigma = (s_{ij})_{i,j=1}^n$ respectively, where $s_{ij} = \sigma_i \sigma_j \rho_{ij}$ for $i \neq j$ and $s_{ij} = \sigma_i^2$ otherwise, i.e.$\sigma_i^2$ is the variance of $y_i$ and $\rho_{ij}$ the correlation between $y_i$ and $y_j$.

Let us denote $\psi_{\mu,\sigma^2}$, $\Psi_{\mu,\sigma^2}$ the density and distribution function of $N(\mu, \sigma^2)$ respectively.

The following result is crucial for the comparison of $\hat{x}_{MV}(R)$ and $\hat{x}_{CVaR}(R)$, as it reveals the relationship between CVaR and variance for assets profit being normally distributed.

**Lemma 5.2.1.** *Let $y$ be normally distributed, $y \sim N(\mu, \Sigma)$. Then*

$$\phi_\alpha(x) = -x^T\mu + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha} x^T \Sigma x.$$

*Proof:* According to Theorem 3.2.2

$$\phi_\alpha(x) = \min_\xi F_\alpha(x, \xi), \tag{5.2.5}$$

where

$$F(x, \alpha) = \xi + \frac{1}{1-\alpha} E[(-x^T y - \xi)^+]. \tag{5.2.6}$$

Since $y$ is normally distributed, the profit $z = x^T y$ of a portfolio defined by the shares $x$ is also normally distributed. Let us denote $m = \mu(z) = x^T\mu$ the mean and $v^2 = \sigma^2(z) = x^T \Sigma x$ the variance of $z$. Then

$$\begin{aligned}
E[(-x^T y - \xi)^+] &= \int_{-\infty}^{-\xi} (-z-\xi) \frac{1}{\sqrt{2\pi v^2}} e^{-\frac{(z-m)^2}{2v^2}} dz \\
&= \int_{-\infty}^{-\xi} (-z+m-m-\xi) \frac{1}{\sqrt{2\pi v^2}} e^{-\frac{(z-m)^2}{2v^2}} dz \\
&= \frac{v}{\sqrt{2\pi}} [e^{-\frac{(z-m)^2}{2v^2}}]_{-\infty}^{-\xi} + (-m-\xi)\Psi_{m,v^2}(-\xi) \\
&= v\psi_{0,1}(\frac{-\xi-m}{v}) + (-m-\xi)\Psi_{0,1}(\frac{-\xi-m}{v})
\end{aligned}$$

Since $F_\alpha(x,\xi)$ is convex (Lemma 3.2.2), $\phi_\alpha(x) = \min_\xi F_\alpha(x,\xi)$ is attained at $\xi$ solving the first order condition

$$\frac{\partial F_\alpha(x,\xi)}{\partial \xi} = 0. \tag{5.2.7}$$

From 5.2.7 it follows

$$\begin{aligned}
\frac{\partial F_\alpha(x,\xi)}{\partial \xi} &= 1 - \frac{1}{1-\alpha}\frac{\xi+m}{v}\psi_{0,1}(\frac{-\xi-m}{v}) + \\
&\quad + \frac{1}{1-\alpha}\frac{\xi+m}{v}\psi_{0,1}(\frac{-\xi-m}{v}) - \frac{1}{1-\alpha}\Psi_{0,1}(\frac{-\xi-m}{v}) \\
&= 1 - \frac{1}{1-\alpha}\Psi_{0,1}(\frac{-\xi-m}{v})
\end{aligned}$$

Therefore the $\min_\xi F_\alpha(x,\xi)$ is attained for $\xi = -m - v\Psi_{0,1}^{-1}(1-\alpha)$ and

$$\begin{aligned}
\phi_\alpha(x) &= F_\alpha(x, -m - v\Psi_{0,1}^{-1}(1-\alpha)) \\
&= \xi + \frac{v}{1-\alpha}\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha)) + (-m-\xi) \\
&= -m + \frac{v}{1-\alpha}\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha)) \\
&= -x^T\mu + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha} x^T \Sigma x.
\end{aligned}$$

□

In other words, Lemma 5.2.1 reveals that for normally distributed asset profits

$$\phi_\alpha(x) = -\mu(x^T y) + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}\sigma^2(x^T y) \tag{5.2.8}$$

Since for any feasible portfolio $\mu(x^T y) = R$, the problem (5.2.4) is for normally distributed assets equivalent to:

$$\left.\begin{array}{lll} \min\limits_{x_i} & \sigma^2(x^T y) & \\ \text{s.t.} & \mu(x^T y) = R & \\ & \sum\limits_{i=1}^{n} x_i = 1 & \\ & x_i \geq 0 & i = 1, \ldots, n. \end{array}\right\} \tag{5.2.9}$$

This only implies following:

**Theorem 5.2.1.** *For normally distributed assets the solution $\hat{x}_{MV}(R)$ of the problem (5.2.3) is equal to the solution $\hat{x}_{CVaR}(R)$ of problem (5.2.4) for any $R \in [\min_i \mu_i, \max_i \mu_i]$.*

Although it may seem that Theorem 5.2.1 proves, that for normally distributed assets are both frameworks equivalent, this is not the case. For the derived result it was crucial to formulate the problem with the constraint on the expected profit given by equality. However, in reality it is more sensible to formulate the problems with the constraint in the form of equality, i.e. as problems (5.1.1) and (5.1.2).

We will show that for this formulation the optimal portfolios of the two frameworks are not necessarily the same.

First, we need to prove the following result:

**Lemma 5.2.2.** *The function $\overline{CVaR}_\alpha$ is convex in $R$.*

*Proof:* Let $R_1, R_2 \in \mathbb{R}$. Then $x = \lambda \hat{x}_{CVaR}(R_1) + (1-\lambda)\hat{x}_{CVaR}(R_2)$ is a feasible portfolio for problem (5.2.4) for $R = \lambda R_1 + (1-\lambda)R_2$. Moreover,

$$\begin{aligned} \overline{CVaR}_\alpha(R) &\leq \phi_\alpha(x) \leq \lambda\phi_\alpha(\hat{x}_{CVaR}(R_1)) + (1-\lambda)\phi_\alpha(\hat{x}_{CVaR}(R_2)) \\ &= \lambda\overline{CVaR}_\alpha(R_1) + (1-\lambda)\overline{CVaR}_\alpha(R_2). \end{aligned}$$

where the first inequality holds because $\overline{CVaR}_\alpha(R)$ is minimal among feasible portfolio and the second follows from the convexity of $\phi_\alpha$ in $x$. $\square$

Therefore, $\overline{CVaR}_\alpha(R)$ is a convex function defined on the compact interval $[\min_i \mu_i, \max_i \mu_i]$. Hence, $\hat{R}_{CVaR} = \arg\min_R \overline{CVaR}_\alpha(R)$ exists, and $\overline{CVaR}_\alpha(R)$ is a decreasing for $R \in [\min_i \mu_i, \hat{R}_{CVaR}]$ and increasing for $R \in [\hat{R}, \max_i \mu_i]$. Therefore, the solution $\bar{x}_{CVaR}$ of the problem (5.1.2) is given by

$$\bar{x}_{CVaR}(R) = \begin{cases} \hat{x}_{CVaR}(\hat{R}_{CVaR}) & R < \hat{R}_{CVaR} \\ \hat{x}_{CVaR}(R) & R \geq \hat{R}_{CVaR} \end{cases} \tag{5.2.10}$$

Moreover, $\hat{x}_{CVaR}(\hat{R}_{CVaR})$ is the solution of a portfolio selection problem without any constraint on the expected profit:

$$\left.\begin{array}{lll} \min\limits_{x_i} & \phi_\alpha(x) & \\ \text{s.t.} & \sum\limits_{i=1}^{n} x_i = 1 & \\ & x_i \geq 0 & i = 1, \ldots, n. \end{array}\right\} \tag{5.2.11}$$

Similarly, for the mean-variance portfolio framework. $\overline{V}(R)$ is convex in $R$, $R \in [\min_i \mu_i, \max_i \mu_i]$. Thus, there exists $\hat{R}_{MV} = \arg\min_R \overline{V}(R)$. Furthermore, the solution $\bar{x}_{MV}$ of the portfolio problem (5.1.1) is given by

$$\bar{x}_{MV}(R) = \begin{cases} \hat{x}_{MV}(\hat{R}_{MV}) & R < \hat{R}_{MV} \\ \hat{x}_{MV}(R) & R \geq \hat{R}_{MV} \end{cases} \tag{5.2.12}$$

Additionally, $\hat{x}_{MV}(\hat{R}_{MV})$ is the solution of

$$\left.\begin{array}{lll} \min\limits_{x_i} & \sigma^2(x^T y) & \\ \text{s.t.} & \sum\limits_{i=1}^{n} x_i = 1 & \\ & x_i \geq 0 & i = 1, \ldots, n. \end{array}\right\} \tag{5.2.13}$$

The relationship between $\hat{R}_{MV}$ and $\hat{R}_{CVaR}$ is described by the following Lemma:

**Lemma 5.2.3.** $\hat{R}_{MV} \leq \hat{R}_{CVaR}$

*Proof:* Proof by contradiction. Let us assume $\hat{R}_{MV} > \hat{R}_{CVaR}$. Using the expression for CVaR from Lemma 5.2.1 we get

$$\begin{aligned} \overline{CVaR}_\alpha(\hat{R}_{CVaR}) &\leq \overline{CVaR}_\alpha(\hat{R}_{MV}) \leq \phi_\alpha(\hat{x}_{MV}(\hat{R}_{MV})) \\ &= -\hat{x}_{MV}(\hat{R}_{MV})^T\mu + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}\sigma(\hat{x}_{MV}(\hat{R}_{MV})) \\ &= -\hat{R}_{MV} + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}\sigma(\hat{x}_{MV}(\hat{R}_{MV})) \\ &< -\hat{R}_{CVaR} + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}\sigma(\hat{x}_{CVaR}(\hat{R}_{CVaR})) \\ &= -\hat{x}_{CVaR}(\hat{R}_{CVaR})^T\mu + \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}\sigma(\hat{x}_{CVaR}(\hat{R}_{CVaR})) \\ &= \phi_\alpha(\hat{x}_{CVaR}(\hat{R}_{CVaR})) = \overline{CVaR}_\alpha(\hat{R}_{CVaR}) \end{aligned}$$

where the first inequality follows from the definition of $\hat{R}_{CVaR}$, the second from definition of $\overline{CVaR}_\alpha$ and the last inequality stems from both the definition of $\hat{R}_{MV}$ and from the assumption $\hat{R}_{MV} > \hat{R}_{CVaR}$. □

Although this proves, that $\hat{R}_{MV} \leq \hat{R}_{CVaR}$, it does not specify, whether there exist a case where $\hat{R}_{MV} < \hat{R}_{CVaR}$. The following Lemma illustrates, that such a case is indeed possible.

**Lemma 5.2.4.** *If $\hat{x}_{MV}(\hat{R}_{MV}) > 0$ and $\mu_i \neq \mu_j$ for some $i, j$, then $\hat{R}_{MV} \neq \hat{R}_{CVaR}$.*

*Proof:* Proof by contradiction. Let us assume $\hat{R}_{MV} = \hat{R}_{CVaR}$. By Lemma 5.2.1, we have $\hat{x}_{CVaR}(\hat{R}_{CVaR}) = \hat{x}_{MV}(\hat{R}_{MV})$.

For $z = (z_1, \dots z_{n-1})^T \in \mathbb{R}^{n-1}$ let us denote $\eta_{MV}(z) = \sigma^2(x^T y)$, where $x = (z_1, \dots, z_{n-1}, 1 - \sum_{i-1}^n z_i)$. Further let us by $\hat{z}_i$ denote the $i$-the coordinate of $\hat{x}_{MV}(\hat{R}_{MV})$, $i = 1, \dots n-1$. We know that $\hat{x}_{MV}(\hat{R}_{MV})$ is the solution of (5.2.13), therefore $\hat{z}$ is the solution of

$$\min_{z \in Z} \eta_{MV}(z) \tag{5.2.14}$$

where $Z = \{z \in \mathbb{R}^{n-1} : z \geq 0, \sum_{i=1}^n z_i \leq 1\}$. The assumption $\hat{x}_{MV}(\hat{R}_{MV}) > 0$ implies that the minimum of $\eta_{MV}(z)$ on $Z$ is attained in the interior of $Z$ and therefore, $\hat{z}$ is a stationary point of $\eta_{MV}(z)$.

Similarly, let us denote $\eta_{CVaR}(z) = \phi_\alpha(x)$, where $x = (z_1, \dots, z_{n-1}, 1 - \sum_{i-1}^n z_i)$. By analogy, $\hat{z}$ is also a stationary point of $\eta_{CVaR}(z)$.

By Lemma 5.2.1 we have

$$\eta_{CVaR}(z) = -\mu_n - z^T \zeta + k \eta_{MV}(z) \tag{5.2.15}$$

where $k = \frac{\psi_{0,1}(\Psi_{0,1}^{-1}(1-\alpha))}{1-\alpha}$ and $\zeta = (\mu_1 - \mu_n, \dots, \mu_{n-1} - \mu_n)$. Thus

$$\begin{aligned} 0 &= \frac{\partial \eta_{CVaR}(z)}{\partial z}|_{z=\hat{z}} && (5.2.16) \\ &= -\zeta + k \frac{\partial \eta_{MV}(z)}{\partial z}|_{z=\hat{z}} && (5.2.17) \\ &= -\zeta && (5.2.18) \end{aligned}$$

This implies $\mu_i = \mu_n$ for any $i$, which yields the contradiction. □

Note, that this condition is fulfilled for example in case $\Sigma = \sigma^2 I$, $\mu \neq 0$, since for independent assets with the same variance the share of each asset in $\hat{x}_{MV}(\hat{R}_{MV})$ is equal to $\frac{1}{n}$.

Lemma 5.2.4 implies, that if all assets are present in $\hat{x}_{MV}(\hat{R}_{MV})$ and the expected profit of the underlying assets is not equal, then $\hat{R}_{MV} < \hat{R}_{CVaR}$. And by 5.2.10 and 5.2.12 we have that for $R < \hat{R}_{CVaR}$ portfolio $\bar{x}_{MV}(R)$ yields profit $\max(R, \hat{R}_{MV})$ whereas the profit of $\bar{x}_{CVaR}(R)$ is $\hat{R}_{CVaR}$. Thus,

$$\bar{x}_{MV}(R) \neq \bar{x}_{CVaR}(R). \tag{5.2.19}$$

Lemma 5.2.3 and 5.2.4 reveal an interesting observation. Let us recall, that $\hat{x}_{MV}(R)$, $\hat{x}_{CVaR}(R)$ are the solutions of (5.2.3) and (5.2.4) respectively, with $\overline{V}(R)$, $\overline{CVaR}_{\alpha}(R)$ being the values of the objective attained it the optimal solution. In turn, $\hat{R}_{MV}$ and $\hat{R}_{CVaR}$ are the arguments, where the global minimum of $\overline{V}(R)$ and $\overline{CVaR}_{\alpha}(R)$ with respect to R is attained, respectively. This result, therefore, reveals that even though the optimal shares of (5.2.3) and (5.2.4) are the same, the the global minimum of the values of the objective can be attained for different arguments.

Furthermore, since both $\hat{x}_{MV}(\hat{R}_{MV})$ and $\hat{x}_{CVaR}(\hat{R}_{CVaR})$ are the solution to the unconstrained portfolio problems (5.2.13), (5.2.11) respectively, they represent the optimal portfolios for investors that choose the portfolio with minimum risk without any concern about the resulting profit. The Lemma 5.2.3 implies, that decisions based on CVaR will always be superior to those based on variance with respect to expected profit.

These results are of particular importance for the comparison of the efficient frontiers of both portfolio frameworks. For the mean-variance framework, the term "efficient frontier" is closely linked to the solutions of problem (5.1.1). Every possible asset combination can be plotted in the variance-profit space, and the collection of all such possible portfolios defines a region in this space. The line along the upper boundary of this region is known as the efficient frontier (sometimes "the Markowitz frontier"). In other words, the efficient frontier is the graph drawn in the variance-profit space representing a set of portfolios for which one cannot improve both risk and profit. We will investigate the set of portfolios constituting the efficient frontier, which we denote by $EF_{MV}$. From the definition of the efficient frontier, $EF_{MV}$ is a set consisting of the solutions to the problem (5.1.1) for varying $R$, i. e. $EF_{MV} = \{\hat{x}_{MV}(\hat{R}), R \geq \hat{R}_{MV}\}$. Similarly, the efficient frontier for the CVaR framework can be defined as a graph depicting the set of the solutions ($EF_{CVaR}$) to the problem (5.1.2) in the CVaR-profit space, i.e. $EF_{CVaR} = \{\hat{x}_{CVaR}(\hat{R}), R \geq \hat{R}_{CVaR}\}$. We conclude this section with the relationship between the efficient frontiers $EF_{MV}$, $EF_{CVaR}$.

**Theorem 5.2.2.** *For normally distributed assets* $EF_{CVaR} \subset EF_{MV}$.

*Proof:* This follows directly. Let $x \in EF_{CVaR}$, then $x = \hat{x}_{CVaR}(\bar{R})$ for some $\bar{R} \geq \hat{R}_{CVaR}$. According to Theorem 5.2.1, $\hat{x}_{MV}(\bar{R}) = \hat{x}_{CVaR}(\bar{R})$. And according to Lemma 5.2.3 $\bar{R} \geq \hat{R}_{CVaR} \geq \hat{R}_{MV}$, which implies $x = \hat{x}_{MV}(\bar{R}) \in EF_{MV}$. $\square$

The results imply that although the solutions of problems (5.2.3) and (5.2.4) are the same for any choice of $R$ , this does not necessarily hold for the solutions of (5.1.1) and (5.1.2) in case of normally distributed assets profit.

## 5.3 Energy Portfolios

The idea of the framework proposed in this section is to combine portfolio optimization with the results derived by the real options model from Chapter 4. The general concept and the underlying assumptions have already been explained in Chapter 2. Let us recall, that the portfolio model is supposed to provide the optimal energy mix under the optimal operation of individual power plants. That is, we use the real options model to find the optimal operation strategy for a given technology chain and its implied profit distribution; the profit distribution is then employed as the input into the portfolio optimization.

We first present the formulation of the CVaR portfolio model to be used to compute the results presented afterwards. Then we discuss the difference between the mean variance framework and the suggested model in case of investment into electricity generating capacities, using the distributions produced by the real options model. Finally, we use the framework to illustrate the impact of different climate policy assumptions on the resulting optimal energy mix.

### 5.3.1 Portfolio model formulation

Let us consider $n$ different assets, which in this case represent the different power generating capacities (e.g. coal plus CCS as the first "chain", biomass plus CCS as the second one, wind as a single technology etc). The profits of the assets are a random vector $y \in \mathbb{R}^n$ with the discrete uniform distribution over values $y^k$, $k = 1, 2, \ldots, N$, where $y^k = (y_1^k, y_2^k, \ldots, y_n^k)^T \in \mathbb{R}^n$. We describe the investment strategy by vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, the scalar value $x_1$, $i = 1, \ldots, n$ standing for the portion of capital invested into the technology chain $i$. Since the assets represents real power plants, we assume that no short positions are possible. A portfolio it thus any element of set $X$, where

$$X = \{x \in \mathbb{R}^n; \sum_{i=1}^{n} x_i = 1, x \geq 0 \text{ for } i = 1, 2, \ldots, n\}. \tag{5.3.20}$$

The profit function $x^T y$ depends on the chosen investment strategy and on the profit of the underlying assets. Because the actual value of the profits is unknown, there is some risk associated with each investment strategy $x$. The investor chooses the "best" portfolio as the one minimizing the risk. As the measure of risk we employ the conditional Value-at-Risk for the loss function $f(x, y)$ defined as negative profits $f(x, y) = -x^T y$. According to Theorem 3.2.2, the problem of minimizing CVaR with respect to the investment strategy is equivalent to the problem of minimizing $F_\alpha(x, \xi)$ with respect to both $x$ and $\xi$. By (3.2.15) for discretely distributed $y$ and $f(x, y)$ given by the portfolio loss function we have

$$F_\alpha(x, \xi) = \xi + \frac{1}{N(1-\alpha)} \sum_{k=1}^{N} (-x^T y^k - \xi)^+. \tag{5.3.21}$$

Let us denote by $m = \frac{1}{N} \sum_{k=1}^{N} y^k$ the vector of expected profit of the individual assets. The problem to find a portfolio minimizing the conditional Value-at-Risk of its losses given a constraint $R$ on its expected profit can be according to 3.2.16 formulated as follows:

$$\begin{array}{lll}
\min\limits_{(x,\xi,u_k)} & \xi + \frac{1}{N(1-\alpha)} \sum\limits_{k=1}^{N} u_k & \\
\text{s.t.} & \sum_{i=1}^{n} x_i = 1 & \\
 & x^T y^k + \xi + u_k \geq 0, & k = 1, 2, \ldots, N \\
 & x^T m \geq R & \\
 & u_k \geq 0, & k = 1, 2, \ldots, N \\
 & x_i \geq 0 & i = 1, 2, \ldots, n
\end{array} \tag{5.3.22}$$

Part $(x^*, \xi^*)$ of the solution of the LP problem (5.3.22) yields the optimal investment strategy $x^*$ with minimal $\alpha$-CVaR. $\xi^*$ is the corresponding threshold of the loss function and an upper bound of VaR. Problem (5.3.22) is a linear programming (LP) problem, with $N + n + 1$ variables, and $N + 1$ and constraints. For the applications presented we use $N = 10,000$ and $n \in \{3, 4\}$. The solution of this problem is computed with the help of GAMS using the simplex CPLEX solver.

### 5.3.2 Comparison with the MV framework

Before presenting the overall results of optimal portfolio composition involving all technologies for different $CO_2$ price scenarios, let us first compare the the model with the results of the mean- variance framework.

Consider only three technology chains for now - coal, gas and biomass (each with the option to add a CCS module). Using the price parameters for

| | $\hat{R}$ | *Gas share* | *Bio share* | *Coal share* |
|---|---|---|---|---|
| Variance | -4387.8 | 13.99 % | 11.99 % | 74.01 % |
| Conditional Value-at-Risk | -4003.1 | 36.47 % | 16.14 % | 47.40 % |

Table 5.1: *Comparison of MV and CVaR approach. Shares and expected profit of the optimal portfolios for the unconstrained problem.*

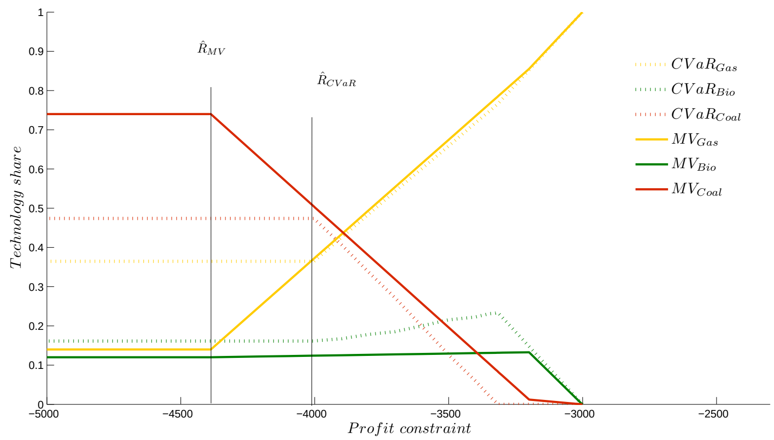


Figure 5.1: *Technology shares - MV and CVaR approach comparison*

the 'B2' 590 ppm scenario, we use the real options model from Chapter 4 to produce the profit distribution for each technology. The descriptive statistics for these distributions were presented in Section 4.2. Let us recall that coal was characterized by both the lowest mean and lowest variance, gas by the highest mean and biomass by the highest variance.

The estimated mean and variance (Table 4.1) of the assets' profit are used as the parameters for the mean-variance portfolio problem (5.1.1). For a range of constraints $R$ on the expected profit $\hat{x}_{MV}(R)$ is calculated analytically according to Appendix A. Also, the solution $\hat{x}_{MV}(\hat{R}_{MV})$ of the unconstrained problem (5.2.13) and the resulting expected portfolio profit $\hat{R}$ is derived. For the same set of constraints we compute the solution $\hat{x}_{CVaR}(R)$ of the CVaR portfolio optimization problem (5.3.22) using the distributions from the real options model. Similarly, also the solution of the unconstrained CVaR optimization problem $\hat{x}_{CVaR}(\hat{R}_{CVaR})$ and the resulting $\hat{R}_{CVaR}$ are computed.

Table 5.1 summarizes the results for unconstrained models, showing both

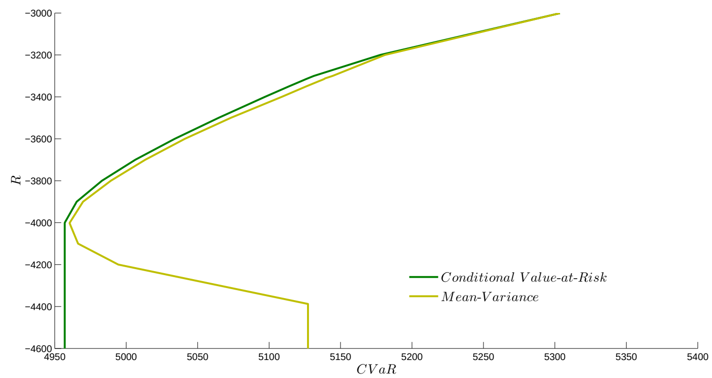


Figure 5.2: *The efficient frontier - MV and CVaR approach comparison*

the the shares and the expected profit.

For the unconstrained problems the difference in the resulting portfolios is significant. We observe a similar feature to the case of normally distributed profits, namely that $\hat{R}_{CVaR} \geq \hat{R}_{MV}$. This means that in case the expected profit constraint is not present in the optimization problem, the investor using CVaR is rewarded by a higher expected profit than the one using the mean-variance framework Apart from this, we see that the shares of individual technologies differ noticeably. Whereas the mean-variance portfolio relies heavily on coal, in case of CVaR one third of the portfolio is constituted of gas. Biomass does not play a major role in either case, which is caused by the extremely high variance without any significant profit. The shift from coal to gas can be explained easily. As has been argued in Chapter 4 (Table 4.1), the gas distribution is characterized by the lowest risk in terms of CVaR, whereas coal is characterized by the lowest variance.

The optimal portfolios for CVaR portfolio problem(5.3.22) and the mean-variance portfolio problem (5.1.1) are shown in Table 5.2 for a range of constraints $R$ on the minimum expected profit. These are also depicted on Figure 5.1.

We see that as the constraint on the minimum expected profit increases, the results of both approaches grow more similar, as was the case for normally distributed asset profits. This is caused partly by the fact, that we consider only three technologies, which implies that as the constraint gets binding ($R \geq \hat{R}_{CVaR}$), the share of one technology is given by the shares of the other two so that the constraint on expected profit is met. However, whereas for normal distributions the optimal portfolios were equal for $R \geq \hat{R}_{CVaR}$, in this case a noticeable discrepancy can be observed.

| $R$ | $CVaR_{Gas}$ | $CVaR_{Bio}$ | $CVaR_{Coal}$ | $MV_{Gas}$ | $MV_{Bio}$ | $MV_{Coal}$ |
|---|---|---|---|---|---|---|
| -3001.07 | 100.00% | 0.00% | 0.00% | 100.00% | 0.00% | 0.00% |
| -3300 | 77.83% | 22.18% | 0.00% | 79.50% | 13.17% | 1.21% |
| -3316 | 76.64% | 23.36% | 0.00% | 78.53% | 13.15% | 7.34% |
| -3317 | 76.57% | 23.41% | 0.03% | 78.47% | 13.15% | 8.38% |
| -3350 | 74.63% | 23.10% | 2.27% | 76.49% | 13.11% | 10.40% |
| -3400 | 71.77% | 22.25% | 5.98% | 73.48% | 13.06% | 13.47% |
| -3600 | 60.11% | 19.99% | 19.90% | 61.43% | 12.84% | 25.72% |
| -3800 | 48.43% | 17.81% | 33.76% | 49.39% | 12.63% | 37.98% |
| -4003.13 | 36.47% | 16.14% | 47.40% | 37.16% | 12.41% | 50.43% |
| -4100 | 36.47% | 16.14% | 47.40% | 31.32% | 12.30% | 56.37% |
| -4387.8 | 36.47% | 16.14% | 47.40% | 13.99% | 11.99% | 74.01% |

Table 5.2: *Comparison of MV and CVaR approach. Shares and expected profit of the optimal portfolios depending on the constraint on the expected profit.*

However, the shares are not the only important result of the optimization frameworks. Comparison of the CVaR attained in the individual optimal portfolios is at least as much important. It shows how much risk can be avoided by employing the more appropriate framework. Figure 5.2 depicts the relationship between the profit constraints and the resulting risk of the optimal portfolio measured by CVaR for the two approaches. The CVaR approach naturally shows better performance, since the objective of the mean-variance approach is different. On the other hand, we also see that in case $R \geq \hat{R}_{CVaR}$, this effect is almost negligible. Therefore, the difference between the results these two frameworks deliver, is significant only for investors, who care mostly about risk and much less about the profits gained by the investment.

### 5.3.3 Climate Policy impact

Now let us analyze the optimal portfolios for different climate policy scenarios, using the full set of technologies for all climate policy scenarios. In this case we introduce the constraint on the use of renewables that was already mentioned in Section 2.4. The renewables share (i.e. wind and biomass) is limited to 50%, because of spatial constraints. Whereas in Section 5.3.2 we investigated the effect of different profit constraints, in this section we focus on the composition of the portfolio under different levels of $CO_2$ price. We compare the results of the $CVaR_{97}$ portfolio model without the constraint on

| *Scenario* | *Target* | *Exp. Profit* | *Gas* | *Bio* | *Wind* | *Coal* |
|---|---|---|---|---|---|---|
| 'B1' | 590 | -4041.76 | 13.71% | 0.00% | 50.00% | 36.29% |
| 'B2' | 590 | -4126.41 | 19.90% | 1.05% | 48.95% | 30.10% |
| 'A2r' | 590 | -4251.33 | 21.47% | 10.03% | 39.97% | 28.53% |
| 'B1' | 520 | -4265.50 | 18.27% | 9.74% | 40.26% | 31.73% |
| 'B2' | 520 | -4013.70 | 25.09% | 16.58% | 33.4% | 24.91% |
| 'A2r' | 520 | -3594.66 | 50.00% | 50.00% | 0.00% | 0.00% |
| 'B1' | 480 | -1314.34 | 50.00% | 50.00% | 0.00% | 0.00% |
| 'B2' | 480 | -2246.74 | 50.00% | 50.00% | 0.00% | 0.00% |
| 'A2r' | 480 | 193.27 | 50.00% | 50.00% | 0.00% | 0.00% |

Table 5.3: *Technology shares given by the solution of the basic model across different climate policy scenarios*

the expected profit, representing an investor interested only in minimizing the risk of investment.

To better understand the results, let us recall that whereas the variance of a profit distribution is independent of its mean, this is not the case of CVaR. Therefore, even though we analyze the results of the CVaR portfolio model without the constraint on expected profit, the expected profit is still accounted for in the objective, a higher expected profit translating to a lower CVaR.

The optimal portfolio composition for each socio-economic scenario and emission concentration target is presented in Table 5.3. Let us recall, that targets measure the stringency of the chosen climate policy. A higher target represents a more lenient policy, whereas lower values lead to a higher $CO_2$ price. Therefore, it is natural that the portfolio moves to less $CO_2$ intensive technologies as the target decreases. We see that in all cases the constraint on the renewables is binding, with biomass playing the leading role for strict targets. We also see the shift from coal fired powerplants to gas as for higher $CO_2$ prices.

It may seem somewhat counterintuitive, though, that the expected profit of the optimal portfolio is higher for stricter targets. This is caused by the combination of two facts. First, a biomass power plant gets significantly more profitable for higher $CO_2$ costs due to its negative emissions property. Second, although the the profit of the fossil-fueled technologies suffers if the policy is strict, the difference in profits is relatively small compared to biomass, since with a CCS module the $CO_2$ emissions drop significantly.

We noted, that although we do not introduce a constraint on the expected profit, the CVaR itself accounts for it. This explains why biomass constitutes such a substantial part of the optimal portfolio for the stricter targets. Although this gain is also reflected in the variance increase, the effect on risk in terms of CVaR is not as pronounced because of the significant expected profit.

When comparing different socio-economic scenarios, we see that the profits and composition of the portfolio for the 'A2r' are usually close to the results of 'B1', or 'B2' for a stricter target, where the difference between the 'B1', or 'B2' portfolios is minor.

In general, we see that the optimal portfolios differ significantly, in particular when comparing results for different targets. Whereas for strict targets the portfolio consists solely of gas and biomass for any scenario considered, high targets rely mostly on a combination of coal and wind. The differences in shares between scenarios for a given target are less pronounced, with the single exception of the 520 ppm target. Concerning the resulting expected profit, we see that again the target is the key driver. For a given target the expected profit if the optimal portfolio stays comparable across scenarios, except for the case of 590 ppm.

This section provided insights as to how a specific climate policy affects the optimal energy mix. However, in reality we do not know, which scenario - target combination best represents the future development of the climate policy. Hence, it is a pertinent question to ask, which portfolio is optimal if at the decision moment there is no information available as to which scenario will materialize. This question can not be answered by the basic portfolio model. Therefore, the model is modified in the next chapter to provide an answer this question.

# Chapter 6

# Robust Portfolios

## 6.1 General formulation

Below we present an extension of the model formulated in Chapter 5. Similar to Chapter 5, let us consider $n$ assets. However, in this case the investment into asset $i$ results in profit $y_i^s$, where $y^s \in \mathbb{R}^n$, is a random vector with known distribution depending on scenario $s \in \{1, 2, \ldots, S\}$. Let us further denote by $x_i$ the share of the asset $i$ in the portfolio without short positions.

The problem we try to answer in this section is how to find the optimal composition of the portfolio, if the distribution of $y^s$ for a given scenario is known, but at the decision moment it is not known which scenario will materialize. Without this knowledge, the investor tends to invest in a combination of the underlying assets performing well in each of them. We suggest the following solution: the investor chooses a portfolio that performs best under the worst scenario possible, performance is measured by the conditional-value-at risk.

Such a portfolio is a solution of the following problem:

$$\left.\begin{array}{rll} \min_x & \max_s \ \phi_\alpha^s(x) & \\ \text{s.t.} & \sum_{i=1}^n x_i = 1 & \\ & x_i \geq 0 & \text{for } i = 1, \ldots, n \end{array}\right\} \tag{6.1.1}$$

where the $\phi_\alpha^s$ denotes the $\alpha$-CVaR associated with the loss function $-x^T y^s$ of the portfolio for scenario $s$.

In other words, we apply a minimax approach (minimizing the maximal possible loss) to value the performance of the portfolio under the different scenarios. This approach is quite common in game theory, originally formulated for two-player zero-sum games. The crucial feature of minimax decision making is that it is non-probabilistic, in contrary to decision based on expected value or expected utility.

Prior to presenting the model for uniformly discretely distributed asset profit, let us analyze the robust portfolio model (6.1.1) in general. It was shown in Chapter 3 that the CVaR portfolio model leads to a convex optimization problem. We show that this pleasant feature is preserved also in this extension. Since the set of feasible solutions is convex, it is sufficient to prove the following.

**Theorem 6.1.1.** *The function* $G(x) = \max_s \phi_\alpha^s(x)$ *for the loss function* $-x^T y^s$ *is convex in* $x$.

*Proof:* We will show that $G(\lambda x_1 + (1-\lambda)x_2) \leq \lambda G(x_1) + (1-\lambda)G(x_2)$ for any $x_1$, $x_2 \in \mathbb{R}^{\ltimes}$ and $\lambda \in [0,1]$. Since for any $s$ the $\phi_\alpha^s(x)$ is convex in $x$ in case of the loss function considered,we have

$$\phi_\alpha^s(\lambda x_1 + (1-\lambda)x_2) \leq \lambda \phi_\alpha^s(x_1) + (1-\lambda)\phi_\alpha^s(x_2) \tag{6.1.2}$$

for any $s$. Also, we know that $G(x) = \phi_\alpha^{\bar{s}}(x)$ for some $\bar{s}$ from the definition of $G$. Hence

$$\begin{aligned} G(\lambda x_1 + (1-\lambda)x_2) &= \phi_\alpha^{\bar{s}}(\lambda x_1 + (1-\lambda)x_2) \\ &\leq \lambda \phi_\alpha^{\bar{s}}(x_1) + (1-\lambda)\phi_\alpha^{\bar{s}}(x_2) \\ &\leq \lambda G(x_1) + (1-\lambda)G(x_2). \end{aligned} \tag{6.1.3}$$

□

## 6.2 Energy Portfolios

Now we can proceed to the case of the assets profit being discretely uniformly distributed. This is also the case of the distributions stemming from the real options model. Let us assume that the assets profit distribution in scenario $s$ is given by the sample $\{y_s^k\}_{k=1}^N$, where $y_s^k \in \mathbb{R}^n$ for $k = 1, \dots, N$. Substituting the formula for $\phi_\alpha$ for such distributions (3.2.15) we get the formulation of model (6.1.1) for this case as:

$$\left.\begin{array}{lll} \min\limits_x & \max_s \min_{\xi_s}(\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^N (-x^T y_k^s - \xi_s)^+) & \\ \text{s.t.} & \sum_{i=1}^n x_i = 1 & \\ & x_i \geq 0 & \text{for } i = 1, 2 \dots, n \end{array}\right\} \tag{6.2.4}$$

We show that this formulation is equivalent to the following problem:

$$\left.\begin{array}{lll} \min\limits_{(x,\xi,u,v)} & v & \\ \text{s.t.} & v \geq \xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^N u_k^s & \text{for } s = 1,\dots,S \\ & \xi_s + x^T y_k^s + u_k^s \geq 0 \quad \text{for } s = 1,\dots,S,\ k = 1,\dots N & \\ & \sum_{i=1}^n x_i = 1 & \\ & x_i \geq 0 & \text{for } i = 1\dots,n \\ & u_k^s \geq 0 \qquad\qquad\quad \text{for } s = 1,\dots,S,\ k = 1,\dots N & \end{array}\right\} \quad (6.2.5)$$

**Theorem 6.2.1.** *The quadruple* $(x^*, \xi^*, u^*, v^*)$, $x^* \in \mathbb{R}^n$, $\xi^* \in \mathbb{R}^S$, $u^* \in \mathbb{R}^N \times \mathbb{R}^S$ *and* $v \in \mathbb{R}$ *is the solution of problem (6.2.5) if and only if* $x^*$ *is the solution of problem (6.1.1). Moreover, one has*

$$v^* = \max_s \min_{\xi_s} (\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^N (-(x^*)^T y_k^s - \xi_s)^+). \qquad (6.2.6)$$

*Proof:* It is important to realize following general rules hold:

$$\begin{aligned} \max_{m\in M} F(m) &= \min_w \{w : w \geq F(m), \forall m \in M\} && (6.2.7) \\ \min_{m\in M} F(m) &= \max_w \{w : w \leq F(m), \forall m \in M\} && (6.2.8) \\ \min_w \{w : w \geq \min_{m\in M} F(m)\} &= \min_{w, m\in M} \{w : w \geq F(m)\} && (6.2.9) \end{aligned}$$

Using rule (6.2.7) yields

$$\begin{aligned} A &= \min_x \max_s \min_{\xi_s} (\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^N (-x^T y_k^s - \xi_s)^+) && (6.2.10) \\ &= \min_x \max_s \min_{\xi_s} (\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^N \min_{u_s^k} u_k^s) && (6.2.11) \end{aligned}$$

with $u_k^s$ satisfying

$$u_k^s \geq -x^T y_k^s - \xi_s, \quad u_k^s \geq 0 \qquad (6.2.12)$$

for $\forall(k, s)$, i.e. $s = 1,\dots,S$ and $k = 1,\dots N$. Furthermore, (6.2.8) and

(6.2.9) imply

$$\begin{aligned}
A &= \min_x \max_s (\min_{u_k^s, \xi_s} [\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^{N} u_k^s]) && (6.2.13)\\
&= \min_x \min\{v : v \geq \min_{u_k^s, \xi_s} [\xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^{N} u_k^s], \forall s\} && (6.2.14)\\
&= \min_x \min_{\xi, u} \{v : v \geq \xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^{N} u_k^s, \forall s\} && (6.2.15)\\
&= \min_{x,\xi,u} \{v : v \geq \xi_s + \frac{1}{N(1-\alpha)} \sum_{k=1}^{N} u_k^s, s = 1, \forall s\} && (6.2.16)
\end{aligned}$$

which yields the desired result. □

This implies that the problem of finding robust portfolios can be reduced to a linear programming problem. In this case the problem comprises $Ns + n + s + 1$ variables and $SN + S + 1$ constraints. The solution for this model in the numerical applications was calculated by GAMS using the CPLEX solver. In the following applications we use the same parameters concerning the technologies and confidence level as in Section 5.3.3, in order to be able to compare back to the results of the basic framework presented there. That means we consider four technologies: biomass, coal and gas (all with the option of CCS) and wind, with the constraint limiting the share of renewables up to 50 %. The confidence level $\alpha$ is assumed 97%. What differs, however, is the set of scenarios over which the portfolio should be robust. Let us further refer to this set as the robust range.

## 6.3 Response to Climate Policy Uncertainty

In Chapter 5 we investigate the optimal portfolio composition of electricity generating capacities for different climate policy scenarios. The results vary greatly depending on the scenario and target chosen, moving from a gas-biomass preference for the strict policies to an energy mix relying on wind, followed by coal and gas. We see that targets have a stronger impact on the optimal energy mix than the socio-economic scenarios. In fact, except for the 520 ppm case, for a fixed target optimal portfolios were rather similar across soico-economic scenarios.

In reality, it is still unclear which scenario and target combination is the best prediction for the future development. However, as has been already

| *Scenario* | *Gas share* | *Biomass share* | *Wind share* | *Coal share* |
|---|---|---|---|---|
| 'B1' | 18.27% | 9.74% | 40.26% | 31.73% |
| 'B2' | 24.06% | 14.44% | 35.56% | 25.94% |
| 'A2r' | 21.47% | 10.03% | 39.97 | 28.53% |
| 480ppm | 50.00% | 50% | 0% | 0% |
| 520ppm | 18.27% | 9.74% | 40.26% | 31.73% |
| 590ppm | 21.47% | 10.03% | 39.97% | 28.53% |
| All | 21.47% | 10.03% | 39.97 | 28.53% |

Table 6.1: *Technology shares across different robust ranges*

mentioned, there is a strong need for the replacement of aging capacities in the OECD countries, where some of the investment will occur before this uncertainty about the direction of both socio-economic conditions and the target is resolved. Therefore, it is of interest to analyze the composition of portfolios of electricity generating capacities that are robust across different climate policy scenarios.

In Section 5.3.3 we present the results of the basic CVaR portfolio model for each of 9 possible combinations of both socio-economic scenarios and targets. In this section we investigate what is the composition of portfolios that are robust across a subset of these 9 combinations. Precisely, we investigate portfolios that are robust across targets for a given socio-economic scenario, as well as portfolios that are robust across socio-economic scenarios for a given target. In addition, we look for the portfolio that is robust across any possible climate policy development, i.e. across all nine combinations of scenarios and targets.

The results of the robust portfolio optimization for different robust ranges are presented in Table 6.1. The rows indicate the robust ranges, the first three rows showing the portfolios robust across scenarios, e.g. 480 ppm denotes the case where the target is fixed at 480 ppm and the robust range consists of 'A2r' 480 ppm, 'B1' 480 ppm and 'B2' 480 ppm. The middle part gives the portfolios robust across targets for a fixed scenario and the last row refers to a case where the solution has to be robust across all nine scenario-target combinations. The shares are also depicted in the Figure 6.1, so that the scale of the differences between the individual robust portfolios is easily observed.

First let us focus on the difference between scenarios for a fixed target. We see that if we incorporate the need to make a robust decisions, the original CVaR portfolio results (Table 5.3) change. The only exception is the case of the 480ppm target, where even the basic portfolio model suggests the

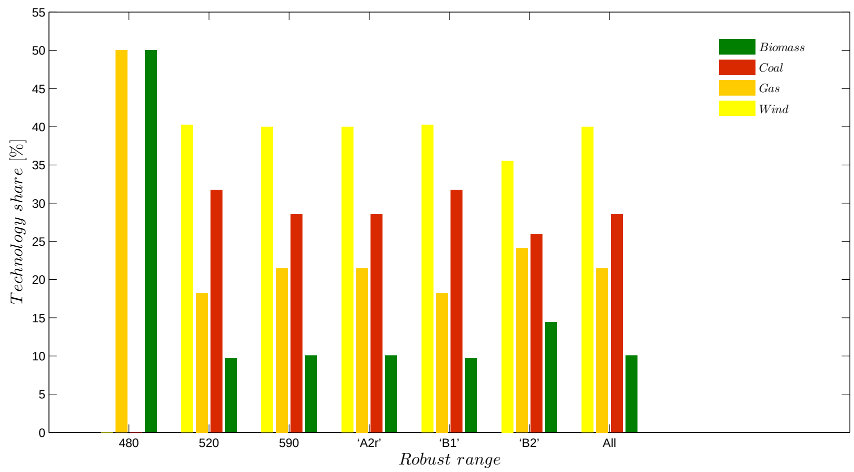


Figure 6.1: *Technology shares across different robust sets*

same optimal portfolio, independent of the scenario choice. For the 590 ppm target, the robust portfolio contains over 10% of biomass, whereas in the basic portfolio framework this is the case only in one socio-economic scenario, the share of biomass in the other two is basically zero. A similar effect is present in case of gas. However, since the basic model results for the 590 ppm target are quite similar across the scenarios, the need to be robust across scenarios does not cause a significant transition. When analyzing the optimal portfolios for a given scenario and target with the basic model in Section 5.3.3, we saw the most significant difference between investment responses to climate policy assumptions ocurred for the 520ppm target. The shares ranged from a half-half combination of gas and biomass on one hand to a coal and wind based energy mix on the other. For the 520 ppm target, the robust framework recommends a coal and wind combination, with a little addition of the other two technologies.

A similar shift can be seen when focusing on the robustness across targets for a given scenario. Even though the basic CVaR model recommends the 50-50% combination of gas and biomass for the 480 ppm target, if we need to choose a portfolio that is robust across all targets, this combination in never optimal. The investor chooses a combination of all technologies considered, preferring wind followed by coal. Precise numbers differ between scenarios, but the general picture is the same. The portfolio that is robust across all 9 possible alternative climate policies considered has the same composition, which is actually the optimal solution of the basic CVaR portfolio model in case of a 'A2r' 590 ppm policy.

Generally, we see that for a given robust range, the robust framework does not recommend a compromise of the results of the basic CVaR portfolio problem for elements of the robust range. It rather suggests one of the limit cases. This is understandable, since the objective of the robust portfolio problem is based on the analysis of extreme cases, not on the combination of the underlying scenarios. In most cases the optimal portfolio is a mix of all technologies relying more on coal and wind. The preference of coal and wind lies mainly in their stability across both scenario and targets. Since the biomass technology is the one that is most sensitive to climate policy, being the most attractive one in case of strict targets but featuring low profit and high risk in others, it is not an adequate choice for a robust decision. Gas shows similar characteristics, although they are by far not as pronounced as in the case of biomass. However, the precise effect varies depending on the robust range considered. In some cases the robust framework does not suggest a composition markedly different from the basic results.

Figure 6.1 shows that the targets have a higher impact on the optimal solution than the socio-economic scenarios. The results for robust ranges consisting of different targets for a given socio-economic scenario are mutually similar, which can not be said in the opposite case. Therefore, the identification of the the right target is more crucial for the investor, enabling him to react optimally. This is best illustrated on the biomass share. In case the target is not known, the share of biomass is kept low, whereas for some targets biomass is the leading technology.

## 6.4 Robust across Time

The second aspect we want to analyze in this chapter is the time structure of the profits, which is a point neglected previously. The technologies considered react differently to the rising $CO_2$ prices. Whereas the profits of the fossil fueled technologies suffer, biomass becomes attractive only later as the prices rise high enough. The only unaffected technology is wind, which performs steadily independent of the climate policy. The motivation for this extension is following: the investor may not be willing to invest into a technology performing best over the whole plant lifetime, if the profits materialize only in the final decade of the planning horizon. Instead, he may feel inclined to substitute a part of his investment by a technology, which does not perform optimally from the point-of-view of overall profits, but is instead especially attractive in the first decades. In other words, the time structure of the profit streams generated by a technology may play an important role in the optimal portfolio selection.

The distributions used in this case are a bit different than the ones used in the previous applications. For a given socio-economic scenario, target and technology, we generate not a single, but six profit distributions. Whereas before the distribution was derived as the sum of the discounted yearly profit for the whole lifetime of the power plant considered, to derive distributions accounting for the time structure the lifetime is formally divided into six 5-year subperiods. The profit is calculated for each of the subperiods separately, as described in the section 4.1.3. Let us recall that the profit for each subperiod is a sum of the discounted the yearly profit in that subperiod, where the profit is discounted to the beginning of the subperiod. Therefore, the profit distributions used in the previous sections is equal to the sum of the discounted profit distributions for the subperiods, each of them discounted from the beginning of the subperiod to year zero. In other words, the profit distribution of the first subperiod reflects the performance of the considered technology during the first 5 years of its lifetime, the second subperiod captures its performance in during the next five years etc.

The robust CVaR portfolio model is then applied to these distributions, where the robust range is comprised of the 6 subperiods. In other words, the investor chooses a portfolio that performs best in terms of CVaR even in the worst of the subperiods considered. By taking into account the changes in the distributions over time (i.e. over 5-year intervals), we can thus also capture such characteristics of the profits' time structure and determine their effects by comparing back to the findings in the previous chapter.

### 6.4.1 Distribution features

Let us first present an illustration of the statistics of the underlying profit distributions, since their characteristics are the major drivers of the results of the CVaR portfolio model robust across the 5-year subperiods. Figure 6.2 shows both the expected costs and the 97%-CVaR of the distributions for all subperiods and technologies in the the B2 590 ppm case (where the profit distribution is the negative of the costs). Although the precise numbers vary across the scenario-target combinations, the general features are the same.

We see the expected costs for biomass decrease sharply, which is caused by the investment into the CCS module. CCS causes a sudden drop in the net yearly emissions of the biomass power plant from zero to a substantial negative amount, the highest in absolute value among the technologies. Therefore, biomass not only gains a lot from a rising $CO_2$ price in terms of expected profit, but is also the most affected by the $CO_2$ price fluctuations. This can be observed when examining its CVaR. Recall that CVaR accounts

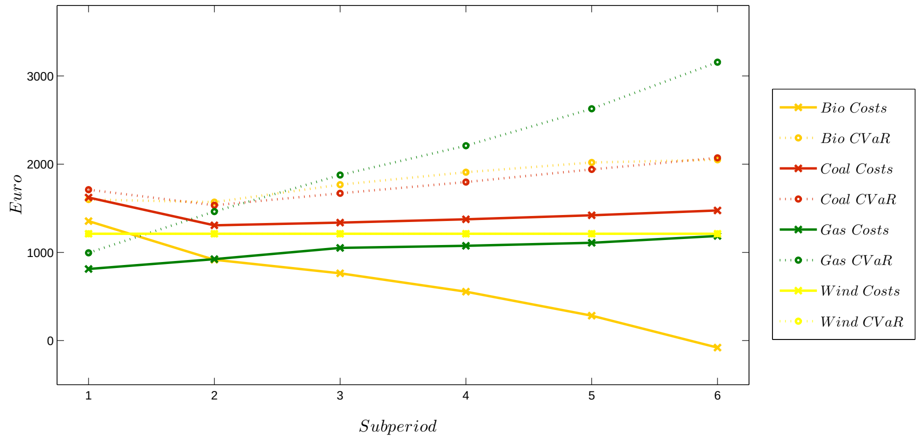


Figure 6.2: *Technology Costs Distribution Statistics - B2 520 ppm*

also for the expected value of the underlying distribution, therefore the CVaR of biomass is particularly high when comparing to the low expected costs. On the contrary, since wind is unaffected by both $CO_2$ and fuel price uncertainty, it is the only technology having constant profits over time. Gas and coal have similar characteristics. Investment into the CCS module can decrease the yearly costs temporarily (because the yearly emissions decrease substantially with the CCS). As the $CO_2$ price is rising, the costs are increasing after the drop caused by the CCS investment, however this increase is slower since CCS captures a large part of the emissions. The CVaR of both coal and gas is therefore affected more by the fuel than by the $CO_2$ price fluctuations. This is also one of the major differences between coal and gas. The gas plant is cheaper than the coal one in terms of both capital and operations costs, but is more fuel intensive, i. e. the requirements for fuel are higher than the ones for coal. Therefore, it is more affected by the rising fuel prices, which results in its faster expected costs increase compared to coal. In addition, the volatility of gas prices is higher that the volatility of coal ones. This is reflected in coal having a much lower CVaR than gas, despite its higher cost. For a stricter target, the difference between the technologies gets more pronounced, with higher costs of coal and gas and lower costs of biomass.

| | Multi-period results [%] | | | | Basic results [%] | | | |
|---|---|---|---|---|---|---|---|---|
| | *Bio* | *Coal* | *Gas* | *Wind* | *Bio* | *Coal* | *Gas* | *Wind* |
| 'B1' 590 | 0.00 | 43.84 | 6.16 | 50.00 | 0.00 | 36.29 | 13.71 | 50.00 |
| 'B2' 590 | 6.88 | 41.25 | 8.75 | 43.12 | 1.05 | 30.10 | 19.90 | 48.95 |
| 'A2r' 590 | 9.16 | 41.99 | 8.01 | 40.84 | 10.03 | 28.53 | 21.47 | 39.97 |
| 'B1' 520 | 4.84 | 44.226 | 5.774 | 45.161 | 9.74 | 31.73 | 18.27 | 40.26 |
| 'B2' 520 | 12.39 | 41.26 | 8.74 | 37.61 | 16.58 | 24.91 | 25.09 | 33.42 |
| 'A2r' 520 | 23.66 | 32.00 | 18.00 | 26.34 | 50.00 | 50.00 | 0.00 | 0.00 |
| 'B1' 480 | 32.03 | 30.51 | 19.49 | 17.98 | 50.00 | 50.00 | 0.00 | 0.00 |
| 'B2' 480 | 18.53 | 40.54 | 9.46 | 31.47 | 50.00 | 50.00 | 0.00 | 0.00 |
| 'A2r' 480 | 50.00 | 0.00 | 50.00 | 0.00 | 50.00 | 50.00 | 0.00 | 0.00 |

Table 6.2: *Comparison of the results for the multi-period and the basic approach. Technology shares across different climate policy scenarios.*

### 6.4.2 Portfolio results

Finally let us present the results of the robust CVaR portfolio model in case of robustness across time. For the rest of the section let us refer to the results that are robust also across time as multi-period results. We investigate two types of robust ranges. First, for a given stabilization targets and socio-economic scenario, we compute the optimal solution of the robust CVaR model, where the robust range is comprised of the 6 subperiods. This represents the case when the target and scenario is known and the investor chooses a portfolio performing well in terms of CVaR in each of the subperiods. Second, we compute portfolios robust not only across time but also across climate policy.

First, let us compare these portfolios to the corresponding results, where the time structure is not considered. All these results were already presented in Section 5.3.3 and Section 6.3. However, we list them also here, so that the difference between the portfolios is easily perceptible.

#### Basic vs. Multi-period

Table 6.2 presents the shares of the individual technologies for the optimal portfolios in case the target and scenario is known. The multi-period results are in the first four columns, whereas the basic results from Section 5.3.3 are shown in the last four columns. Each row represents a different scenario-target combination.

The general effect caused by the introduction of the time structure is the

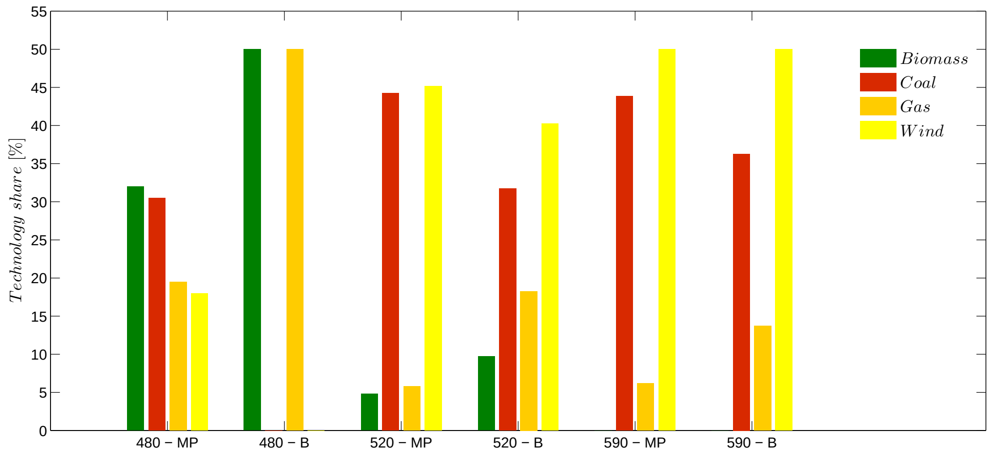


Figure 6.3: *Comparison of the results for the multi-period and the basic approach for the 'B1' scenario.*

same across the socio-economic scenarios, but its significance varies, being most and least prominent in the 'B2' and 'A2r' case respectively. Let us illustrate this by examining the results for middle case, i.e. the 'B1' scenario. The comparison between the multi-period shares and the basic shares for this scenario are depicted in Figure 6.3; the shares are shown for each target, where the multi-period and basic results are denoted by MP and B respectively.

We see a pattern that is common for all targets. Coal gains at the expense of gas, and wind at the expense of biomass. This is of course most noticeable in the 480 ppm target, since the basic results recommend a pure gas and biomass mix. In the multi-period case, the results suggest a more than 30% share of coal and almost 20% share of wind. The cause of this shift from gas to coal lies in the distribution features. Although the overall statistics of gas make it attractive for the basic model; when considering time structure, we see that its CVaR is increasing faster than the one for coal (due to both higher fuel requirements and higher volatility of fuel prices). Therefore, the high levels of risk in the later subperiod disqualify the gas technology in favor of coal, and similarly also biomass in favor of wind.

For the 'B1' scenario, this pattern is common to all targets. The situation for 'B2' is similar, but the shift from gas to coal is even more pronounced. In the most stringent target, the coal comprises more than 40% of the multi-period portfolio, whereas in the basic case it was not present at all. Also,

| | Multi-period results [%] | | | | Robust results [%] | | | |
|---|---|---|---|---|---|---|---|---|
| | *Bio* | *Coal* | *Gas* | *Wind* | *Bio* | *Coal* | *Gas* | *Wind* |
| 'B1' | 1.97 | 43.52 | 6.48 | 48.03 | 18.27 | 9.74 | 40.26 | 31.73 |
| 'B2' | 6.88 | 41.25 | 8.76 | 43.12 | 24.06 | 14.44 | 35.56 | 25.94 |
| 'A2r' | 9.16 | 42.00 | 8.01 | 40.84 | 21.47 | 10.03 | 39.97 | 28.53 |
| 480 | 18.53 | 40.54 | 9.46 | 31.47 | 50.00 | 50.00 | 0.00 | 0.00 |
| 520 | 8.38 | 42.07 | 7.93 | 41.62 | 18.27 | 9.74 | 40.26 | 31.73 |
| 590 | 2.81 | 43.03 | 6.97 | 47.19 | 21.47 | 10.03 | 39.97 | 28.53 |
| All | 6.54 | 42.74 | 7.26 | 43.46 | 21.47 | 10.03 | 39.97 | 28.53 |

Table 6.3: *Comparison of the results for the multi-period and the robust approach. Technology shares across different robust ranges.*

biomass suffers much more for this target than in the 'B1' case, its share decreases by more than 30%. For the 'A2r' scenario, the results differ depending on the target considered. The 480 ppm is the only case when both approaches deliver the same results. This is the case when the underlying $CO_2$ price is the highest, therefore the gas and biomass still dominate the portfolio. For other targets, we see an overall decrease of the share of gas and a slightly lower decrease of biomass.

Whereas for the basic model the results vary greatly and are mainly driven by targets, the gas dominating over coal for the stricter targets and the vice versa for the loose targets, the results for the multi-period model are more similar amongst themselves. We see, that an investor accounting for time structure of the profit chooses an energy mix consisting of a significantly larger coal share and a slightly lower biomass share than an investor indifferent to the time structure. Still, there are still differences between the portfolios suggested, depending on the climate policy. However, in this case they mostly impact the biomass share, the coal share remains stable in the majority of cases.

### Robust vs. Multi-period

Now let us turn to the results robust across different climate policies, comparing the optimal energy mix given by the multi-period approach back to the approach neglecting time structure presented in Section 6.3. The robust ranges considered were based on the experiments from that section. For each target, we computed the results for a robust range consisting of the subperiods for each socio-economic scenario, i.e. in total of 18 possible scenarios. Similarly, for each scenario we analyzed the robust range comprising a com-

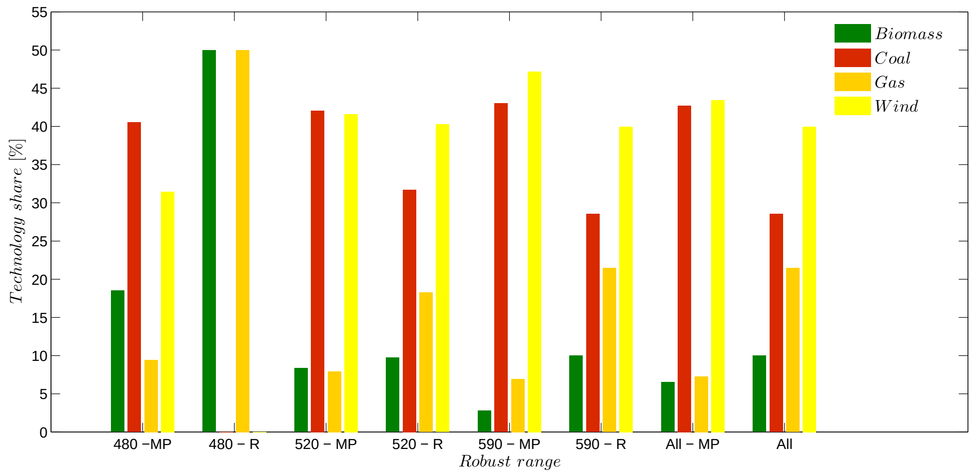


Figure 6.4: *Comparison of the results for the multi-period and the robust approach for different robust ranges.*

bination of all targets and subperiods. Finally, the portfolio robust across all scenarios, targets and subperiods was computed, the robust range consists of 54 elements.

The shares of the individual technologies for the multi-period portfolios accounting for time structure are in the first four columns of Table 6.3, the results neglecting the time structure are listed in the last four columns, the rows showing the robust ranges denoted in the same way as in Section 6.3. To foster the visualization, the shares are also shown in Figure 6.4, where in this case for simplicity the robust ranges for fixed scenarios are omitted. The results from Section 6.3 are denoted by R, the multi-period results by MP.

These experiments represent the case when the stabilization target is known but the socio-economic scenario is unknown. The case when the investor does not care about the time structure of the profit was already presented in Section 6.3. We saw that if we need to be robust across the different socio-economic scenarios, apart from the strictest target, the investor prefers a portfolio consisting of all four technologies, relying mostly on wind followed by coal and gas. We see a similar impact of the introduction of the concept of time structure as in as in the case of the basic framework, which is most noticeable in the 480 ppm case. Whereas the robust decision not accounting for the time structure recommended a 50-50% combination of gas and wind, the portfolio that is robust also across subperiods consists of more that 40% coal and 30 % wind. In general, the multi-period portfolio, when compared to the respective case neglecting time structure, is characterized by a major shift from gas to coal. In addition, also the wind share increases noticeably at the expense of biomass, which makes the coal and wind even more dominant than in the robust case neglecting time structure. Both these effects persist also in case when the decision needs to be robust across both scenario and target.

#### Climate Policy impact

In Section 6.3 we discuss the impact of the uncertainty with respect to the development of climate policy, using the results from the basic model and results derived with the robust framework (the robust ranges consisting of a combination of scenarios and targets). Now we can investigate the same question, assuming the investor cares about the time structure of the profit by comparing the shares presented in the first four columns of Tables 6.2 and 6.3. In Section 6.3 we did not discover any common pattern in the change of the technology shares. However, if the investor cares about the time structure, the situation is different. For a given target, the multi-period portfolios that are robust across socio-economic scenarios always prefer the portfolios with the larger share of both coal and biomass than the multi-period results for each of the socio-economic scenarios. That means that even if in some instances of the multi-period approach the optimal portfolio consists of a significant share of gas or biomass (as for example the 'B1' 480 ppm case), the portfolio that is robust also across this policy does not have this feature anymore. This can be easily noticed, since both coal and wind are dominant for any robust set of climate policies in the multi-period results from Table 6.3, whereas in the case when the climate policy is known, it was not always the case.

## 6.5 Conclusion

The portfolio model extension suggested in this chapter is a novel contribution to the literature on power generation technology portfolios. Not only does it enable us to capture relevant features, in the case of discrete distributions it is also shown to be equivalent to a linear programming problem, which makes it widely applicable. Moreover, even in the case of general distributions, the model was shown to have favorable characteristics. Leading to a convex optimization problem, it makes the powerful tools of convex analysis applicable.

The proposed extension enables us to find portfolios robust across a set of scenarios considered, i.e. performing best in the worst scenario possible. This makes it extremely suitable to investigate the uncertainty surrounding the future climate policy development, represented by a scenario-target combination. Since there is currently little knowledge as to which scenario and target combination is the most likely one, the investors will need to invest into portfolios that perform well in each of them. As was explained, the scenarios represent the future socio-economic development of the world, trying to capture different rates of population growth, technology transfer to developing countries etc. On the other hand, the target represents the $CO_2$ concentration level at which emissions need to be stabilized. Whereas the socio-economic conditions are of a more global nature and therefore can not be resolved easily by the policy makers, the situation is different for the stabilization targets. We want to emphasize. that we do not assess the targets by their desirability. Rather, in this thesis the target represents the beliefs of the investor about the stringency of the future climate policy. If the policy makers set clear signals promising a strict policy, the investor will assume a strict target. On the other hand, if the policy makers fail to give an indication concerning the stringency of the policy, he chooses a robust portfolio that performs well across the targets. Therefore, analyzing the difference between robust portfolios and portfolios where the target is known provides insights also for the policy makers concerning the importance of setting clear guidelines for their intent.

Independent of climate policy uncertainty, we have shown that the time structure of the profit flows varies greatly over the underlying technologies. We do not argue that this is crucial for all investors, still, some of them will not be willing to invest in a portfolio that is attractive from the overall point of view, but suffers losses in the first 20 years, getting profitable only in the last decade. This motivation led us to study portfolios that behave robustly across time.

In total this provides the incentive to study robust decisions across two dimensions - across time and climate policy. The effects of both were investigated. We have shown that both of these dimensions bring substantial changes in the investor's behavior. The introduction of time structure revealed a uniform pattern, where the resulting portfolio comprises a substantial share of coal and wind, even if the original results neglecting time structure suggested otherwise. This was observed in almost all of the cases, with the only exception being the case of the strictest $CO_2$ policy considered ('A2r' 480 ppm). The effects of robustness across different climate policy alternatives was noticeable, but in case of neglecting the time structure not so unambiguous. However, we saw that the key climate policy parameters driving the portfolio composition are targets, not the scenarios. In combining both the climate policy and time structure this was confirmed, in addition we saw that the effects of climate policy are more significant. The coal and wind shares are even more substantial in this setting, effectively eliminating a gas and biomass based mix.

This analysis not only shows how important it is for the investor to account for climate policy uncertainty correctly. It also illustrates the importance of strong policy signals. If the signals from the policy makers are not clear, the investor has to choose a portfolio that is far from the optimal one, with respect to the technology shares. This is best seen in the comparison of robust results for the 'A2r' scenario across different targets. Whereas in case the target is known, the investment response is in both of the stricter targets a 50-50% biomass and gas mix, the robust results prefer the optimal portfolio for the 'A2r' 590 ppm, which is markedly different. Moreover, since the biomass and gas produce less emissions than wind and coal respectively, the resulting portfolio will be much more emission intensive mix.

Also, the analysis of the time structure provided interesting insights for the investment. It has helped to understand the importance of considering the time profile of technologies. In particular, the multi-period framework can explain why power plant owners hold on to coal-fired capacity and plan even more of the same, even though they know that they will be facing some sort of $CO_2$ policy in the medium to long run. This is because coal-fired capacity will eventually be less risky than gas-fired power plants, which suffers from higher fuel price volatility. Also the riskiness of biomass increases over the sub-periods, a fact which is not taken into account in the single-period framework, where only overall expected profits count.

Although the analysis presented is still an admittedly stylized exercise, with a limited number of technologies, it still manages to provide insights to the effect that uncertainty has on decision-making when there is no information about the probability of the occurrence of events. This extension provides a new perspective on such investment decisions, illustrated by the numerical application to electricity-generating technologies.

# Chapter 7

# Dynamic Portfolios

Models discussed both in Chapter 5 and Chapter 6 suffered from one shortcoming. The discussion remained inherently static insofar as the investor would allocate his funds once in the beginning of the planning period, using information about the optimal, dynamic behavior of individual plants from the real options model. In other words, we disregarded the fact that this investment may be followed by others in the coming years. As we mentioned in the introductory chapter, considerable investment into new capacities is due in the OECD countries in the coming years [36]. Naturally, this investment will be spread over the years and will not happen at once, as we assumed so far.

This chapter seeks to remedy this deficiency by taking into account the possibility to diversify not only over assets, but also over *time*. In other words, we should take into account that the option to alter the portfolio in the future might affect the present portfolio decisively. This is achieved by reformulating the basic, static framework, so that it considers not only current portfolio shares, but also future sub-portfolios. As before, the investor is choosing his portfolio so as to minimize risk in terms of CVaR.

## 7.1 Formulation

We investigate a case, where the investor has the possibility to invest into specific portfolios of sizes $b_t$ at different time points $t = 1, 2, \dots, T$, $\sum_{t=1}^{T} b_t = 1$. We therefore implicitly assume that there are pre-specified capacities that have to be installed at pre-specified time instants. Furthermore, we assume the decisions about the investment has to be taken today. This is, however, not an extremely unrealistic assumption, since investment into new power generating capacities requires thorough planning and is not carried out at short notice. As we assume the investor is bound by demand constraints, it is also natural to assume the amount $b_t$ that has to be invested at a given point in time $t$ to be exogenously given. This setup corresponds to the situation outlined in the introduction of this chapter.

Therefore in this case we consider $nT$ underlying assets, $n$ for each decision point. The return of these assets is a random vector $y \in \mathbb{R}^{nT}$, where $y_t \in \mathbb{R}^n$ is the return on investment into the technologies considered, when invested at time $t$. This is the only exception, where we characterize the investment into technologies by returns, rather than profits. Since we will need to compare the profitability of investments at different points in time, the returns are more convenient.

A portfolio is then defined by its shares $x \in \mathbb{R}^{nT}$ , where $x^t = (x^t_1, \ldots, x^t_n)^T \in \mathbb{R}^n$ is a vector comprising the shares of the technologies subject to investment at time $t$. The return of the portfolio is then given by $\sum_{t=1}^T (x^t)^T y_t$. The problem of the investor can be hence formulated as follows:

$$\left.\begin{array}{l} \min_x \phi_\alpha(x) \\ \qquad\quad \text{s.t.} \quad \sum_{i=1}^n x_i^t = b_t, \quad t = 1, \ldots, T, \\ \qquad\qquad\quad\;\; x \geq 0 \end{array}\right\} \tag{7.1.1}$$

where $\phi_\alpha(x)$ is the $\alpha-$CVaR of the loss function given by $-\sum_{t=1}^T (x^t)^T y_t$.

For the application presented, the returns are derived by the real options model from Chapter 4 and have a discrete uniform distribution over values $y^k \in \mathbb{R}^{nT}$, $k = 1, 2, \ldots, N$, where by $y^k_t = \in \mathbb{R}^n$ we denote the sample of returns of the investment into technologies at time $t$. Let us recall that the return is calculated as the total discounted profit on a unit of investment, where the profit is discounted to the moment when the investment is carried out. The convenience of measuring profitability by returns compared to profits lies in the following: the profit is expressed in [€] at the time when the investment is carried out. Therefore the current value of future investment is not equal to the profit, but to the discounted value of this profit (discounted from the time the investment is carried out to the current moment). Since return is calculated as profit per 1 € of investment, this measure is indifferent to the moment when the future investment is valued.

The problem (7.1.1) takes according to 3.2.16 following form:

$$\left.\begin{array}{rl} \min_{(x,\xi,u)} & \xi + \frac{1}{N(1-\alpha)} \sum_{k=1}^N u_k \\ s.t. & \sum_{i=1}^n x_i^t = b_t, \quad t = 1, \ldots, T \\ & \sum_{t=1}^T (x^t)^T y_t^k + \xi + u_k \geq 0, \; k = 1, \ldots, N \\ & x \geq 0 \\ & u_k \geq 0 \quad k = 1, \ldots N \end{array}\right\} \tag{7.1.2}$$

where $u^k \in \mathbb{R}$ are auxiliary variables. The solution $(\hat{x}, \hat{\xi}, \hat{u})$ of (7.1.2) yields the optimal $\hat{x}^*$ for which the minimum of the corresponding $\alpha$-CVaR is attained. This is a linear programming problem, with $N + nT + 1$ variables, and $N + T$ and constraints. In the application we use $N = 10,000$, $T = 3$ and $n = 2$ and solve the model in GAMS with the use of the CPLEX solver.

## 7.2 Case study

Let us consider a case, where the investment is planned for the next decade, happening in five years steps (i.e. we have three different investment time point, in years 0, 5 and 10), where the capacity installed at decision point $i$ is given by $b_i$, $b_1 + b_2 + b_3 = 1$. This is a realistic perspective when we think of the current situation of many OECD countries, which will have to replace part of their existing capacity over the next decade. A ten-year planning horizon, where investment can happen in five-year steps therefore seems a reasonable case.

For simplification and easier interpretation of results we concentrate on two technologies only - coal (with the option of adding CCS) as the typical representative of the fossil-fueled technologies, and biomass (also with the option of adding CCS) as the representative of renewables. The analysis is performed for the 'B2' 590 ppm, which is a scenario with a relatively loose climate policy. The sample of returns $y^k$, $k = 1, \ldots, N$ is derived as described in Section 4.1.3. As we discussed in Section 6.4.1 (e.g. see Figure 6.2), these two technologies are affected by the rising $CO_2$ price differently. Whereas currently coal is the more attractive technology, the profitability of biomass increases substantially later in time.

Before we present the results of the portfolio model(7.1.2), let us first inspect the characteristics of these distributions (see Table 7.1). The table lists the expected profit, standard deviation, –VaR and –CVaR for each asset considered. We report –CVaR, since the CVaR is defined for a loss function, i.e in this case for the negative of the return. Hence –CVaR describes how much return can be secured at the given confidence level. We see the results are consistent with the expectations. Coal has the highest expected return when installed today (i.e. in year 0). In addition, it is less risky both in terms of variance and CVaR compared to its biomass counterpart. However, this relationship between the technologies changes if we consider a later investment time. The later the coal-fired power plant is installed, the more do the expected return and also the –CVaR and –VaR fall. This implies that less return can be secured at the confidence level of 97%. In contrast, biomass gains both on the investment safety side (in terms of –CVaR) and in terms

| *Installation time* | | *Exp. return* | *Std. deviation* | *–VaR* | *–CVaR* |
|---|---|---|---|---|---|
| **Coal** | **0** | 1.421 | 0.043 | 1.345 | 1.333 |
| | **5** | 1.313 | 0.052 | 1.225 | 1.209 |
| | **10** | 1.223 | 0.047 | 1.145 | 1.129 |
| **Biomass** | **0** | 1.414 | 0.111 | 1.239 | 1.216 |
| | **5** | 1.583 | 0.163 | 1.321 | 1.287 |
| | **10** | 1.821 | 0.241 | 1.444 | 1.394 |

Table 7.1: *Descriptive statistics of the return distributions of coal-fired and biomass-fired power plants for the 97% confidence level*

of expected returns as we move further into the future. However, we see the variance of biomass is higher than the variance of coal, independent of the investment time. Since the biomass-fired power plant gains from a rising $CO_2$ price, it gets more and more profitable the later it is installed. However since the profitability is caused by the substantial amount of negative emissions, it is also connected with a high sensitivity to the $CO_2$ price.

## 7.3 Portfolio results

The next step in the analysis is to determine the optimal portfolios, given as the solutions of the problem (7.1.2) for a range of constraints $b_t$. Let us recall that $b_t$ is the fraction of total investment that has to be carried out at decision point $t$. As the investment is planned for the next decade, happening at decision points 1, 2 and 3 (referring to year 0, 5 and 10 respectively), the optimal portfolio $\bar{x}$ consists of 3 sub-portfolios $\bar{x}^t$ for $t = 1, 2, 3$. We will refer to these as to the dynamic sub-portfolio shares and to the constraints $b_t$ as sub-portfolio sizes. For the dynamic portfolio optimization we expect the future to play a vital role. While the static analysis insofar considered a portfolio of coal and biomass that are installed now and used for the coming 30 years, the dynamic analysis will also allow for funding to be spread across time, i.e. portfolios can be composed of coal installed today and bio installed five years later. The option of having bio in the portfolio at a later point in time should then affect the decisions made today. For comparison, the corresponding static portfolios $\tilde{x}^t$ are computed at each point in time individually, without taking future options of having other portfolios into account. This situation reflects the way how we calculated the optimal energy mix in the previous chapters, disregarding future investment plans.

As the portfolios $\tilde{x}^t$ are chosen independently for for every decision point,

| *Inst. time* | *Coal share* [%] | *Biomass share*[%] | *Exp. return* | *–CVaR* |
|---|---|---|---|---|
| **0** | 89.5 | 10.5 | 1.42 | 1.337 |
| **5** | 28.8 | 71.2 | 1.506 | 1.29 |
| **10** | 0 | 100 | 1.821 | 1.394 |

Table 7.2: *Portfolio results for the static setup.*

each of them is a solution of the problem 5.3.22 for two technologies without the constraint on the expected return. Then $\tilde{x} = (b_1\tilde{x}^1, b_2\tilde{x}^2, b_3\tilde{x}^3)$ is the total portfolio if the the sub-portfolios of size $b_t$ are chosen independently in each time point. We will refer to the portfolio $\tilde{x}$ as the static solution, with $b_t\tilde{x}^t$ being the static sub-portfolio for time $t$.

The results $\tilde{x}^t$ of the static optimization for sub-portfolio $t$ are presented in Table 7.2. We see that for example, if the investor only has the option to invest in power plants, which are installed today, he will choose for over 89.5% of the capacity to be coal-fired and invests 10.5% into biomass. This gives an expected return of 1.42 and a –CVaR of 1.337. However, in case the investment will be realized 5 years later, the biomass is preferred, having a share of over 70% , in case of the decision for year 10 we observe a total dominance of biomass.

The complete dynamic portfolio results for all sub-portfolio sizes considered are presented in Tables 7.3, 7.4 attached at the end of the chapter. It lists on the left-hand side the results of model (7.1.2) for a range of constraints $b$, starting with case $b = (1, 0, 0)$ (i.e. all investment goes into plants installed in year 0), and reducing the share $b_1$ as we go down in the table. Next to the sub-portfolios, the expected return, $R$, the –VaR, the –CVaR and the shares of coal and biomass at the different time points are displayed. The Table lists also the shares and statistics of the static solution, given in the right hand side of the tables.

Let us first describe some general observations that hold for all sub-portfolio sizes considered. The results of the portfolio optimization show that dynamic portfolios always outperform the corresponding static portfolios not only in terms of CVaR (which is natural since the portfolios that solve the static framework are feasible in the dynamic case), but also in terms of expected return. When the difference is computed, gains in excess of 3% can be observed for returns and also –CVaR and –VaR often improve by around 1 percentage point. In fact, the lower $b_1$, the higher the gains are. This is intuitively plausible, since higher returns can be realized if diversification flexibility over time opens up new opportunities. The fact that we observe the static portfolios to be inferior in terms of expected return and risk underlines the usefulness of an integrated optimization process and, thus, a dynamic framework for the assessment of investment opportunities at different points in time.

Apart from the gains in the objective, we are interested also in the difference in the portfolio shares between the dynamic and static results. Since the primary motivation for this extension was to investigate the impact of planned investments in the future on the composition of capacities to be installed now, one of the most important results is the difference in the shares of coal installed in decision point 1, i.e. year 0 between the two frameworks. More precisely we calculate the *relative difference*, i.e. the $\frac{1}{b_1}\bar{x}^1_{Coal} - \tilde{x}^1_{Coal}$. In this the way, the shift in the first sub-portfolio is expressed independently of $b_1$, reflecting which fraction of the investment done in the first decision point is used differently in the dynamic framework than in the static one. Of course, it is also interesting to see how the overall resulting mix of technologies changes, i.e. what is the *absolute difference* in the coal technology share between the dynamic and static portfolio $(\sum_{t=1}^{3}(\bar{x}^t_{Coal} - b_t\tilde{x}^t_{Coal}))$.

Concerning the investment planned for the other decision points, the results show that $\bar{x}^3 = b_3\tilde{x}^3$ independently of the choice of $b_3$, where the investment planned for year 10 consists always of 100 % biomass. This is not true for the second decision point. In this case the dynamic results never recommend a higher share than the share of the static results. A closer look reveals, that $\bar{x}^2_{Coal}$ is positive only in cases where both $b_1$ and $b_3$ are low, i.e. where the size of the second sub-portfolio dominates the others. Otherwise $\bar{x}^2$ consists solely of biomass, whereas we saw that for the static case we would invest almost 30 % of the sub-portfolio size into coal.

Although this cannot be observed from Tables 7.3, 7.4 directly, the total share of the coal technology is generally lower for the dynamic framework (i.e. the *absolute difference* is negative). Exceptions to this rule are only the cases where the sub-portfolio size $b_2 = 0$, i.e. the investment is planned only for year 0 and 10. This can be observed in Figure 7.1 depicting the *absolute difference* for the case of $b_1 = 0.3$. This effect can be explained as follows: since we know that the biomass is dominating coal in that case, in the dynamic framework the investor reacts and diversifies by increasing the share of coal in year 0. In case the $b_2 > 0$ this diversification is not needed because it is substituted by the diversification over time, into the sub-portfolio $\bar{x}^2$. To the contrary if $b_2 = 0$, the *absolute difference* is positive independent of the $b_1$ considered (see Figure 7.2).

Figure 7.1 reveals also an interesting result concerning the *relative difference* of the current investment into coal. Since the relative share of coal in the static results $\tilde{x}^1_{Coal}$ is independent of the sub-portfolio sizes $b$, the

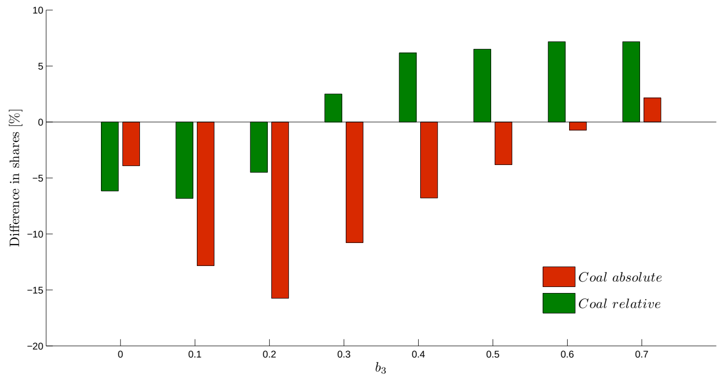


Figure 7.1: *The comparison of coal shares between the dynamic and static framework in case* $b_1 = 0.3$

trend in the *relative difference* is in fact the trend in the relative share of coal invested in year 0 in the dynamic framework. We see that if $b_1 = 0.3$ the relative share of coal in the dynamic results increases with an increasing $b_3$, eventually reaching higher values than the static results. This effect is present for any $b_1$, if $b_1$ does not dominate the whole portfolio. The effect grows in magnitude with decreasing $b_1$, where eventually the coal constitutes all 100 % of the first sub-portfolio when $b_1 = 0.1$ and $b_3 \geq 0.4$ resulting in the relative difference of 10.5 %. This effect is illustrated for the limit case $b_2 = 0$.

Let us recall that the relative share of coal for the first sub-portfolio $\tilde{x}^1_{Coal}$ in the static model reflects the optimal share of coal assuming we consider only the current investment. This corresponds to the situation presented in the previous chapters. We see that if the third sub-portfolio size is significant, the share of current coal $\bar{x}^1_{Coal}$ from the dynamic results is higher than the one in the static results. This implies that an investor accounting for the future investment will build more coal capacity now, even though the total share of coal will be lower than in the static case, if some intermediate investment is planned (i.e. $b_2 \geq 0$).

## 7.4 Conclusion

The portfolio framework suggested in this chapter analyzes the optimal current investment decision, where the inclusion of future sub-portfolios makes it possible to capture the effect from the flexibility to change the mix of tech-

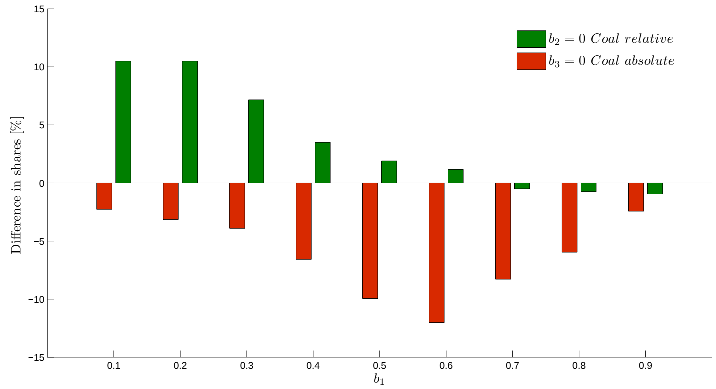


Figure 7.2: *The comparison of coal shares between the dynamic and static framework depending on $b_1$ for the limit cases $b_2 = 0$, $b_3 = 0$.*

nologies at a future point in time. As the model leads to a linear programming problem, it enables the use of the wide range of solution algorithms. The outcome of the new framework is then compared to the portfolios, which are optimized for each decision point separately. By comparing these outcomes, we can establish whether the option to diversify in the future has an impact on the composition of today's portfolio.

The findings show, indeed, that accounting for future investments has an effect on today's portfolio investment decisions. Including the future opportunities leads to diversification not only over technologies but also over time. The analysis conducted and presented in this chapter has clearly shown that also in portfolio applications dynamics matter. While this has not been widely acknowledged in the existing literature on portfolios of electricity generating capacities, which largely rests on mean-variance Markowitz-style implementations of portfolio optimization, we believe that a richer framework taking into account the option of future portfolio investment can deliver important insights for large-scale electricity planning and therefore also for policy-makers, who are interested to learn about the impact of their actions on investment behavior. In the energy sector, where most equipment is long-lived, such information can be of great value, since large-scale investment into particular technologies or a particular family of technologies can lead to further lock-in for decades.

The interpretation of these results is that the dynamic optimization takes into account the value of flexibility that the future opportunity offers, while the static optimization fails to do so. As a result, the return and the –CVaR are lower as well, making the dynamic portfolio superior in terms of both returns and risk. Moreover we saw that accounting for future investment may result in a higher share of coal fired capacities installed today. This may explain why even though the climate policy is getting more realistic, a lot of investment into new fossil fueled technologies is planned in then OECD countries, also in Slovakia. The results further suggest that this may be only temporarily, resulting in a higher share of renewables in total.

A more detailed comparison of the results shows that the investor prefers more coal, the lower the first sub-portfolio and the higher the the third sub-portfolio sizes are, where the future investment shifts completely to biomass in the future and most diversification is taken care of in the current period. The conclusion from this is that for the dynamic version of the portfolio optimization, there is not only a diversification effect across technologies (coal versus biomass), but also a benefit to be reaped from diversification over time.

| SP0 | SP5 | SP10 | Return | -VaR | -CVaR | coal at 0 | bio at 0 | coal at 5 | bio at 5 | coal at 10 | bio at 10 | Return* | -VaR* | -CVaR* | coal at 0 | bio at 0 | coal at 5 | bio at 5 | coal at 10 | bio at 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1.42 | 1.349 | 1.337 | 0.895 | 0.105 | 0 | 0 | 0 | 0 | 1.42 | 1.349 | 1.337 | 0.895 | 0.105 | 0 | 0 | 0 | 0 |
| 0.9 | 0.1 | 0 | 1.437 | 1.367 | 1.354 | 0.81 | 0.09 | 0 | 0.1 | 0 | 0 | 1.429 | 1.361 | 1.349 | 0.806 | 0.094 | 0.029 | 0.071 | 0 | 0 |
| 0.9 | 0 | 0.1 | 1.46 | 1.383 | 1.37 | 0.797 | 0.103 | 0 | 0 | 0 | 0.1 | 1.46 | 1.383 | 1.37 | 0.806 | 0.094 | 0 | 0 | 0 | 0.1 |
| 0.8 | 0.2 | 0 | 1.453 | 1.374 | 1.359 | 0.714 | 0.086 | 0 | 0.2 | 0 | 0 | 1.437 | 1.368 | 1.355 | 0.716 | 0.084 | 0.058 | 0.142 | 0 | 0 |
| 0.8 | 0.1 | 0.1 | 1.477 | 1.4 | 1.386 | 0.711 | 0.089 | 0 | 0.1 | 0 | 0.1 | 1.469 | 1.396 | 1.382 | 0.716 | 0.084 | 0.029 | 0.071 | 0 | 0.1 |
| 0.8 | 0 | 0.2 | 1.5 | 1.402 | 1.386 | 0.71 | 0.09 | 0 | 0 | 0 | 0.2 | 1.501 | 1.402 | 1.386 | 0.716 | 0.084 | 0 | 0 | 0 | 0.2 |
| 0.7 | 0.3 | 0 | 1.469 | 1.375 | 1.358 | 0.63 | 0.07 | 0 | 0.3 | 0 | 0 | 1.446 | 1.369 | 1.355 | 0.627 | 0.073 | 0.086 | 0.214 | 0 | 0 |
| 0.7 | 0.2 | 0.1 | 1.493 | 1.407 | 1.393 | 0.633 | 0.067 | 0 | 0.2 | 0 | 0.1 | 1.477 | 1.4 | 1.387 | 0.627 | 0.073 | 0.058 | 0.142 | 0 | 0.1 |
| 0.7 | 0.1 | 0.2 | 1.517 | 1.419 | 1.402 | 0.625 | 0.075 | 0 | 0.1 | 0 | 0.2 | 1.509 | 1.414 | 1.397 | 0.627 | 0.073 | 0.029 | 0.071 | 0 | 0.2 |
| 0.7 | 0 | 0.3 | 1.541 | 1.415 | 1.395 | 0.623 | 0.077 | 0 | 0 | 0 | 0.3 | 1.541 | 1.415 | 1.395 | 0.627 | 0.073 | 0 | 0 | 0 | 0.3 |
| 0.6 | 0.4 | 0 | 1.486 | 1.372 | 1.353 | 0.532 | 0.068 | 0 | 0.4 | 0 | 0 | 1.454 | 1.366 | 1.351 | 0.537 | 0.063 | 0.115 | 0.285 | 0 | 0 |
| 0.6 | 0.3 | 0.1 | 1.509 | 1.408 | 1.392 | 0.542 | 0.058 | 0 | 0.3 | 0 | 0.1 | 1.486 | 1.401 | 1.387 | 0.537 | 0.063 | 0.086 | 0.214 | 0 | 0.1 |
| 0.6 | 0.2 | 0.2 | 1.533 | 1.426 | 1.409 | 0.542 | 0.058 | 0 | 0.2 | 0 | 0.2 | 1.518 | 1.419 | 1.402 | 0.537 | 0.063 | 0.058 | 0.142 | 0 | 0.2 |
| 0.6 | 0.1 | 0.3 | 1.557 | 1.431 | 1.41 | 0.535 | 0.065 | 0 | 0.1 | 0 | 0.3 | 1.549 | 1.425 | 1.405 | 0.537 | 0.063 | 0.029 | 0.071 | 0 | 0.3 |
| 0.6 | 0 | 0.4 | 1.581 | 1.423 | 1.399 | 0.544 | 0.056 | 0 | 0 | 0 | 0.4 | 1.581 | 1.423 | 1.399 | 0.537 | 0.063 | 0 | 0 | 0 | 0.4 |
| 0.5 | 0.5 | 0 | 1.486 | 1.365 | 1.346 | 0.433 | 0.067 | 0.059 | 0.441 | 0 | 0 | 1.463 | 1.363 | 1.345 | 0.448 | 0.052 | 0.144 | 0.356 | 0 | 0 |
| 0.5 | 0.4 | 0.1 | 1.526 | 1.406 | 1.387 | 0.437 | 0.063 | 0 | 0.4 | 0 | 0.1 | 1.495 | 1.399 | 1.384 | 0.448 | 0.052 | 0.115 | 0.285 | 0 | 0.1 |
| 0.5 | 0.3 | 0.2 | 1.549 | 1.428 | 1.409 | 0.447 | 0.053 | 0 | 0.3 | 0 | 0.2 | 1.526 | 1.419 | 1.403 | 0.448 | 0.052 | 0.086 | 0.214 | 0 | 0.2 |
| 0.5 | 0.2 | 0.3 | 1.573 | 1.439 | 1.417 | 0.456 | 0.044 | 0 | 0.2 | 0 | 0.3 | 1.558 | 1.43 | 1.41 | 0.448 | 0.052 | 0.058 | 0.142 | 0 | 0.3 |
| 0.5 | 0.1 | 0.4 | 1.597 | 1.437 | 1.414 | 0.451 | 0.049 | 0 | 0.1 | 0 | 0.4 | 1.589 | 1.431 | 1.409 | 0.448 | 0.052 | 0.029 | 0.071 | 0 | 0.4 |
| 0.5 | 0 | 0.5 | 1.621 | 1.429 | 1.401 | 0.457 | 0.043 | 0 | 0 | 0 | 0.5 | 1.621 | 1.428 | 1.401 | 0.448 | 0.052 | 0 | 0 | 0 | 0.5 |
| 0.4 | 0.6 | 0 | 1.486 | 1.357 | 1.337 | 0.347 | 0.053 | 0.118 | 0.482 | 0 | 0 | 1.471 | 1.355 | 1.337 | 0.358 | 0.042 | 0.173 | 0.427 | 0 | 0 |
| 0.4 | 0.5 | 0.1 | 1.538 | 1.401 | 1.379 | 0.337 | 0.063 | 0.014 | 0.486 | 0 | 0.1 | 1.503 | 1.394 | 1.377 | 0.358 | 0.042 | 0.144 | 0.356 | 0 | 0.1 |
| 0.4 | 0.4 | 0.2 | 1.566 | 1.426 | 1.406 | 0.343 | 0.057 | 0 | 0.4 | 0 | 0.2 | 1.535 | 1.417 | 1.4 | 0.358 | 0.042 | 0.115 | 0.285 | 0 | 0.2 |
| 0.4 | 0.3 | 0.3 | 1.59 | 1.442 | 1.419 | 0.37 | 0.03 | 0 | 0.3 | 0 | 0.3 | 1.566 | 1.431 | 1.411 | 0.358 | 0.042 | 0.086 | 0.214 | 0 | 0.3 |
| 0.4 | 0.2 | 0.4 | 1.613 | 1.448 | 1.422 | 0.375 | 0.025 | 0 | 0.2 | 0 | 0.4 | 1.598 | 1.437 | 1.413 | 0.358 | 0.042 | 0.058 | 0.142 | 0 | 0.4 |
| 0.4 | 0.1 | 0.5 | 1.637 | 1.442 | 1.416 | 0.372 | 0.028 | 0 | 0.1 | 0 | 0.5 | 1.629 | 1.436 | 1.41 | 0.358 | 0.042 | 0.029 | 0.071 | 0 | 0.5 |
| 0.4 | 0 | 0.6 | 1.661 | 1.431 | 1.401 | 0.372 | 0.028 | 0 | 0 | 0 | 0.6 | 1.661 | 1.43 | 1.401 | 0.358 | 0.042 | 0 | 0 | 0 | 0.6 |
| 0.3 | 0.7 | 0 | 1.485 | 1.348 | 1.327 | 0.25 | 0.05 | 0.181 | 0.519 | 0 | 0 | 1.48 | 1.347 | 1.327 | 0.269 | 0.031 | 0.201 | 0.499 | 0 | 0 |
| 0.3 | 0.6 | 0.1 | 1.54 | 1.393 | 1.37 | 0.248 | 0.052 | 0.065 | 0.535 | 0 | 0.1 | 1.512 | 1.388 | 1.369 | 0.269 | 0.031 | 0.173 | 0.427 | 0 | 0.1 |
| 0.3 | 0.5 | 0.2 | 1.582 | 1.423 | 1.399 | 0.255 | 0.045 | 0 | 0.5 | 0 | 0.2 | 1.543 | 1.414 | 1.395 | 0.269 | 0.031 | 0.144 | 0.356 | 0 | 0.2 |
| 0.3 | 0.4 | 0.3 | 1.606 | 1.442 | 1.417 | 0.276 | 0.024 | 0 | 0.4 | 0 | 0.3 | 1.575 | 1.43 | 1.409 | 0.269 | 0.031 | 0.115 | 0.285 | 0 | 0.3 |

Table 7.3: *Dynamic vs. static portfolio results.* Note: Columns 4-12 correspond to the dynamic framework, columns 13-21 to the static one.

| SP0 | SP5 | SP10 | Return | -VaR | -CVaR | coal at 0 | bio at 0 | coal at 5 | bio at 5 | coal at 10 | bio at 10 | Return* | -VaR* | -CVaR* | coal at 0 | bio at 0 | coal at 5 | bio at 5 | coal at 10 | bio at 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.3 | 0.4 | 1.63 | 1.45 | 1.425 | 0.287 | 0.013 | 0 | 0.3 | 0 | 0.4 | 1.606 | 1.439 | 1.415 | 0.269 | 0.031 | 0.086 | 0.214 | 0 | 0.4 |
| 0.3 | 0.2 | 0.5 | 1.653 | 1.454 | 1.424 | 0.288 | 0.012 | 0 | 0.2 | 0 | 0.5 | 1.638 | 1.443 | 1.415 | 0.269 | 0.031 | 0.058 | 0.142 | 0 | 0.5 |
| 0.3 | 0.1 | 0.6 | 1.677 | 1.447 | 1.416 | 0.29 | 0.01 | 0 | 0.1 | 0 | 0.6 | 1.669 | 1.441 | 1.41 | 0.269 | 0.031 | 0.029 | 0.071 | 0 | 0.6 |
| 0.3 | 0 | 0.7 | 1.701 | 1.434 | 1.4 | 0.29 | 0.01 | 0 | 0 | 0 | 0.7 | 1.701 | 1.435 | 1.4 | 0.269 | 0.031 | 0 | 0 | 0 | 0.7 |
| 0.2 | 0.8 | 0 | 1.493 | 1.338 | 1.315 | 0.164 | 0.036 | 0.214 | 0.586 | 0 | 0 | 1.489 | 1.337 | 1.315 | 0.179 | 0.021 | 0.23 | 0.57 | 0 | 0 |
| 0.2 | 0.7 | 0.1 | 1.545 | 1.383 | 1.359 | 0.169 | 0.031 | 0.107 | 0.593 | 0 | 0.1 | 1.52 | 1.38 | 1.359 | 0.179 | 0.021 | 0.201 | 0.499 | 0 | 0.1 |
| 0.2 | 0.6 | 0.2 | 1.598 | 1.418 | 1.39 | 0.173 | 0.027 | 0 | 0.6 | 0 | 0.2 | 1.552 | 1.409 | 1.387 | 0.179 | 0.021 | 0.173 | 0.427 | 0 | 0.2 |
| 0.2 | 0.5 | 0.3 | 1.622 | 1.44 | 1.412 | 0.165 | 0.035 | 0 | 0.5 | 0 | 0.3 | 1.583 | 1.426 | 1.404 | 0.179 | 0.021 | 0.144 | 0.356 | 0 | 0.3 |
| 0.2 | 0.4 | 0.4 | 1.646 | 1.452 | 1.424 | 0.2 | 0 | 0 | 0.4 | 0 | 0.4 | 1.615 | 1.439 | 1.413 | 0.179 | 0.021 | 0.115 | 0.285 | 0 | 0.4 |
| 0.2 | 0.3 | 0.5 | 1.67 | 1.458 | 1.428 | 0.198 | 0.002 | 0 | 0.3 | 0 | 0.5 | 1.646 | 1.446 | 1.417 | 0.179 | 0.021 | 0.086 | 0.214 | 0 | 0.5 |
| 0.2 | 0.2 | 0.6 | 1.694 | 1.458 | 1.425 | 0.2 | 0 | 0 | 0.2 | 0 | 0.6 | 1.678 | 1.446 | 1.415 | 0.179 | 0.021 | 0.058 | 0.142 | 0 | 0.6 |
| 0.2 | 0.1 | 0.7 | 1.717 | 1.451 | 1.415 | 0.2 | 0 | 0 | 0.1 | 0 | 0.7 | 1.709 | 1.445 | 1.409 | 0.179 | 0.021 | 0.029 | 0.071 | 0 | 0.7 |
| 0.2 | 0 | 0.8 | 1.741 | 1.437 | 1.399 | 0.2 | 0 | 0 | 0 | 0 | 0.8 | 1.741 | 1.437 | 1.399 | 0.179 | 0.021 | 0 | 0 | 0 | 0.8 |
| 0.1 | 0.9 | 0 | 1.498 | 1.328 | 1.303 | 0.071 | 0.029 | 0.255 | 0.645 | 0 | 0 | 1.497 | 1.327 | 1.303 | 0.09 | 0.01 | 0.259 | 0.641 | 0 | 0 |
| 0.1 | 0.8 | 0.1 | 1.551 | 1.373 | 1.348 | 0.084 | 0.016 | 0.147 | 0.653 | 0 | 0.1 | 1.529 | 1.372 | 1.347 | 0.09 | 0.01 | 0.23 | 0.57 | 0 | 0.1 |
| 0.1 | 0.7 | 0.2 | 1.609 | 1.41 | 1.38 | 0.078 | 0.022 | 0.019 | 0.681 | 0 | 0.2 | 1.56 | 1.402 | 1.378 | 0.09 | 0.01 | 0.201 | 0.499 | 0 | 0.2 |
| 0.1 | 0.6 | 0.3 | 1.638 | 1.436 | 1.405 | 0.082 | 0.018 | 0 | 0.6 | 0 | 0.3 | 1.592 | 1.423 | 1.398 | 0.09 | 0.01 | 0.173 | 0.427 | 0 | 0.3 |
| 0.1 | 0.5 | 0.4 | 1.662 | 1.452 | 1.42 | 0.1 | 0 | 0 | 0.5 | 0 | 0.4 | 1.623 | 1.438 | 1.41 | 0.09 | 0.01 | 0.144 | 0.356 | 0 | 0.4 |
| 0.1 | 0.4 | 0.5 | 1.686 | 1.461 | 1.428 | 0.1 | 0 | 0 | 0.4 | 0 | 0.5 | 1.655 | 1.446 | 1.416 | 0.09 | 0.01 | 0.115 | 0.285 | 0 | 0.5 |
| 0.1 | 0.3 | 0.6 | 1.71 | 1.466 | 1.43 | 0.1 | 0 | 0 | 0.3 | 0 | 0.6 | 1.686 | 1.451 | 1.417 | 0.09 | 0.01 | 0.086 | 0.214 | 0 | 0.6 |
| 0.1 | 0.2 | 0.7 | 1.734 | 1.462 | 1.425 | 0.1 | 0 | 0 | 0.2 | 0 | 0.7 | 1.718 | 1.45 | 1.414 | 0.09 | 0.01 | 0.058 | 0.142 | 0 | 0.7 |
| 0.1 | 0.1 | 0.8 | 1.757 | 1.454 | 1.414 | 0.1 | 0 | 0 | 0.1 | 0 | 0.8 | 1.749 | 1.446 | 1.407 | 0.09 | 0.01 | 0.029 | 0.071 | 0 | 0.8 |
| 0.1 | 0 | 0.9 | 1.781 | 1.441 | 1.397 | 0.1 | 0 | 0 | 0 | 0 | 0.9 | 1.781 | 1.441 | 1.396 | 0.09 | 0.01 | 0 | 0 | 0 | 0.9 |
| 0 | 1 | 0 | 1.506 | 1.316 | 1.29 | 0 | 0 | 0.288 | 0.712 | 0 | 0 | 1.506 | 1.316 | 1.29 | 0 | 0 | 0.288 | 0.712 | 0 | 0 |
| 0 | 0.9 | 0.1 | 1.559 | 1.364 | 1.336 | 0 | 0 | 0.177 | 0.723 | 0 | 0.1 | 1.537 | 1.361 | 1.335 | 0 | 0 | 0.259 | 0.641 | 0 | 0.1 |
| 0 | 0.8 | 0.2 | 1.616 | 1.4 | 1.369 | 0 | 0 | 0.057 | 0.743 | 0 | 0.2 | 1.569 | 1.394 | 1.368 | 0 | 0 | 0.23 | 0.57 | 0 | 0.2 |
| 0 | 0.7 | 0.3 | 1.655 | 1.428 | 1.396 | 0 | 0 | 0 | 0.7 | 0 | 0.3 | 1.6 | 1.418 | 1.39 | 0 | 0 | 0.201 | 0.499 | 0 | 0.3 |
| 0 | 0.6 | 0.4 | 1.678 | 1.45 | 1.415 | 0 | 0 | 0 | 0.6 | 0 | 0.4 | 1.632 | 1.433 | 1.404 | 0 | 0 | 0.173 | 0.427 | 0 | 0.4 |
| 0 | 0.5 | 0.5 | 1.702 | 1.462 | 1.425 | 0 | 0 | 0 | 0.5 | 0 | 0.5 | 1.663 | 1.446 | 1.413 | 0 | 0 | 0.144 | 0.356 | 0 | 0.5 |
| 0 | 0.4 | 0.6 | 1.726 | 1.469 | 1.43 | 0 | 0 | 0 | 0.4 | 0 | 0.6 | 1.695 | 1.452 | 1.417 | 0 | 0 | 0.115 | 0.285 | 0 | 0.6 |
| 0 | 0.3 | 0.7 | 1.75 | 1.471 | 1.43 | 0 | 0 | 0 | 0.3 | 0 | 0.7 | 1.726 | 1.455 | 1.417 | 0 | 0 | 0.086 | 0.214 | 0 | 0.7 |
| 0 | 0.2 | 0.8 | 1.774 | 1.466 | 1.424 | 0 | 0 | 0 | 0.2 | 0 | 0.8 | 1.758 | 1.453 | 1.413 | 0 | 0 | 0.058 | 0.142 | 0 | 0.8 |
| 0 | 0.1 | 0.9 | 1.797 | 1.457 | 1.412 | 0 | 0 | 0 | 0.1 | 0 | 0.9 | 1.79 | 1.451 | 1.405 | 0 | 0 | 0.029 | 0.071 | 0 | 0.9 |
| 0 | 0 | 1 | 1.821 | 1.444 | 1.394 | 0 | 0 | 0 | 0 | 0 | 1 | 1.821 | 1.444 | 1.394 | 0 | 0 | 0 | 0 | 0 | 1 |

Table 7.4: *Continued*

# Chapter 8

# Conclusions

This thesis presents a combined real options and portfolio optimization framework that can be used to analyze the investment in new power generation capacities under climate policy uncertainty. It enables to account both for the optimization on the plant level and on the larger scale.

The real options model formulated in 4 is used to derive the optimal management strategy under stochastic $CO_2$ and fuel prices on the power plant level for each electricity generating technology considered. Following this strategy implies a distribution of profit flows representing the profitability of the technology in case it is operated optimally. These profit distributions are used as an input for the portfolio model.

The portfolio model analyzes the optimal investment at the larger scale, the objective of the investor being the minimization of risk, risk measured by the conditional Value-at-Risk. Several portfolio models are proposed. The basic CVaR portfolio model as formulated in [60] is discussed in 5. Its results are used as a benchmark for the comparison of the models proposed in Chapter 6 and 7.

## 8.1 Main contribution

The contribution of this thesis can be divided into following points:

First, it is the formulation of the combined framework optimizing the decision both on the plant level and at a larger scale. In particular with respect to the literature on investment into new electricity capacities, this is an original addition. Until now, the decisions have been studied separately, neglecting the effect of the optimization of the operation and incremental investment on the optimal choice of the resulting energy mix. At the same time, the results of the optimization on plant level presented in Chapter 4 justify the choice of conditional Value-at-Risk as the measure of risk, instead of the standard Markowitz approach used in the literature so far. As all portfolio models discussed lead to linear programming problems, the proposed framework is able to formulate quite a complex modeling problem in an effective way.

Second lies in formulation of the robust and dynamic portfolio models presented in Chapters 6 and 7 respectively. The formulation is motivated by the need to account for issues neglected in the literature on portfolios of electricity generating capacities so far. At the same time, the models are shown to lead to linear programming problems, preserving the advantageous feature of the basic CVaR portfolio framework.

Third, by testing and comparing the results of the individual portfolio models, we can validate the motivation for the extension of the basic model both with respect to robustness and dynamics. Even though the models remain still highly stylized, they provide an important insight for the decision on the aggregate level. We see that the results of the modified portfolio models are substantially different from the ones derived by the basic framework. The comparison between the models enables us to identify the key driving forces of the portfolio composition, providing policy implications as well. The concrete results and effects are discussed in the conclusion of Chapter 6 and 7 respectively. In general, we see that accounting for robustness in respect to both time structure of the profit flows and climate policy uncertainty leads to a significant shift in the portfolio composition. The effect of the introduction of dynamics is not as pronounced, however, the proposed dynamic model can capture the effect of diversification over time. We show that accounting for future investment affects the composition of investment carried out today. We see that analyzing both current and future investment within one framework results in an increased current investment in fossil-fuel capacities for some cases, even though the total share of fossil-fueled capacities is generally lower when compared to the case when these investments are analyzed separately.

Not the least, the comparison of the portfolio model minimizing conditional Value-at-Risk to the classic Markowitz portfolio framework minimizing variance presented in Section 5.2 provides an interesting insight for the portfolio theory as well. In particular, we analyze portfolios or real assets for normally distributed asset profits. We show that in case the constraint on the expected profit is given by equality, these approaches are equivalent. However, if the constraint is formulated in form of inequality, which is more natural, this feature is not necessarily preserved, implying the two frameworks are not equivalent. Moreover, we show that the portfolios based on the minimization of CVaR dominate the mean-variance portfolios in terms of expected profit.

## 8.2 Further research

With the respect to the presented applications, there is definitely still a lot of scope for future development of the presented framework. The assumptions formulated in Section 2.4 neglected some of the technical characteristics of the technologies considered. In particular, the load structure of the technologies was not accounted for, as the time structure of demand and its uncertainty were disregarded. However, to provide precise recommendations for the optimal energy mix, these issues would need to be incorporated. In addition, more technologies would need to be considered and regionally specific conditions (e.g. resource constraints) taken into account.

Another area that warrants further research is the comparison between the classic Markowitz portfolio framework and the basic CVaR model. Section 5.2 provided the analysis for the case of normally distributed assets, showing that under some constraints the approaches are equivalent. It is thus pertinent to ask whether this result holds true also for different distributions.

# Appendix A: Price parameters

| | Scenario | Target | Price $CO_2$ | Price Elec. | Fuel Costs Coal | Fuel Costs Bio | Fuel Costs Gas |
|---|---|---|---|---|---|---|---|
| *Initial Condition* | 'B1' | 590 | 4.39 | 3.07 | 87.97 | 161.65 | 101.10 |
| | 'B2' | 590 | 8.00 | 2.29 | 85.12 | 193.77 | 94.32 |
| | 'A2r' | 590 | 19.66 | 2.62 | 90.01 | 253.72 | 94.64 |
| | 'B1' | 520 | 14.57 | 3.16 | 88.79 | 161.65 | 99.49 |
| | 'B2' | 520 | 15.19 | 2.38 | 85.94 | 195.91 | 93.35 |
| | 'A2r' | 520 | 35.69 | 2.81 | 89.60 | 250.51 | 94.64 |
| | 'B1' | 480 | 35.80 | 3.25 | 81.05 | 175.57 | 91.09 |
| | 'B2' | 480 | 25.20 | 2.48 | 84.71 | 182.00 | 92.38 |
| | 'A2r' | 480 | 58.01 | 3.05 | 84.31 | 267.64 | 94.97 |
| *Trend* | 'B1' | 590 | 4.88 | 0.16 | -0.16 | 2.45 | 1.72 |
| | 'B2' | 590 | 4.88 | 1.22 | 0.16 | 0.77 | 1.47 |
| | 'A2r' | 590 | 4.88 | 1.31 | -0.32 | 0.82 | 1.58 |
| | 'B1' | 520 | 4.88 | 0.15 | -0.64 | 2.78 | 1.47 |
| | 'B2' | 520 | 4.88 | 1.28 | 0.06 | 0.95 | 1.57 |
| | 'A2r' | 520 | 4.88 | 1.49 | -0.05 | 1.58 | 1.36 |
| | 'B1' | 480 | 4.88 | 0.33 | -0.60 | 2.81 | 1.38 |
| | 'B2' | 480 | 4.88 | 1.51 | -0.27 | 1.81 | 1.65 |
| | 'A2r' | 480 | 4.88 | 1.64 | 0.45 | 2.13 | 1.46 |
| *Volatility* | All | All | 0.05 | – | 0.089 | 0.1 | 0.145 |

Table A.1: *Electricity, $CO_2$ price and fuel costs data across scenarios.* Initial conditions are in [€ /t$CO_2$] for the $CO_2$ price, in [€ /MWh] for the electricity price and in [€ /year] for all fuel costs. Price trends are given in [%].

# Appendix B: Distribution statistics

| | *Scenario* | *Target* | *Expec. Cost* | *Std. dev.* | *–VaR* | *–CVaR* |
|---|---|---|---|---|---|---|
| *Coal* | 'B1' | 590 | -4180.45 | 314.94 | -4861.55 | -5071.93 |
| | 'B2' | 590 | -4656.28 | 314.25 | -5336.24 | -5554.39 |
| | 'A2r' | 590 | -5214.79 | 301.68 | -5871.84 | -6086.07 |
| | 'B1' | 520 | -5067.27 | 282.93 | -5682.84 | -5883.27 |
| | 'B2' | 520 | -5045.23 | 302.74 | -5705.41 | -5926.23 |
| | 'A2r' | 520 | -5644.1 | 325.84 | -6352.26 | -6569.17 |
| | 'B1' | 480 | -5456.33 | 277.77 | -6053.76 | -6235.11 |
| | 'B2' | 480 | -5229.58 | 290.81 | -5861.28 | -6062.35 |
| | 'A2r' | 480 | -6168.91 | 359.93 | -6938.49 | -7165.63 |
| *Biomass* | 'B1' | 590 | -5177.91 | 935.76 | -7291.87 | -7990.93 |
| | 'B2' | 590 | -4349.12 | 883.69 | -6310.44 | -6910.7 |
| | 'A2r' | 590 | -2822.31 | 1296.14 | -5536.61 | -6355.47 |
| | 'B1' | 520 | -3142.86 | 1097.55 | -5500.02 | -6239.11 |
| | 'B2' | 520 | -2874.57 | 1016 | -5012.04 | -5653.91 |
| | 'A2r' | 520 | 352.22 | 1740.12 | -3032.47 | -3972.54 |
| | 'B1' | 480 | 1212.01 | 1626.89 | -1894.85 | -2704.97 |
| | 'B2' | 480 | -780.75 | 1272.39 | -3280.75 | -3995.59 |
| | 'A2r' | 480 | 4597.51 | 2462.4 | -17.46 | -1051.98 |
| *Gas* | 'B1' | 590 | -2903.08 | 770.67 | -4764.7 | -5471.89 |
| | 'B2' | 590 | -3001.08 | 694.9 | -4662.04 | -5303.49 |
| | 'A2r' | 590 | -3635.36 | 705.9 | -5332.1 | -5983.58 |
| | 'B1' | 520 | -3498.27 | 731.29 | -5255.73 | -5923.74 |
| | 'B2' | 520 | -3423.11 | 696.4 | -5100.25 | -5737.17 |
| | 'A2r' | 520 | -3891.4 | 680.9 | -5529.28 | -6153.01 |
| | 'B1' | 480 | -3840.68 | 656.85 | -5418.05 | -6022.32 |
| | 'B2' | 480 | -3712.74 | 695.6 | -5392.2 | -6030.61 |
| | 'A2r' | 480 | -4210.96 | 698.41 | -5880 | -6525.78 |
| *Wind* | All | All | -4253.25 | -4253.25 | -4253.25 | 0 |

Table B.1: *Profit distribution statistics of the individual technologies for all scenarios and targets considered.*

# Appendix C:
# Solution of the mean-variance portfolio problem

## C.3 Formulation

Let us consider the mean-variance portfolio choice problem 5.1.1, where $y \sim N(\mu, \Sigma)$ with $\mu = (\mu_1, \mu_2, \mu_3)^T$ and

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \tag{C.1}$$

Let us analyze the optimal portfolio composition in case $\mu_1 > \mu_2 > \mu_3$ and $\sigma_i \neq \sigma_j$ for $i \neq j$.

In section 5.2 we considered three different formulations for the mean-variance portfolio problem, distinguished by the constraint on the expected profit: problem 5.2.13 where the constraint was not present, 5.2.3 where the constraint was given as equality and finally 5.1.1 where the constraint was in form of an inequality with the expected profit bounded below. Although, ultimately, only the solution to problem 5.1.1 is of interest, its solution can be derived by analyzing the former two.

For the purpose of the appendix we will simplify the notation used in 5.2. Let us denote the solution to the unconstrained problem 5.2.13 $x^*$ and the resulting expected profit of the portfolio as $R^* = \mu^T x^*$. As already discussed in Section 5.2, the solution to the problem 5.1.1 is equal to $x^*$ in case $R \leq R^*$ and to the solution of problem 5.2.3 for $R > R^*$. Therefore to derive the solution to 5.1.1 it is sufficient to derive the solution to 5.2.3 and 5.2.13. The solutions of both of these are presented in the following section.

## C.4 Solution

### C.4.1 Unconstrained problem

First let us first derive $x^*$. In case of only three assets the problem 5.2.13 can be reformulated as follows

$$\begin{aligned} \min_{x_1,x_2} \quad & \sigma(x_1,x_2) = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + (1-x_1-x_2)^2\sigma_3^2 \\ & +2x_1x_2\sigma_{12} + 2x_1(1-x_1-x_2)\sigma_{13} + 2x_2(1-x_1-x_2)\sigma_{23} \\ \text{s.t.} \quad & (x_1,x_2) \in X \end{aligned} \tag{C.2}$$

where $X = \{x_1, x_2; x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1\}$. This is in principle a problem of minimizing a convex function on a compact set. Therefore the minimum exists and is equal either to the global minimum of the objective, or is attained at the boundary of $X$.

First let us compute the global minimum $\bar{x}$ of the function $\sigma(x_1, x_2)$. Since it is a convex function the solution is attained in those $\bar{x}_1$, $\bar{x}_2$ that fulfil the first-order conditions. Since $\sigma$ is a quadratic function in both $x_1$ and $x_2$, the first order conditions yield a linear system for $x_1$, $x_2$:

$$\frac{\partial \sigma}{\partial x_1}|_{x=\bar{x}} = A\bar{x}_1 + B\bar{x}_2 + D = 0 \tag{C.3}$$

$$\frac{\partial \sigma}{\partial x_2}|_{x=\bar{x}} = B\bar{x}_1 + C\bar{x}_2 + E = 0 \tag{C.4}$$

where

$$\begin{aligned} A &= \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} \\ B &= \sigma_{12} + \sigma_3^2 - \sigma_{13} - \sigma_{23} \\ C &= \sigma_2^2 - 2\sigma_{23} + \sigma_2^2 \\ D &= \sigma_{13} - \sigma_3^2 \\ E &= \sigma_{23} - \sigma_3^2 \end{aligned}$$

The solution to the first order conditions is thus

$$\bar{x}_1 = \frac{AE - BD}{B^2 - AC} \tag{C.5}$$

$$\bar{x}_2 = \frac{CD - BE}{B^2 - AC}. \tag{C.6}$$

In case $\bar{x} \in X$ the solution $x^* = (\bar{x}_1, \bar{x}_2, 1 - \bar{x}_1 - \bar{x}_2)^T$ and $R^* = \mu^T x^*$.

Otherwise the solution $x^*$ is attained at the boundary of $X$. In such case $x^*$ belongs to one of the lines given by $x_1 = 0$, $x_2 = 0$, $x_1 + x_2 = 0$. Let us denote $x^*_{23}$, $x^*_{13}$,$x^*_{12}$ the solution of $\min \sigma(x_1, x_2)$ on the lines $x_1 = 0$, $x_2 = 0$, $x_1 + x_2 = 0$, respectively. It is important to realize that these lines correspond to the case of portfolio composition where one share is equal to zero.

Let us derive $x^*_{12}$. By definition it is the solution of $\min \sigma(x_1, x_2)$ on the line $x_1 + x_2 = 1$, i.e. it is the solution of the unconstrained mean-variance portfolio problem 5.2.13 for two assets. This problem can be reformulated as

$$\min_{x_1 \in \mathbb{R}} x_1^2\sigma_1^2 + 2x_1(1 - x_1)\sigma_{12} + (1 - x_1)^2\sigma_2^2. \tag{C.7}$$

This is a problem of minimizing a quadratic function and therefore its solution can be derived as the solution of the first-order condition (which is in fact a linear equation) as

$$x_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - \sigma_{12} + \sigma_2^2} \tag{C.8}$$

and therefore by symmetry

$$x_{12}^* = (\frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - 2\sigma_{12} + \sigma_2^2}, \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - 2\sigma_{12} + \sigma_2^2}) \tag{C.9}$$

$$x_{13}^* = (\frac{\sigma_3^2 - \sigma_{13}}{\sigma_1^3 - 2\sigma_{13} + \sigma_3^2}, 0) \tag{C.10}$$

$$x_{23}^* = (0, \frac{\sigma_3^2 - \sigma_{23}}{\sigma_2^2 - 2\sigma_{23} + \sigma_3^2}) \tag{C.11}$$

$$\tag{C.12}$$

If $\tilde{X} = X \cap \{x_{23}^*, x_{13}^*, x_{12}^*\} \neq \emptyset$, then $x^* = (\tilde{x}_1, \tilde{x}_2, 1 - \tilde{x}_1 - \tilde{x}_2)$, where $\tilde{x} = \arg\min_{x \in \tilde{X}} \sigma(x)$. Otherwise

$$x^* = \arg\min\{\sigma(x) | x \in \{e_1, e_2, e_3\}\}, \tag{C.13}$$

$e_i$ being the $i$-th coordinate unit vector.

It should be noted that the by-product of this solution is the optimal portfolio for the unconstrained mean-variance portfolio problem 5.2.13 for two assets, given by C.8.

## C.4.2 Constrained case

Now let us turn to the constrained problem 5.2.3. Since the constraint is given by equality, and $\mu_1 > \mu_2 > \mu_3$, the set of feasible solutions

$$X = \{x \in \mathbb{R}^3; x_1 + x_2 + x_3 = 1, \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 = R, x_1, x_2, x_3 \geq 0\}$$

is not empty only for $R \in [\mu_3, \mu_1]$ . Moreover, any feasible solution $x$ is can be written as

$$x = f(x_2) = \{(\frac{R - \mu_3}{\mu_1 - \mu_3} - x_2 \frac{\mu_2 - \mu_3}{\mu_1 - \mu_3}, x_2, \frac{\mu_1 - R}{\mu_1 - \mu_3} - x_2 \frac{\mu_1 - \mu_2}{\mu_1 - \mu_3}), \}$$

for $x_2 \in X_2 = [0, \min(\frac{R-\mu_3}{\mu_2-\mu_3}, \frac{\mu_1-R}{\mu_1-\mu 2})]$. This implies that the problem 5.2.3 is equivalent to the problem of minimizing $\sigma(x_2) = f(x_2)^T \Sigma f(x_2)$ on a compact interval, where $\sigma(x_2)$ is a convex quadratic function of $x_2$. Therefore its global minimum $\hat{x}_2$ is the solution of the first order condition, after some rearranging it reads

$$\frac{\partial \sigma}{\partial x_2} |_{x_2 = \hat{x}_2} = 2\hat{x}_2 F - 2G = 0 \tag{C.14}$$

where

$$
\begin{aligned}
F &= \frac{(\mu_2-\mu_3)^2}{(\mu_1-\mu_3)^2}\sigma_1^2 - 2\frac{\mu_2-\mu_3}{\mu_1-\mu_3}\sigma_{12} + \sigma_2^2 \\
&\quad -2\frac{\mu_1-\mu_2}{\mu_1-\mu_3}\sigma_{23} + \frac{(\mu_1-\mu_2)^2}{(\mu_1-\mu_3)^2}\sigma_3^2 + 2\frac{(\mu_1-\mu_2)(\mu_2-\mu_3)}{(\mu_1-\mu_3)^2}\sigma_{13} \\
G &= \frac{(R-\mu_3)(\mu_2-\mu_1)}{(\mu_1-\mu_3)^2}\sigma_1^2 - \frac{R-\mu_3}{\mu_1-\mu_3}\sigma_{12} - \frac{\mu_1-R}{\mu_1-\mu_3}\sigma_{23} \\
&\quad +\frac{(\mu_1-\mu_2)(\mu_1-R)}{(\mu_1-\mu_3)^2}\sigma_3^2 + \frac{(R-\mu_3)(\mu_1-\mu_2)+(\mu_2-\mu_3)(\mu_1-R)}{(\mu_1-\mu_3)^2}\sigma_{13}.
\end{aligned}
$$

If $\hat{x}_2 \in X_2$, then $x_R^* = f(\hat{x}_2)$. Otherwise the minimum of $\sigma(x_2)$ on $X_2$ is attained at the border of $X_2$ and therefore $x^* = f(\check{x}_2)$, where

$$
\check{x}_2 = \arg\min \sigma(x_2), \quad x_2 \in \{0, \min(\frac{R-\mu_3}{\mu_2-\mu_3}, \frac{\mu_1-R}{\mu_1-\mu 2})\}. \tag{C.15}
$$

# Bibliography

[1] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, *Coherent measures of risk*, Mathematical Finance, 9 (1999), pp. 203–228.

[2] S. Awerbuch, *Portfolio-based electricity generation plannning: Policy implications for renewables and energy security*, Mitigation and Adaptation Strategies for Global Change, 11 (2006), pp. 693–710.

[3] S. Awerbuch and M. Berger, *Applying portfolio theory to eu electricity planning and policy making*, Working Paper EET/2003/03, International Energy Agency, 2003.

[4] S. Awerbuch and R. Sauter, *Exploiting the oil-gdp effect to support renewables deployment*, Energy Policy, 34 (1976), pp. 2805–2819.

[5] C. Azar and H. Rodhe, *Targets for stabilization of athmospheric* $co_2$, Science, 276 (1999), p. 1818.

[6] D. Bar-Lev and S. Katz, *A portfolio approach to fossil fuel procurement in the electric utility industry*, Journal of Finance, 31 (1976), pp. 933–942.

[7] Basel Committee on Bankingon Supervision, *Trends in Risk Integration and Aggregation*, Bank for International Settlements, 2003.

[8] ——, *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, 2006.

[9] P. Bertsekas and S. Shreve, *Stochastic Optimal Control: The Discrete Time Case*, Academic Press, New York, 1978.

[10] W. Blyth, M. Yang, and R. Bradley, *Climate Policy Uncertainty and Investment Risk*, IEA Publications, Paris, 2007.

[11] A. Botterud, *Long-term planning in Restructured Power Systems*, dissertation, The Norwegian University of Science and Technology, 2003.

[12] J. Campbell, *Strategic asset allocation: Portfolio choice for long-term investors*, nber research summary in program report on asset pricing, National Bureau of Economic Research, 2000.

[13] Z. Chladná, *Real Options Applications for Strategic Investment Decisions*, dissertation, University of Vienna, 2007.

[14] O. Costa and A. Paiva, *Robust portfolio selection using linear-matrix inequality*, Journal of Economic Dynamics & Control, 26 (2002), pp. 889–909.

[15] X. Deng, Z. Li, and S. Wang, *A minimax portfolio selection strategy with equilibrium*, European Journal of Operational Research, 166 (2005), pp. 278–292.

[16] A. Dixit and R. Pindyck, *Investment under Uncertainty*, Princeton University Press, Princeton, 1994.

[17] J. Doege, P. Schiltknecht, and H.-J. Lüthi, *Risk management of power portfolios and valuation of flexibility*, OR Spectrum, 28 (2006), pp. 267–287.

[18] E. Eberlein and G. Stahl, *Both sides of the fence: a statistical and regulatory view of electricity risk*, Energy & Power Risk Management, 8 (2006), p. 34–38.

[19] P. Embrechts, C. Klüppelberg, and T. Mikosch, *Modelling Extremal Events for Insurance and Finance*, Springer, New York, 1997.

[20] E. Fama, *Multiperiod consumption-investment decisions*, American Economic Review, 60 (1973), pp. 163–174.

[21] W. Fichtner, M. Göbelt, and O. Rentz, *Ein modell zur strategischen investitions- und produktionsplanung bei energieversorgern unter berücksichtung von unsicherheiten*, ZfE - Zeitschrift für Energiewirtschaft, 26 (2002), pp. 181–192.

[22] I. Fortin, S. Fuss, J. Hlouskova, N. Khabarov, M. Obersteiner, and J. Szolgayová, *An integrated cvar and real options approach to investments in the energy sector*, Journal of Energy Markets, 1 (2008).

[23] S. Fuss, *Sustainable Energy Development under Uncertainty*, dissertation, University of Maastricht, 2008.

[24] S. Fuss, D. Johansson, J. Szolgayová, and M. Obersteiner, *Impact of climate policy uncertainty on the adoption of electricity generating technologies*, Energy Policy, 37 (2009), pp. 733–743.

[25] S. Fuss, N. Khabarov, J. Szolgayova, and M. Obersteiner, *Valuing climate change uncertainty reductions for robust energy portfolios*, in Conference Proceedings of the 33rd International Symposium on Remote Sensing of Environment, May 2009, Stresa, 2009.

[26] S. Fuss and J. Szolgayová, *Fuel price & technological uncertainty in a real options model for electricity planning*, Applied Energy, 86 (2009).

[27] S. Fuss, J. Szolgayova, N. Khabarov, and M. Obersteiner, *The effects of climate policy on the energy technology mix: An integrated cvar and real options approach*, in Modeling Environment-Improving Technological Innovations under Uncertainty, A. Golub and A. Markandya, eds., Routledge, New York, 2009.

[28] S. Fuss, J. Szolgayová, N. Khabarov, and M. Obersteiner, *Renewables &climate change mitigation: Irreversible energy investment under uncertainty & portfolio effects*. submitted to Energy Policy, 2010.

[29] S. Fuss, J. Szolgayová, M. Obersteiner, and M. Gusti, *Investment under market and climate policy uncertainty*, Applied Energy, 85 (2008), pp. 708–721.

[30] D. Goldfarb and G.Iyengar, *Robustportfolioselectionproblems*, Mathematics of Operations Research, 28 (2003), pp. 1–38.

[31] M. Halická, P. Brunovský, and P. Jurča, *Optimálne riadenie*, EPOS, Bratislava, 2009.

[32] B. Hobbs, *Optimization methods for electric utility resource planning*, European Journal of Operational Research, 83 (1995), pp. 1– 20.

[33] H. Humphreys and K. McClain, *Reducing the impacts of energy price volatility through dynamic portfolio selection*, Energy Journal, 19 (1998).

[34] M. Ilic, F. Galiana, and L. Fink, *Power Systems Restructuring: Engineering and Economics*, Kluwer Academic Publishers, Cambridge/Mass, 1996.

[35] Intergovernmental Panel on Climate Change, *Climate change*, fourth assessment report, United Nations, 2007.

[36] International Energy Agency, *Projected costs of generating electricity (2005 update)*, tech. report, OECD/IEA, 2005.

[37] ——, *Energy technology perspectives 2006: Scenarios and strategies to 2050*, tech. report, OECD/IEA, 2006.

[38] ——, *World energy outlook*, tech. report, OECD/IEA, 2007.

[39] ——, *$Co_2$ emissions from fuel combustion*, tech. report, OECD/IEA, 2009.

[40] ——, *Key world energy statistics*, tech. report, OECD/IEA, 2009.

[41] International Institute of Applied Systems Analysis, *Ggi scenario database*, 2009. available at http://www.iiasa.ac.at/Research/GGI/DB/.

[42] C. Jarque and A. Bera, *A test for normality of observations and regression residuals*, International Statistical Review, 55 (1987), pp. 1–10.

[43] S. Kilianová, *Stochastic Dynamic Optimization Models for Pension Planning*, dissertation, Comenius University, Bratislava, 2007.

[44] H. Lilliefors, *On the kolmogorov–smirnov test for normality with mean and variance unknown*, Journal of the American Statistical Association, 62 (1967), p. 399–402.

[45] H. Markowitz, *Portfolio selection*, Journal of Finance, 7 (1952), pp. 77–91.

[46] ——, *Portfolio Selection: Efficient Diversification of Investments*, Wiley, 1959.

[47] R. Merton, *Lifetime portfolio selection under uncertainty: The continuous-time model*, Review of Economics and Statistics, 51 (1969), pp. 247–257.

[48] ——, *Optimum consumption and portfolio rules in a continuous-time model*, Journal of Economic Theory, 3 (1971), pp. 373–413.

[49] ——, *An intertemporal capital asset pricing model*, Econometrica, 41 (1973), pp. 86–87.

[50] H. J. Mo, B. and I. Waagensteen, *Stochastic generation expansion planning by means of stochastic dynamic programming*, IEEE Transactions on Power Systems, 6 (1991), pp. 662–668.

[51] N. e. a. Nakicenovic, *Special Report on Emissions Scenarios: A Special Report of Working Group III of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, U.K., 2000.

[52] R. Newell and D. Hall, *U.s. climate mitigation in the context of global stabilization*, tech. report, The Climate Policy Forum, Resources for the Future, 2007.

[53] M. Oppenheimer and A. Petsonk, *Article 2 of the unfccc: Historical origins, recent interpretations*, Climatic Change, 73 (2005), pp. 195–226.

[54] G. Pflug, *Some remarks on the value-at-risk and the conditional value-at-risk*, in Probabilistic Constrained Optimization: Methodology and Applications, S. Uryasev, ed., Kluwer Academic Publishers, Dordrecht, 2000.

[55] R. Pindyck, *The long-run evolution of energy prices*, The Energy Journal, 20 (1999).

[56] A. Prekopa, *Stochastic Programming*, Kluwer Academic Publishers, Dordrecht, 1995.

[57] K. Riahi, A. Gruebler, and N. Nakicenovic, *Scenarios of long-term socio-economic and environmental development under climate stabilization*, Greenhouse Gases - Integrated Assessment, Special Issue of Technological Forecasting and Social Change, 74 (2007), pp. 887–935.

[58] R. Rockafellar, *Convex Analysis*, vol. 28 of Princeton Mathematics, Princeton University Press, 1970.

[59] R. Rockafellar and S. Uryasev, *Optimization of coditional value at risk*, Journal of Risk, 2 (2000), pp. 21–42.

[60] ——, *Conditional value at risk for general loss distributions*, Journal of Banking and Finance, 26 (2002), pp. 1443–1471.

[61] R. Rockafellar, S. Uryasev, and M. Zabarankin, *Generalized deviations in risk analysis*, Finance and Srochastics, 10 (2006), pp. 51–74.

[62] F. ROQUES, D. NEWBERY, AND W. NUTTALL, *Fuel mix diversification incentives in liberalized electricity markets: A mean-variance portfolio theory approach*, Energy Economics, forthcoming (2008).

[63] P. SAMUELSON, *Lifetime portfolio selection by dynamic stochastic programming*, Review of Economics and Statistics, 51 (1969), pp. 239–246.

[64] M. SPANGARDT, G. LUCHT AND E. HANDSCHIN, *Applications for stochastic optimization in the power industry*, Electrical Engineering, 88 (2006), pp. 177–182.

[65] J. SZOLGAYOVÁ, S. FUSS, N. KHABAROV, AND M. OBERSTEINER, *A dynamic cvar-portfolio approach using real options: An application to energy investments*, in Proceedings of the 6th International Conference on the European Energy Market, 27-29 May 2009, Louvain, 2010.

[66] ——, *A dynamic cvar-portfolio approach using real options: An application to energy investments.* accepted in European transactions on Electrical Power, 2010.

[67] ——, *Robust long-term energy portfolios - a climate mitigation analysis.* submitted to Applied Energy, 2010.

[68] J. SZOLGAYOVÁ, S. FUSS, AND M. OBERSTEINER, *Assessing the effects of co2 price caps on electricity investments - a real options analysis*, Energy Policy, 36 (2008), pp. 3974–3981.

[69] S. UDDIN AND L. BARRETO, *Biomass-fired cogeneration systems with* $co_2$ *capture and storage*, Renewable Energy, 32 (2007), pp. 1006–1019.

[70] G. UNGER AND H.-J. LÜTHI, *Power portfolio optimization and the importance of operational flexibility*, tech. report, IFOR, ETH Zentrum, 2002.

[71] M. VAN DEN BROEK AND W. TURKENBURG, *Planning for an electricity sector with carbon capture and storage: Case of netherlands*, International Journal of Greenhouse Gas Control, 2 (2008).

[72] A. VAN ZON AND S. FUSS, *Risk, embodied technocal change and irreversible investment*, in Analytical Methods for Energy Diversity and Security, B. M. and F. Roques, eds., Elsevier, 2008.

[73] I. WANGENSTEEN, *Power markets - resources and the environment*, lecture notes, Dept. of Electrical Power Engineering, Norwegian University of Science and Technology, 2005.

# Resumé

V tejto práci sa zaoberáme primárne formuláciou matematických modelov voľby portfólia zloženého z reálnych aktív, s využitím pomerne nového konceptu miery rizika. Pojem podmienenej hodnoty rizika (CVaR) a jeho využitie pri problémoch voľby portfólia boli prvý krát formulované v [60]. Tento koncept je v tejto práci aplikovaný na voľbu optimálnej skladby investícií v energetickom sektore pod vplyvom neistej ceny emisií.

Obsahovo je práca rozdelená na dve časti. Kým prvá časť poskytuje motiváciu a formuláciu skúmaných problémov, spolu s prehľadom súčasného stavu problematiky, druhá časť je venovaná vlastným výsledkom a navrhuje riešenie nastolených problémov.

Prvá kapitola dizertačnej práce uvádza stručný prehľad energetického sektora s dôrazom na stranu producentov elektriny pre distribučnú sieť. Porovnáva základné charakteristiky jednotlivých technológií a objasňuje relevantnosť otázok skúmaných v tejto práci. Druhá kapitola je venovaná presnejšiemu popisu problému, naznačuje základný koncept jeho riešenia a navrhnutý prístup dáva do súvislosti s doterajšími prácami v tejto oblasti. V neposlednej miere uvádza súhrn predpokladov uvažovaných pri formulácii modelov v ďalších kapitolách. Tretia kapitola uzatvára prehľadovú časť práce a poskytuje potrebné poznatky o podmienenej hodnote rizika, najmä s ohľadom na jej aplikáciu pri formulácii problémov tvorby portfólia.

Ostatné kapitoly tvoria jadro práce a obsahujú vlastné výsledky. Hlavný prínos práce sa skladá z dvoch línií.

Prvou líniou je tvorba modelu pre určenie optimálnej skladby investícií do reálnych aktív, s aplikáciou v energetike. Základná motivácia pre tvorbu spomínaných modelov môže byť zjednodušene vyjadrená nasledovnou otázkou: Ako optimálne voliť skladbu investícií do nových kapacít v prípade neistej ceny emisií, uvažujúc že jednotlivé typy technológií budú využívané optimálne?

Navrhnutým riešením je model zložený z dvoch prepojených úrovní. Prvá úroveň reprezentuje optimalizáciu prevádzky jednotlivých reálnych aktív za účelom maximalizácie zisku. Formulácia tejto úlohy vedie na problém stochastického optimálneho riadenia. Jej primárnym výstupom je náhodný vektor reprezentujúci zisk spojený s investíciou do daného reálneho aktíva, za predpokladu že je dané aktívum využívané optimálne. Formulácia a riešenie tohto problému, spolu s charakteristikou jednotlivých výstupov je prezentovaná v kapitole 4.

Na druhej úrovni sa rieši optimalizačný problém voľby portfólia, kde vs-

tupmi sú zisky jednotlivých aktív, t.j. vstup je tvorený výstupom z prvej optimalizačnej úrovne pre jednotlivé aktíva. V tejto práci sú navrhnuté tri rozdielne modely voľby portfólia zohľadňujúce špecifiká spojené s investovaním do reálnych aktív v oblasti energetiky.

Kapitola 5 formuluje základný model minimalizujúci risk portfólia, kde risk je definovaný podmienenou hodnotou rizika. Výsledky tohto modelu slúžia ako referenčná hodnota pre porovnanie výsledkov zložitejších modelov predstavených v kapitolách 6 a 7.

Kapitola 6 sa bližšie zaoberá problémom voľby robustného portfólia, t.j. portfólia ktoré vykazuje žiaduce charakteristiky vo viacerých uvažovaných scenároch. Formulácia tohto problému vedie rovnako ako v prípade základného modelu voľby portfólia na problém lineárneho programovania.

Kapitola 7 naopak rozširuje základný model o možnosť diverzifikácie rizika nielen medzi jednotlivými aktívami, ale aj v čase. Tento model je schopný zohľadniť, aký vplyv má plánované rozšírenie kapacít v budúcnosti na rozhodnutia urobené v súčasnosti. Riešenia jednotlivých modelov sú porovnané na reálnych dátach a ukazujú, že obe rozšírenia majú výrazný vplyv na výslednú skladbu portfólia.

Druhou líniou vlastného prínosu je teoretické porovnanie klasickej Markowitzovej teórie voľby portfólia so základným modelom portfólia z kapitoly 5 pre normálne rozdelené výnosy reálnych aktív. Analýza ukazuje, že vo všeobecnosti uvedené prístupy vedú na rovnaké portfóliá, ak je vo formulácii problému prítomná podmienka na očakávaný výnos portfólia vo forme rovnosti. Avšak v prípade že uvedená podmienka je formulovaná ako nerovnos?, dané prístupy nie sú ekvivalentné, navyše výnos portfólia minimalizujúceho CVaR je vo všeobecnosti zdola ohraničený výnosom portfólia minimalizujúceho varianciu.