Fakulta hospodárskej informatiky Ekonomickej Univerzity v Bratislave

Faculty of Economic Informatics University of Economics Bratislava

Participácia doktorandov na vedecko-výskumnej činnosti

IV. medzinárodný vedecký seminár doktorandov

ZBORNÍK

Participation of doctoral students in science and research

4th International scientific seminar of doctoral students

PROCEEDINGS

21. máj 2004
May 21st 2004
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Min-max calibration method for interest rate models and its application to Central European financial markets

Alexandra Urbánová Csajková

The article is organized as follows. In the next section we briefly review the CIR model. Section II presents the transformation of parameters and section III describes the two step optimization method. In section IV we discuss the results of calibration. Section V concludes the article.

I. Cox-Ingersoll-Ross interest rate model

Recall that the Cox-Ingersoll-Ross interest rate model is derived from a basic assumption made on the form of the stochastic process driving the overnight (or short rate) interest rate r_t, $t \in [0,T]$. In this model we assume that the overnight process satisfies the following mean reverting process of the Orstein-Uhlenbeck type:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\,dw_t \tag{1}$$

where $\{w_t, t \geq 0\}$ denotes the standard Wiener process. Positive constants κ, θ, and σ related to formula (1) denote the speed of reversion, the limiting interest rate and volatility of the process. In the CIR theory the price $P=P(t,T,r)$ of the zero coupon bond is assumed to be a function of the present time $t \in [0,T]$, expiration time $T>0$ and the present value of the short interest rate $r=r_t$. The crucial step in deriving of any one-factor model, including CIR model in particular, consists in construction of a risk-less portfolio containing of two bonds with different exercise times. Next, as a consequence of the Itô lemma, we obtain a parabolic partial differential equation for the price of the zero coupon bond $P=P(t,T,r)$ of the form:

$$\frac{\partial P}{\partial t} + (\kappa(\theta - r) - \lambda r)\frac{\partial P}{\partial r} + \frac{1}{2}\sigma^2 r \frac{\partial^2 P}{\partial r^2} - rP = 0 \qquad t \in (0,T), \quad r > 0. \tag{2}$$

The parameter $\lambda \in R$ represents the so-called market price of risk. A solution P of (2) is subjected to a terminal condition $P(T,T,r)=1$ for any $r>0$. It is well known that PDE (2) with such a terminal condition admits an explicit solution:

$$P(T-\tau, T, r) = A_\tau e^{-B_\tau r}, \quad \tau = T - t \in [0,T] \tag{3}$$

where $$B_\tau = \frac{2(e^{\eta\tau} - 1)}{(\kappa + \lambda + \eta)(e^{\eta\tau} - 1) + 2\eta}, \quad A_\tau = \left(\frac{2\eta e^{(\kappa+\lambda+\eta)\tau/2}}{(\kappa + \lambda + \eta)(e^{\eta\tau} - 1) + 2\eta}\right)^{\frac{2\kappa\theta}{\sigma^2}} \tag{4}$$

and $\eta=\sqrt{(\kappa+\lambda)^2+2\sigma^2}$. If we fill $\frac{\partial P}{\partial r}=-PB$ into (2) then the multiplier of rP will be the risk premium: $r^*=(1-\lambda B)$.

II. Transformation of parameters

The idea of reducing the four dimensional parameter space into essential three parameters consists in introducing the following new variables:

$$\beta=e^{-\eta}\,,\ \xi=\frac{\kappa+\lambda+\eta}{2\eta}\ ,\ \rho=\frac{2\kappa\theta}{\sigma^2} \tag{5}$$

where $\eta=\sqrt{(\kappa+\lambda)^2+2\sigma^2}$. Conversely, for the original CIR parameters we have

$$\eta=-\ln\beta\ ,\ \kappa=\eta(2\xi-1)-\lambda\ ,\ \sigma=\eta\sqrt{2\xi(1-\xi)}\ ,\ \theta=\frac{\rho\sigma^2}{2\kappa}. \tag{6}$$

Then the functions A_τ, B_τ can be expressed in terms of new variables β,ξ,ρ as follows:

$$B_\tau=-\frac{1}{\ln\beta}\frac{1-\beta^\tau}{\xi(1-\beta^\tau)+\beta^\tau}\,,\ A_\tau=\left(\frac{\beta^{(1-\xi)\tau}}{\xi(1-\beta^\tau)+\beta^\tau}\right)^\rho \tag{7}$$

III. Two step optimization method

III.1. Finding minimizer of the cost functional

In order to measure quality of approximation of the set of real market yield curves by CIR yield curves corresponding to present values of the short rate interest rate value we introduce the following cost functional:

$$\begin{aligned}U(\beta,\xi,\rho)&=\frac{1}{m}\sum_{j=1}^{m}\frac{1}{n}\sum_{i=1}^{n}(R_j^i-\overline{R}_j^i)^2\,\tau_j^2=\frac{1}{m}\sum_{j=1}^{m}\frac{1}{n}\sum_{i=1}^{n}(R_j^i\tau_j-B_jR_0^i+\ln A_j)^2=\\&=\frac{1}{m}\sum_{j=1}^{m}(\tau_jE(R_j)-B_jE(R_0)+\ln A_j)^2+D(\tau_jR_j-B_jR_0)\end{aligned} \tag{8}$$

It measures the time-weighted distance of the real market yield curves $\{R_j^i$, j=1, ..., m$\}$ and the set of CIR yield curves $\{\overline{R}_j^i$, j=1, ..., m$\}$ at time i=1, ..., n, determined from the bond price - yield curve relationship $A_je^{-B_jr^i}=e^{-\overline{R}_j^i\tau_j}$ where $r^i=R_0^i$ is the overnight interest rate at time i=1, ...,n.

III.2. Genetic algorithm

Evolution strategy (ES) [1] is one of the most successful stochastic algorithm. Many different versions and applications of ESs have been developed. In our case there has been used a soft modification of (p + c) ES the (p + c + d) ES. The (p + c) ES has *p* parents and *c* children per population, among which the p best individuals are selected to be next generation's parents by their fitness value. The modification (p + c + d) ES comprise selection on wider set. For this multimembered ES for the initialization of the starting population $P^{(0)}$ there are given lower and upper bounds for all object parameters. These parameters are denoted by: (β,ξ,ρ). By using mutation and recombination the p parents "produce" c children. The modification is in the randomly generated wild population with d individuals. The lower and upper bounds are valid for them like for the starting population. Each of the c offsprings together with the d individuals of the wild population is assigned a fitness value depending on its quality. They are sorted by their fitness value and the first p individuals are selcelted to be next generation parents (c must be greater or equal to p, in our case $p = c = 10^5$). The termination criterion is the determined number of generations equal to 300. This method finds minimum of the cost functional U(β,ξ,ρ) for any given λ in the first step.

III.3. Restricted maximum likelihood function

In this section we analyze how to measure goodness of term structure estimation by means of CIR model. If we have short rate process in form (1), then the maximum likelihood function is defined as:

$$\ln L(\kappa,\sigma,\theta) = -\frac{1}{2}\max_{a,b,c,\sigma}\sum_{t=2}^{N}\left(\ln v_t^2 + \frac{\varepsilon_t^2}{v_t^2}\right) \tag{9}$$

where: $v_t^2 = \frac{\sigma^2}{2\kappa}(1-e^{2\kappa})r_{t-1}$, $\varepsilon_t = r_t - e^{-\kappa}r_{t-1} - \theta(1-e^{\kappa})$. Following the maximum likelihood approach, the estimator of model parameters is the argument $(\kappa^u, \sigma^u, \theta^u)$ of the maximum of $\ln L^u = \ln L(\kappa^u, \sigma^u, \theta^u)$ taken over the whole feasible 3D set. Since the maximum likelihood function *ln L* is determined from equation (1) both the value *ln* L^u as well as the estimated parameters $(\kappa^u, \sigma^u, \theta^u)$ are independent of the market price of risk λ. On the other hand, the rest of the yield curve {R_j^i, j=1, ..., m} depends on λ and is not taken into account as far as calculation of the maximum likelihood is concerned. In this approach we optimize *ln L* over a one dimensional subset of parameter values. This restricted set is consisted of triples $(\kappa_\lambda, \sigma_\lambda, \theta_\lambda) \subset (0,\infty)^3$ for which the utility function *U* attains the global minimum. Next we find a global maximum of the maximum likelihood function *ln L* over λ. Now the parameters are estimated for CIR model by the two step min-max optimization method.

IV. Results

We have used BRIBOR (Slovakia), PRIBOR (Czech Republic), WIBOR (Poland), BUBOR (Hungary), LIBOR (in EURO and USD) data for 2001-2003. We have calibrated these data quarterly with presented two step optimization method. For testing of the accuracy of our calibration we have used two tests, the nonlinear R^2 ratio defined as: $R^2 = 1 - \frac{U(\beta_{opt}, \xi_{opt}, \rho_{opt})}{U(1,1,1)}$ and the maximum likelihood ratio defined as: $MLR = \frac{ML_{restricted}}{ML_{unrestricted}}$.

Below there are presented some results for the R^2 ratio, MLR, parameters θ, σ and the risk premium r^* for these data.

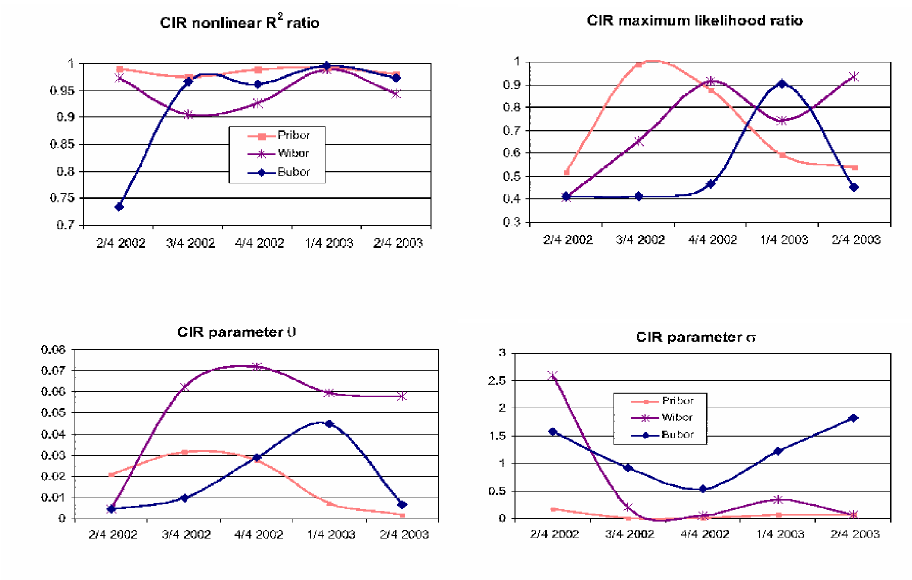

Fig.1 Results of calibration for term structures with maturities up to 10 years. R^2 ratio, maximum likelihood ratio for Pribor, Wibor and Bubor. Estimated parameters θ and σ are shown on these charts.

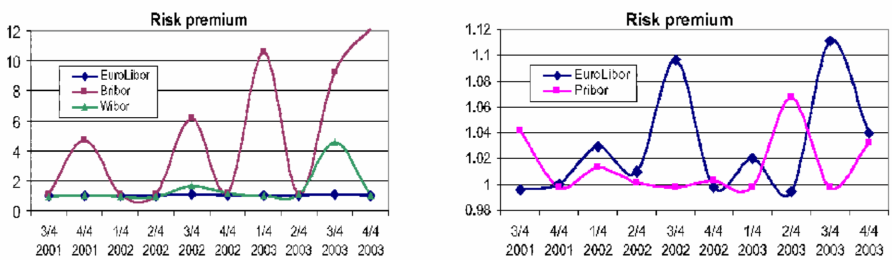

Fig.2 Comparison of risk premium factors $(1-\lambda B)$ for EuroLibor and Pribor (right) and EuroLibor, Wibor and Bribor (left).

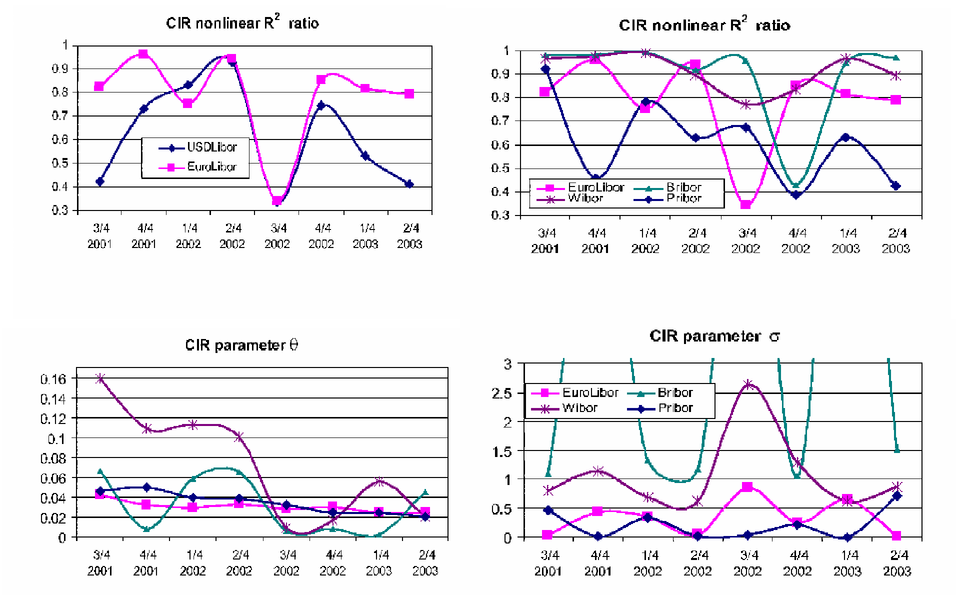

Fig.3 Results of calibration for various term structures. R^2 ratio for EuroLibor and USDLibor. Comparison of the same ratio for EuroLibor, Bribor, Pribor and Wibor and estimated parameters θ and σ are shown below.

V. Conclusion

In this article we have presented a nonlinear regression method for calibration of well-known Cox-Ingersoll-Ross model. By transformation of four „old“ parameters (κ,λ,θ,σ) into three „new“ (β,ξ,ρ), in the first optimization step by ES, we have found the minimum of the cost functional for fixed λ. In the second optimization step we have maximized the maximum likelihood function over λ for given (β,ξ,ρ). We have calibrated real market data from various economies (stable western and emerging) for years 2001-2003. The accuracy of the calibration was tested with two tests. The MLR was ≈ 0,7-0,9 for the Western Europe markets data and ≈ 0,4-0,6 for the emerging economies. The nonlinear R^2 ratio was mostly higher for the western markets data, but there were exceptions like BRIBOR in 2003 ≈ 0,97. The R^2 ratio tells us something about the accuracy of our calibration. If this ratio is close to 1, the calibration is more precise. MLR describes the appropriateness of the CIR model for the data. The data are better described with CIR model, if MLR close to 1. From our results we can tell, that the Western Europe markets data are better described with CIR model. In emerging economies we can also use CIR model but only for the Czech data.

Summary

We would like to introduce a two-phase min-max optimization method for calibration of the well known Cox-Ingersoll-Ross interest rate model (CIR). In the first optimization step we determine four CIR parameters by minimizing sum of squares of differences of theoretical

CIR yield curve and real market yield curve data. It turns out that the minimum is attained on a one dimensional curve in the four dimensional CIR parameter space. Next we find global maximum of the maximum likelihood function computed over this curve. We compare results of calibration for stable Western Europe markets to those of emerging economies like e.g. Slovakia, Hungary, Czech Republic and Poland.

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